CLASS DESCRIPTIONS—WEEK 3, MATHCAMP 2015

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9:10 Classes

Absolute Values: All the Other Ones. (🌶🌶, J-Lo, Tuesday–Saturday)
Can you cut up a square into an odd number of triangles, all of which have equal area? Or find a set of at least two consecutive unit fractions (e.g. 1/4, 1/5, 1/6, and 1/7) that add up to a whole number? These questions turn out to be related to a strange way of measuring the size of numbers. A way which shows that all triangles are isosceles, allows $1 + 2 + 4 + 8 + 16 + \ldots$ to converge, and gives us a new way to talk about the viral Gangnam Style phenomenon.

In this class, we will discuss what it means for a function to count as an absolute value, classify all possible absolute values you can use on the rational numbers, and study a few of them for their applications to number theory, geometry, computer science, and more. We'll also get a sneak preview of the p-adic numbers, an alternate and very different world from the real numberse you know and love.

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Algorithms in Number Theory (W1)

Continued Fractions (Week 2 of 2). (🌶🌶🌶, Susan, Tuesday–Saturday)
This is a continuation of the week 2 class.

Homework: Recommended

Prerequisites: Week 1 of the same class or equivalent.

Functions of a Complex Variable (Week 2 of 2). (🌶🌶🌶, Mark, Tuesday–Saturday)
Continuation of the week 2 class. If you want to join now, talk to me about what you know and what we've done.

Homework: Recommended

Prerequisites: Week 1 of the same class, or equivalent.

Related to (but not required for): Abel's Theorem (W1–5); The Factorial Function (W3); Laurent Phenomenon (W4)

Lebesgue Measure. (🌶🌶🌶🌶, Alfonso + Steve, Tuesday–Saturday)
In your calculus class you probably learned to integrate using Riemann sums. This is wrong. There are functions that can't be integrated this way, but should have integrals: for example, the Dirichlet function (the characteristic function of the rationals). Since there are only countably many rational numbers at all, we might intuitively say that its integral should be zero. This is backed up by the fact that (exercise!) we can write the Dirichlet function as the pointwise limit of a sequence of continuous functions whose integrals tend towards zero. However, any attempt to integrate the Dirichlet function in the usual way — i.e., via Riemann sums — fails completely.

In this class we will present the *right* way to integrate things: Lebesgue integration.

Homework: Required

Prerequisites: Basic real-line topology or metric spaces; calculus

Related to (but not required for): Differentiation under the Integral Sign (W1); Measure and Martin's Axiom (W1); The Banach–Tarski Paradox (W2); The Ham Sandwich Theorem and Friends (W2); Almost All You Want (W3)

Mathematical Magic. (, Don, Tuesday–Saturday)

> It is the unspoken ethic of all magicians to not reveal the secrets.
>
> *David Copperfield*
>
> The secret impresses no one. The trick you use it for is everything.
>
> *"The Prestige" (2006)*
>
> For centuries, magicians have intuitively taken advantage of the inner workings of our brains.
>
> *Neil Degrasse Tyson*

This will be a course in which you learn magic.

The problem is, I'm a magician. I cannot tell you my secrets.

What I can do is ask you questions. Answer enough questions, and you'll have a sufficient understanding of the inner workings of a deck of cards to create your own tricks. In class, you will present both your answers to these questions, and the magic tricks based on the answers.

A mathematical magic trick is a trick that requires no sleight of hand or misdirection; rather, it is based on potentially unintuitive mathematical principles. Because all of your tricks will be mathematical in nature, you won't need to explain them; your classmates can, and will, solve them.

Because the class will be Moore Method, you'll need to work on a few problems (and try to think of at least one trick based on them) before the first day of classes. You can pick up the homework due on the first day from me or in the office, starting on Sunday.

Homework: Required

Prerequisites: None

Related to (but not required for): Error-Correcting Codes (W3)

10:10 Classes

Abel's Theorem (Week 3 of 5). (, Mira + Ruthi, Tuesday–Saturday)
We continue our explorations this week, looking at either Riemann surfaces or more advanced group theory. If you want to join the class this week, please come to speak with us.

Homework: Required

Prerequisites: None.

