REUNION CLASS DESCRIPTIONS—SATURDAY, WEEK 3, MATHCAMP 2013

Contents

9:10 AM Classes

Collective Rationality. (🌶, Ari)
In traditional games, such as chess, go, and Sea Dracula, there is only one winner, and the players are in direct competition. However, there is also an entire class of "games" including economic competition, marriage, and global thermonuclear war, wherein more than one player can win, or all players can lose. Optimal strategies for the former class of games always exist. However, it is far from obvious what constitutes an optimal strategy even for very simple games of the latter class. In this class we will talk about the Prisoners Dilemma and other nonzero-sum games, and why our naive notions of rationality are inadequate. Well also discuss the relevance of the golden rule and Kant's categorical imperative to game theory.
Prerequisites: None.

Math Golf. (🌶🌶, *Nathan Benjamin & Alex Arkhipov*)
In golf, the winner is the person who scores in the fewest number of moves. We like to think the same about math.

Here's one problem we'll look at. An ant starts at point 0 on the number line. Every second, he randomly moves one unit left or right with equal likelihood. What is the probability he reaches 3 before he reaches -7? You could solve this using recurrence relations, but we'll give you a very short proof you could figure out in your head.

In this class, we'll pose a bunch of questions and puzzles that seem intractable at first, but all have beautiful, intuitive one[1]-sentence explanations. These short proofs get to the essence of why the result is true.

This class will probably feature your favorite characters, like prisoners with hats, rational dinosaurs, and pirates doing pirate stuff. Also it will be totally awesome.
Prerequisites: None.

Multilinear algebra: Exterior Algebras and Determinants. (🌶🌶🌶, Asi and Waffle)
This class is the final day of a weeklong class, but will be mostly understandable to anyone familiar with linear algebra and ring theory. In this class we will construct the exterior algebra on a vector space, and then use the exterior algebra to give a very elegant and simple coordinate-free definition of the determinant of a linear map. Unlike other definitions you may have seen before, this definition will make it very easy to verify the basic properties of determinants.

[1] plus ϵ for some $\epsilon \in \mathbb{N}$

Prerequisites: You should feel comfortable thinking about abstract vector spaces and rings.

Wedderburn's Theorem. (🌶🌶🌶–🌶🌶🌶🌶, Mark)
Have you seen the quaternions? They form an example of a division ring that isn't a field. (A *division ring* is a set like a field, but in which multiplication isn't necessarily commutative. Specifically, the quaternions form a four-dimensional vector space over $\mathbb{R}$, with basis $1, i, j, k$ and multiplication rules

$$i^2 = j^2 = k^2 = -1, ij = k, ji = -k, jk = i, kj = -i, ki = j, ik = -j.)$$

Have you seen any examples of *finite* division rings that aren't fields? No, you haven't, and you never will, because Wedderburn proved that any finite division ring is commutative (and thus a field). In this class we'll see a beautiful proof of this theorem, due to Witt, using cyclotomic polynomials (polynomials whose roots are complex roots of unity).
Prerequisites: Group theory and ring theory; some familiarity with complex roots of unity would help

10:10 AM CLASSES

Linguisticsless Linguistics Problem Solving. (🌶, Pesto)
Given are 6 Kurdish sentences and their English translations in random order. Which is which?

A. You see Bear.	1. Ez h'irc'e dibinim
B. You see me.	2. Tu dir'evi
C. Bear runs.	3. Tu min dibini
D. You run.	4. H'irc' dir'eve
E. I see Bear.	5. Ez dir'evim
F. I run.	6. Tu h'irc'e dibini

Second, what does the Kurdish sentence "H'irc' min dibine" mean?

We'll solve pure logic puzzles like these without assuming any knowledge of linguistics, but secretly teach some linguistics in the process.
Prerequisites: None.

Math of the Rubik's Cube. (🌶, *Lucas Garron*)
You might have heard that if you take apart a Rubik's cube an reassemble it randomly, you have only a 1/12 chance of getting a solvable state. This happens because of three kinds of parity present in the standard cube. I will explain the permutation and parity, and how they can be applied to puzzles like the Rubik's cube