Almost All You Want. (, *mmmmmike Hall*, Tuesday–Saturday)
"It might be terrible, but at least it fits in a breadbox." – I made this quote up

Most functions you meet in your first calculus class are quite nicely behaved, but in general a function can do some nasty things. Like manage to be increasing but with a discontinuity at every rational number. Or be continuous everywhere but differentiable nowhere (!). Is there a compromise somewhere among the good, the bad, and the ugly?

Many classic results in real analysis revolve around cleverly partitioning space up into some pieces where a function is nice, and others where it's allowed to misbehave, but which limit the extent of its malfeasance. Our goal will be to show, when possible, that we can shove all the bad behavior into a tiny, tiny region – a set of "measure 0". The title of this class is a reference to properties that hold "almost everywhere" – that is, everywhere except on a set of measure 0.

As we'll see, these sets still leave enough room to go wild – but just a little.

Homework: Optional

Prerequisites: Familiarity with limits, infinite sums, derivatives, integrals.

Related to (but not required for): Lebesgue Measure (W3), Time-Frequency Analysis (W3)

Graphs on Surfaces. (🌶🌶, Marisa, Tuesday–Saturday)
You've heard the joke about the topologist at breakfast, who can't tell the difference between a coffee cup and a donut. Maybe you've seen some topological spaces before, like the sphere and the torus and the double-torus (a donut with two holes); and maybe some non-orientable ones, too, like the Klein bottle. Sure, a Klein bottle looks weird. But how differently does it behave from the sphere? After all, if you were an ant walking around any of these surfaces, everything around you would look just like the plane.

One way to see just how weird a surface can be is to draw graphs on it, and that's what we'll do in this class. We'll be able to fully classify the surfaces on which some families of graphs can live, and quickly get to the frontier of what graph theorists don't know about the behavior of surfaces as simple as the double-torus.

Homework: Optional

Prerequisites: Basic graph theory or Coloring Maps. We'll cover the relevant topology in class.

Related to (but not required for): Point-set Topology (W1); Intro Knot Theory (W2); Coloring Maps (W2)

Induced Matchings from Szemeredi's Regularity Lemma. (🌶🌶🌶, *Po-Shen Loh*, Tuesday–Wednesday)
Start with a set of N vertices. Successively lay down induced matchings $M_1, M_2, \ldots, M_N$, where at each turn, M_k is a (not necessarily perfect) matching within this vertex set, which does not include any edge whose endpoints are in the same M_j for an earlier $j < k$. Let G be the resulting graph. Note that for each k, the subgraph of G induced by the vertices of M_k is precisely M_k, and nothing more. What's the maximum number of edges G can have at the end?

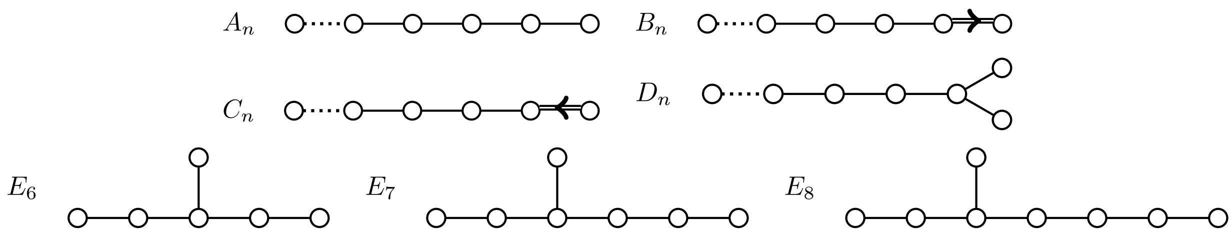
Homework: Recommended

Prerequisites: None.

Related to (but not required for): Linear Algebra (W1); A (re)Introduction to Polynomials (W1)

Lie Algebras. (🌶🌶🌶🌶, Asilata + Kevin, Tuesday–Saturday)
The product of two matrices with trace 0 does not necessarily have trace 0. But there is another operation that actually preserves the set of $n \times n$ matrices with trace 0 — just send $[A, B]$ to $AB - BA$! We call this the *bracket* of A and B, and it makes the set of $n \times n$ traceless matrices into a *Lie algebra*. The bracket has many properties that we are not used to. For example, it is anti-commutative: $[A, B] = -[B, A]$. In general, a Lie algebra is a vector space that has a "bracket" operation with similar properties to the one we just saw.

In this class, we will start out with some fundamental examples which will then lead us into the rigid and beautiful structure of the world of Lie algebras. We'll see how the seemingly complicated algebra implied by this mysterious bracket can be distilled down to an elegant description given by the strange pictures drawn below.



Homework: Required
Prerequisites: Group theory, ring theory
Related to (but not required for): Reflection Groups (W2)

Ordinal Arithmetic. (, Jalex, Thursday–Saturday)
If you were here on the first day of camp, you know that ordinals are the things you get when you count by *shvoomp*ing. For the purposes of counting, ordinals uniquely extend the natural numbers — in other words, infinite counting works just the way we want it to. Exponentiation is iterated multiplication is iterated iterated addition, so as long as we come up with an appropriate notion of iterating iteration past the finite numbers, we can do all sorts of arithmetic with ordinals.

In this class, we'll look at several different notions of "iterating iteration". For multiplication and addition, we'll discover that we can't get nice algebraic properties unless we sacrifice topological ones. For exponentiation, we can't have the nice algebraic properties at all — a result proven just this March! Along the way, we'll make a little detour to prove this fun fact:
There is a set of points in $\mathbb{R}^2$ which intersects each line exactly twice. (If that doesn't weird you out, think about it a little harder.)
Homework: Recommended
Prerequisites: Counting.
Related to (but not required for): Infinitesimals (W1)

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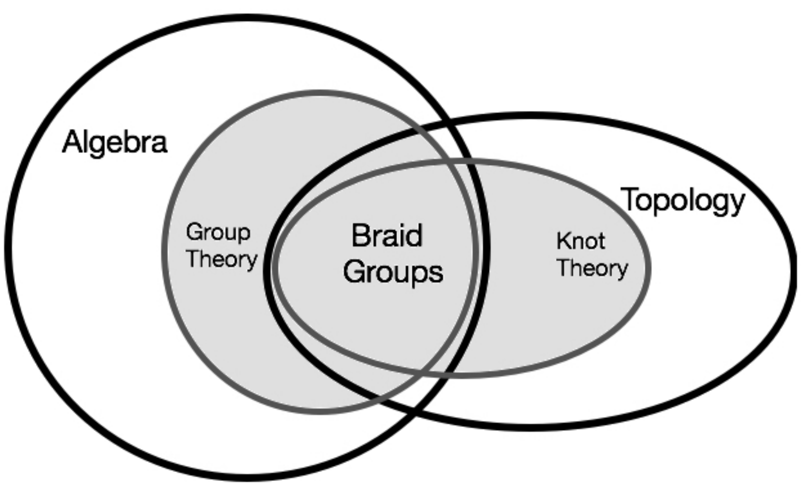
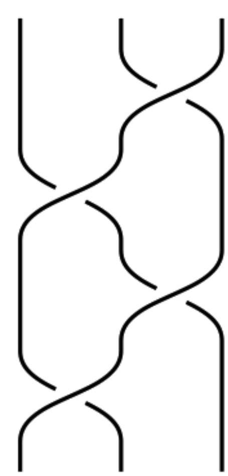
Braid Groups. ($\to$, Nancy, Wednesday–Saturday)





example braid

A braid group is an infinite group generated by stacking tangled diagrams like the one shown above. For every natural number n, there is a braid group on n strands. Braid groups arise in many different topological settings including knot theory and much much more!

Homework: None

Prerequisites: group theory (need to know definition of group, homomorphism, quotient group, group presentation, symmetric group, and understand statement of Cayley's Theorem)

Related to (but not required for): Fundamental Group (W2); Intro Knot Theory (W2)

Classifying Spaces. (🌶🌶🌶, Chris, Wednesday–Saturday)
Imagine there are mathematical objects that you really cherish and care about. Let's call them vector bundles over a space M. It is really easy for you to come up with many examples of those vector bundles, but unfortunately it is very hard to decide if two vector bundles are essentially the same or not. Wouldn't it be great if there was one space — let's call it the classifying space $BGl(n)$ — and one bundle — let's call it the universal bundle $EGl(n)$ — such that *every* vector bundle over *any* space can be constructed out of a (essentially unique) map from X to $BGl(n)$ and $EGl(n)$. This way you could reduce your study of those cool and complicated objects to the study of maps from X to a fixed space which is relatively easy to understand.