Modular Forms. (🌶🌶🌶🌶, Dan Gulotta)
A modular form is a (sufficiently well-behaved) function from the upper half complex plane to the complex numbers that satisfies

$$f(z) = (cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right)$$

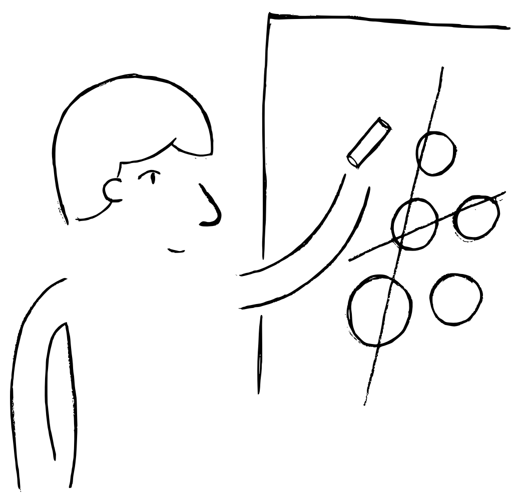
for some integer k and all integers a, b, c, d satisfying $ad - bc = 1$. It turns out that modular forms have many applications to number theory, including the proof of Fermat's last theorem. That proof is way beyond the scope of this course, but I will use modular forms to prove that every nonnegative integer is the sum of four squares. Surprisingly, almost no knowledge of number theory is required!
Prerequisites: Group Theory, Functions of a Complex Variable

A New Mathematical Pathway: How Underserved Students Can Enter Mathematics. (🌶, *Daniel Zaharopol*)
Why is it so hard for low-income and minority students to become scientists and mathematicians? Looking around you, have you wondered at the challenges they face, or thought about what it must be like to be a smart kid interested in math who just can't seem to penetrate into the ecosystem of opportunities that we all know and were a part of? What can we do, as a community of people who love math, to help these students succeed?

We'll discuss the challenges that students face in their school experiences, opportunity to learn, and home lives. Then I'll share my experiences running the Summer Program in Mathematical Problem Solving for underserved NYC middle school students with talent in math and what we've discovered about creating a viable pathway to becoming mathematicians and scientists. It's my hope that this pathway will even bring some of them through Mathcamp!

Prerequisites: None

Proofs Without Words. . . Without Words. (🌶, Zach)



Prerequisites: None

11:10 AM Classes

Finiteness with Choice and Categories. (🌶🌶🌶🌶, Don)
Finite sets are nice: unlike infinite sets, they stop after a while. To rigorously define what "stopping after a while" means, we need to appeal to the natural numbers; a set X is finite if there is a bijection $f : X \to \{1, 2, 3, ..., n\}$ for some $n \in \mathbb{N}$. Unfortunately, many categories don't have a nice object like $\mathbb{N}$ to define finiteness, which means that this definition doesn't generalize well.

A definition that does generalize well is: a set X is finite if every injection $f : X \to X$ is a bijection. The only problem is that if we don't assume the axiom of choice, this definition isn't equivalent to the original definition!

In this class, we'll discuss exactly what goes wrong, and why in the setting of a general category, it's still pretty darn useful. We'll end by talking about a few open problems in the field of category-theoretic finiteness.

Prerequisites: Category of Sets, or knowing the definitions of "Functor" and "Monomorphism"

Math 55 in 55 Minutes. (🌶🌶🌶🌶, *Aaron Landesman*)
Math 55 is a year long class at Harvard designed to give Freshman an introduction to undergraduate mathematics. In this class, I'll attempt to present a year's worth of material in 55 minutes, along with some stories from the class.

Prerequisites: Formally, there are no prerequisites. It might help to have had exposure to abstract math. This class will be fast-paced.

Möbius Strips Like You've Never Seen. (🌶, *Yasha Berchenko-Kogan*)
Presumably, you've already made Mobius strips out of strips of paper, and then cut them up in cool ways. That's only the beginning.

We'll see what the Mobius strip has to do with musical intervals, the torus, well-ventilated hats, the real projective plane, the real projective line, and the Klein bottle. Along the way, I'll show you a model I built of a Mobius strip whose boundary is a real honest-to-goodness round circle, not just a topological circle the way you usually see it.

Prerequisites: It's best to have some idea of what the objects in the blurbs are, but we'll briefly review them.

The Oregon Health Study: a Statistical Adventure Story. (🌶, Mira)
From 2008 until 2012, I worked on a landmark project in health economics: the Oregon Health Study, the first ever randomized controlled trial of the effects of extending health insurance to the low-income uninsured. The subject of the study – medical insurance for the poor – is currently a burning topic in US politics, so our results have received a lot of attention in the press. However, my talk will barely touch on either the results or the controversy surrounding this whole topic.

Instead, I'll tell you why randomized controlled trials in economics are so rare and so tricky; how we got this once-in-a-lifetime chance at a beautiful statistical design in an incredibly messy research area; and how we struggled for four years to protect the purity of our design (and hence the validity of our results) from bungling bureaucrats, inept subcontractors, and the forward march of history. For almost a year, the study was constantly on the brink of disaster, and it was only our frantic and increasingly complicated statistical trickery that allowed us to keep our precious randomization intact. Who knew statistics could be such an adrenaline-filled adventure! And at the end of the talk, I'll tell you how and why we wrote most of our paper before we ever looked at our results ... all for statistical reasons.

Prerequisites: A little probability and/or statistics would be useful at times, but not required. You should be able to follow the story even without getting all the mathy details.