All of this is indeed possible and in this course I will explain what vector bundles are, why you should care about them, and how you can classify them.

Homework: Recommended

Prerequisites: Linear Algebra (at least the definitions of vector spaces, linear maps, injective maps.) Topology (understand this statement: "A map is continuous if preimages of open subsets are open.")

Related to (but not required for): Introduction to Groups (W1); Fundamental Group (W2); Homotopy Theory (W4)

Error-Correcting Codes. (🌶🌶, Tim!, Wednesday–Saturday)
Don and I are secretly planning to take over the camp. Shhh, don't tell anyone; it's a secret! Of course, we have a secret code. Don sends me messages by knocking or tapping four times on my door. For instance, knock-tap-knock-tap means "I've hacked the class schedule and replaced every class with Category Theory", tap-tap-tap-knock means "Tonight is the night to steal Susan's idol of power", and so on.

One night, Don knock-knock-knock-knocks on my door, but I mishear it as knock-tap-knock-knock. So, instead of the message "Let me in", I respond to the message "May Day! Burn down the dorms!". This is a setback.

The problem is that if I mishear even one of the knocks, I get the wrong message-phrase. But there is a solution! There are codes that are *error-detecting* — if I mishear one of the knocks/taps, I'll know just from what I heard that something has gone wrong. Even more amazingly, there are codes that are *error-correcting* — if I mishear one of the knocks/taps, then the knocks/taps I do hear will tell me exactly what I misheard and what the correct message was supposed be. It seems too good to be true, but the simplest error-correcting codes are easy to construct, and the best ones are used in real-life computers and computer systems all over the world (and all over the solar system — the New Horizons spacecraft that just sped by Pluto really wants to make sure its photos and data get transmitted back to earth correctly).

Come check out the power and magic of these codes!

Homework: Recommended

Prerequisites: Linear algebra

Related to (but not required for): Mathematical Magic (W3)

Representation Theory of Finite Groups (Week 1 of 2). (, Mark, Wednesday–Saturday)
It turns out that you can learn a lot about a group by studying homomorphisms from it to groups of linear transformations (if you prefer, groups of matrices). Such a homomorphism is called a *representation* of the group; representations of groups have been used widely in areas ranging from quantum chemistry and particle physics to the famous classification of all finite simple groups. For example, Burnside, who was one of the pioneers in this area along with Frobenius and Schur, used representation theory to show that the order of any finite simple group that is not cyclic must have at least three distinct prime factors. (The smallest example of such a group, the alternating group A_5 of order $60 = 2^2 \cdot 3 \cdot 5$, is important in understanding the unsolvability of quintic equations by radicals.) We may not get that far, but you'll definitely see some unexpected, beautiful, and important facts about finite groups in this class, along with proofs of most or all of them. The first week of the class should get you to the point of understanding character tables, which are relatively small, square tables of numbers that encode *all* the information about the representations of particular finite groups; these results are quite elegant and very worthwhile, even if you go no further. In the second week, the chili level will ramp up a bit (from about π to 4) as we start introducing techniques from elsewhere in algebra (such as algebraic integers, tensor products, and possibly modules) to get more sophisticated information.

Homework: Recommended

Prerequisites: Linear algebra, group theory, and general comfort with abstraction.

Related to (but not required for): Ring Theory (W1); Reflection Groups (W2)

The Factorial Function. (, Sachi, Wednesday–Saturday)
Factorials show up everywhere in combinatorics, whether you're counting the number of combinations of LN_2 ice cream flavors or the number of ways to form TPS teams. What you might not realize is that the factorial function shows up many places in number theory, too. For example, the number of polynomial functions from the integers $\mathbb{Z}$ to the integers modulo n, $\mathbb{Z}/n\mathbb{Z}$ is

$$\prod_{k=0}^{n-1} \frac{n}{\gcd(n, k!)}.$$

Fields medalist Manjul Bhargava published a paper in 1997 which generalized the factorial function to subsets of the integers. With this super-powered factorial, we can expand our original theorems to cover arbitrary subsets of the integers.

Homework: Recommended

Prerequisites: Basic number theory (modular arithmetic)

Related to (but not required for): Algorithms in Number Theory (W1); Functions of a Complex Variable (W2–3)

1:10 Classes

Automated Proofs in Geometry. (, Misha, Tuesday–Saturday)

Many proofs in geometry rely on cleverly spotting the right similar triangle or cyclic quadrilateral. On the other hand, even a computer can convert a geometry problem into a system of equations and then provide a coordinate proof. Such a proof is often unsatisfying: it will be too long to read, and having read it, you will gain no intuition for why it works.

As a compromise, we will develop a systematic method for solving geometry problems (once again, one that a computer can implement) that manipulates natural geometric quantities such as areas. This method provides solutions to a wide class of geometry problems, and often (since we avoid coordinates) they turn out to be elegant ones that do not require a computer to read.

Homework: Required

Prerequisites: None.

Related to (but not required for): Algorithms (W1); Non-classical Constructions (W1)

Cryptography. (, Pesto, Wednesday–Saturday)
Does cryptography exist, and why is it a thriving branch of mathematics if we're not sure?

Suppose you want to {send a secret message on the internet, prove that you are who you say are to someone on the internet, run a long-distance poker tournament in which no one trusts any one else or any third party to draw cards fairly}. But, there's an evilly eavesdropping enemy Eve who can see everything you say, knows how you're trying to communicate, and has {infinitely much computational power, a few supercomputers' worth of computational power} available. Can you do it safely despite Eve {no matter what, iff P $\neq$ NP, iff you can do any cryptography at all}?

We'll start with questions as abstract as "is there an easily-computable function that is indistinguishable from random to anyone without lots of computer power?" (no one knows, so we'll assume so) and work our way up to protocols to solve some of the above.

Homework: Recommended

Prerequisites: None, but familiarity with computer science reductions helpful. Intro complexity theory useful.

Related to (but not required for): Introduction to Complexity (W2); P vs NP (W4)

Cryptography: What is it and Why Do we Care? (, *Kristin Lauter*, Tuesday)
Cryptography is the science of keeping secrets. This is useful for e-commerce and in general for secure and authentic communication and transactions. For example, how do you buy something on the internet without an eavesdropper learning your credit card number? How do you know the that "Amazon" is really Amazon? How do you send private email over the internet? The security of many cryptosystems is linked to hard problems in mathematics. This class will explore some of those hard problems and explain what research relevant to implementing and attacking these cryptosystems looks like.

Homework: Recommended

Prerequisites: None.

Exploring Equality via Homotopy and Proof Assistants. (, *Jason Gross*, Tuesday–Saturday)

What does it mean for two things to be equal? What if the things are themselves proofs of equality? Enter homotopy type theory, an exciting new branch of mathematics, which gives us a new way to think about mathematical objects. When proofs of equality are fundamentally paths between points in a space, we can use ideas about shapes (topology!) to study them. In this class we will explore the nature of mathematical object-hood and of equality using Coq, an interactive theorem prover.

Homework: Recommended

Prerequisites: Proof by induction, basic exposure to formal logic (comfort with modus ponens, and the difference between axioms and theorems). Helpful but not required: programming.

Fundamental Theorem of Calculus in Dimension n. (🌶🌶🌶🌶, Jeff, Tuesday–Saturday)
Legend has it that if you take the ancient tome "*On Certain Differential Expressions and the Pfaff Problem, 1899*" and hold the binding over a fire, you will see the following script written by Dark Lord Cartan from when he forged the book:

Fashion Four Derivatives for Dimension Three
Called Total, Partial, Curl and Grad;
In the realms in Dim Two, Only Two Shall be;
On the Domain of Dim One, only $\frac{d}{dx}$ *be had.*

One Derivative to rule them all, One Derivative intertwines them
One Integral to take that One, and in the Darkness bind it.

In this class, we will explore the exterior derivative, the One True Derivative of Calculus, which combines all of the above derivative operations into a single object that extends to any dimension. On our journey, we will:

- Create a language to describe higher dimensional analogues of vector fields.
- Prove a Fundamental Theorem of Calculus For Dimension n that can be stated in just 9 characters.
- Glimpse into the future on how we can do calculus on manifolds.