Why I Ditched Lecturing. (🌶, Alfonso)
I have always loved lecturing. Based on my students reviews, they like it when I lecture, too. And yet, for the last decade, I have grown disenchanted with lecturing as a tool for teaching math. Last term, I decided to go nuts and taught a course in which I never lectured or explained anything on the board, using instead the Moore Method. While Mathcampers may be familiar with this format, this was perceived as a bold gamble at my university, and a lot could have gone wrong.

Here is how my Moore Method worked. Students did not use any external resources (no textbooks, no internet, no nothing) and relied entirely on their brains and on each other. I provided them with my own set of notes, which contained only motivation, basic definitions and axioms, and a sequence of exercises that lead to building the theory of point-set topology (the topic of the course). Their homework, before coming to class, was to work those sets of problems and build that theory entirely by themselves. Class time was spent with students presenting their results on the board, criticizing each other's presentations, discussing, and coming to consensus.

In this presentation I will explain why I chose to do it, how I did it, and how it turned out. I will also tell some "tales from the classroom".

1:30 PM Classes

Arrow's Impossibility Theorem. (🌶, Alfonso)
You may have heard Arrow's Impossibility Theorem before described as "The only fair voting system is a dictatorship." In this lecture, I will explain what this actually means. I will present the statement and the proof in a way that is rigorous and convincing, but also clean and understandable (which is more than can be said of most places online who claim to explain this result, particularly Wikipedia). The whole hour will be a single exercise on how innocent-looking axioms have unexpected consequences.

Also, for the record, the only fair voting system is to have Alfonso as a dictator. Thank you.

Prerequisites: None.

Fractal Sundials. (🌶🌶, *Mike Hall*)
Sundials are great for when you're lounging outside in the summertime, but it's such a pain to go over and see exactly where the shadow is pointing, and you don't feel like putting down your lemonade. Wouldn't it be nicer if you could dangle some complicated shape above you whose shadow projected the exact time with big clear numbers and blinking colons in between, updated every second just by the movement of the sun? Turns out there *could be* such a thing, assuming light traveled in idealized straight lines and didn't interact with anything not on its path. I'm not going to say this is feasible on any realistic physical sense, but hey, you could always buy a watch!

Prerequisites: None

Learning as Program Synthesis. (🌶🌶, *Josh Tenenbaum*)
What do people know about the world, and how do they come to know it? I will talk about recent work in cognitive science attempting to answer these questions in computational terms – terms suitable for both reverse-engineering human intelligence and also building more human-like AI systems. This work follows in a long tradition of viewing knowledge as some kind of program, where learning then becomes a kind of program induction or program synthesis. By formalizing this classic idea using new tools for probabilistic programming, it becomes more powerful both as a machine-learning approach and as a framework for describing and explaining human learning. I will discuss working models in several domains, including learning word meanings, learning the structural form of a data set or function for regression, learning motor programs, learning physical laws, and bootstrap learning (or "learning to learn").

Prerequisites: It would be good if students know what a normal distribution is, and are comfortable with elementary probability.

Nonclumpiness of the Parallel Chip-Firing Game. (🌶🌶🌶, *Ziv Scully*)
The parallel chip-firing game—which, sadly, isn't really much of a game—is an automaton on graphs. We start by putting a number of chips on each vertex of a graph. Then, each turn of the (solitaire) game, each vertex "fires" one chip to each of its neighbors if and only if it has enough chips to do so. All parallel chip-firing games are eventually periodic.

In this class, we'll show that the periodic behavior of individual vertices in the parallel chip-firing game is severely limited by a simple rule. What's interesting is that this rule is not locally prohibited; we could construct examples "zoomed in" on a part of the graph where this rule is violated and see nothing wrong. However, by carefully picking the correct global properties to measure, we can show

that only a small subset of vertex firing patterns—those which we'll call "nonclumpy"—can occur. To do so, we'll need a surprise guest: the Bellman-Ford algorithm!
Prerequisites: Know what a graph is.
Related to (but not required for): Chip-firing, Sandpiles, and All That Jazz ...

The Theorem Formerly Known as the Mordell Conjecture. (🌶🌶🌶🌶, Ruthi)
The premise of arithmetic geometry is this: we can turn questions from algebraic number theory into problems about geometry, and use our geometric knowledge to solve the problem there. The beautiful and surprising way these interplay changed the way people think about number theory and has formed modern research in the field. One of the key theorems is Faltings' Theorem (né the Mordell Conjecture), which states that no curve of genus 2 or higher has infinitely many rational points. In this talk we will discuss what this means, some results we can get from it, and some of the key elements of a proof first presented by Bombieri (perhaps touching upon the tour de force which landed Faltings the Fields Medal in 1986).
Prerequisites: Should know what a finitely-generated abelian group is. The more algebra you've seen the easier this will be to follow.