Homework: Required

Prerequisites: Multivariable Calculus, Linear Algebra. You should be able to prove that V is isomorphic to the space of linear maps from $V \to \mathbb{R}$.

Related to (but not required for): Differentiation under the Integral Sign (W1); Multivariable Calculus (W1); Shortest Distance (W4)

History of Math. (🌶 $\rightarrow$ 🌶🌶, *Moon Duchin*, Tuesday–Thursday)
Notation: What we write and what we write on and why it matters. Symbols, pictures, and diagrams. Special attention to the history of paper and the curious story of the complex plane.

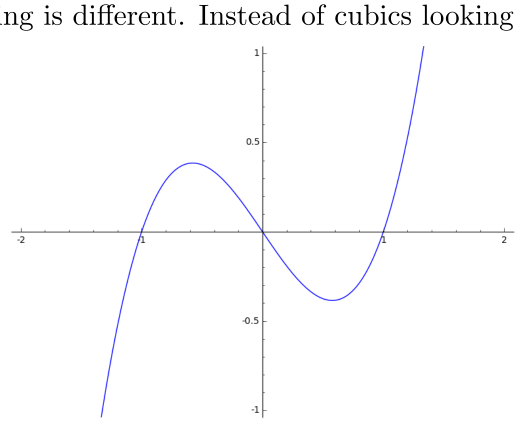
Parts of the whole: Ways of dealing with what's less than one. Special attention to the surprising complexity of Egyptian fractions.

Competition: Duels, flame wars, and contests. Hobbes and Wallis. Tartaglia the stutterer. Special attention to the singular culture of the Cambridge Tripos exam.

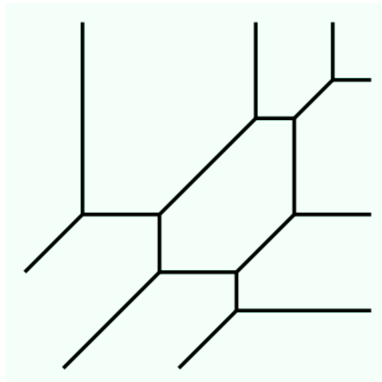
Homework: None

Prerequisites: None.

Tropical Curves. (🌶🌶, Ruthi, Friday–Saturday)
In the tropical world, everything is different. Instead of cubics looking like this:



They look like something like this:



You may think that addition and multiplication work like this:

$$1 + 1 = 2 \quad 1 \times 1 = 1$$

But in tropical world, they work like this:

$$1 + 1 = 1 \quad 1 \times 1 = 2$$

We can make rigorous what I mean by this, and these worlds are not unrelated: tropical world makes it easier to solve problems in our world (and vice versa!). Tropical geometry is all about curves and surfaces are related to polytopes – how algebra and geometry relate to combinatorics. In this class we'll try to understand where these things come from and why we might study this weird world.

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Ring Theory (W1)

Colloquia

Secure Genomic Computation: a Contest. (*Kristin Lauter*, Tuesday)
Over the last 10 years, the cost of sequencing the human genome has come down to around $1,000 per person. Human genomic data is a gold-mine of information, potentially unlocking the secrets to human health and longevity. As a society, we face ethical and privacy questions related to how to handle human genomic data. Should it be aggregated and made available for medical research? What are the risks to individual's privacy? This talk will describe a mathematical solution for securely handling computation on genomic data, and highlight the results of a recent international contest in this area.

Triples. (*Po-Shen Loh*, Wednesday)
What's the longest sequence of triples $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots$ that satisfies the following properties?

- Each number is an integer between 1 and N inclusive.
- For every $j < k$, if we compare the triples (x_j, y_j, z_j) and (x_k, y_k, z_k), there are at least two coordinates in which the latter triple strictly exceeds the former triple.

It turns out that this simple-sounding problem is equivalent to a question from Ramsey Theory, inspired by a question from k-majority tournaments, and related to deep question involving induced matchings and Szemeredi's Regularity Lemma.

No knowledge of Ramsey Theory, k-majority tournaments, induced matchings, or Szemeredi's Regularity Lemma is required for this colloquium. The talk will be a tour of Combinatorics which introduces many of these concepts along the way.

Can You Hear the Shape of a Drum? (*Moon Duchin*, Thursday)
For every geometric object, there is an infinite list of numbers called its spectrum that describes the frequencies at which it can vibrate– let's interpret this as the sounds it can make when you strike it. Suppose there are some differently shaped drumheads that are hidden from view and you get your friends to smack them melodiously in an effort to help you tell them apart. Is it possible that two differently shaped drumheads are isospectral, or sonically indistinguishable?

A closely related question asks you to consider a surface and study the length spectrum– the list of all possible lengths of curves that are pulled tight on the surface. Can two differently shaped surfaces support all of the same curve-lengths? What geometric features of a surface can you "read off" of the length spectrum?

I'll explain the connection between these, describe some fun related problems, and tell you about some of my own research in shape-hearing. (I proved that, for a suitable way of making all the words precise, you can hear the shape of a billiard table!)

My Favorite Prime. (*Zach Abel*, Friday)
Meet my favorite prime number! All twelve thousand(ish) digits of it.[1] *Wait, that's a massive number.*

[1] 999999999999999999999999999999999999999999999999999999999999999999998999998989889989899999999999999999
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How do you know it's actually prime? My computer said so. *But how does the computer know?* It asked the Fibonacci numbers. *And how did you* find *a prime so large, anyway?* By setting Pascal's triangle on fire. *OK, but why is* this *prime your favorite?* Because I'm its favorite! It told me so. *What?!* Seriously, this prime is mine and no one else's, and it'll tell you the same. We'll discuss the above questions in more detail, and all the prime-number mischief they unlock.

Visitor Bios

Jason Gross. Jason is a second year Computer Science PhD student at MIT. His mathematical interests include category theory, topology, homotopy type theory, logic, and type theory. Computers like to tell him that he's making the universe inconsistent. Jason was a camper in 2007–2009 and he's excited to be back! Jason also enjoys sailing, hiking, skydiving, reading science fiction and fantasy, rationality, dancing (squares, contra, ceilidh, among others), and quantum mechanics.

Kristin Lauter. Kristin Lauter leads the Cryptography Research Group at Microsoft Research. She loves working on hard math problems which are used in modern cryptosystems, focusing mostly on algorithmic number theory problems. Besides working on challenges such as constructing elliptic curves for cryptography and attacking lattice-based cryptosystems, she has proposed new cryptosystems based on the hardness of routing in certain graphs, for example. Kristin is currently President of the Association for Women in mathematics and she is passionate about supporting careers for women and girls in mathematics. She was a co-founder of WIN, a research network of women in Number Theory – ask her about Research Collaboration Conferences for Women!

Po-Shen Loh. Po-Shen Loh studies questions that lie at the intersection of two branches of mathematics: combinatorics (the study of of discrete systems) and probability theory. Randomness can manifest itself in the construction of a combinatorial system, as in the case of a so-called "random graph", but may also be artificially introduced as a proof technique to solve problems about purely deterministic systems, as was pioneered by Paul Erdos in what is now known as the Probabilistic Method.

mmmmmike Hall. Mike studies the relationship between quantum systems and their classical analogues using techniques from symplectic geometry, harmonic analysis, and partial differential equations. He is also interested in inverse problems and black hole type phenomena for wave equations. Outside of math, Mike likes donuts and other sorts of confections, but mostly donuts. He also likes to run, swim, bike, and jump.

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Noah Snyder. Noah Snyder is an Assistant Professor and Indiana University in Bloomington. He was a mentor in 2006-2008, AC in 2009, and faculty in 2012. He's sad to be breaking his streak of being full-time staff for 3 straight Tacoma Mathcamps. He works on planar and 3-dimensional algebra and its relationships to representation theory, topology, and operator algebras.

Zach Abel. Zach works somewhere between the intersection and union of discrete geometry and theoretical computer science. He often thinks about the ancient and beautiful art of origami from an algorithmic perspective, exploring just how powerful (or useless) computers can be at folding-related tasks relating to robotics, nanomanufacturing, architecture, and (of course) recreational paper folding. He is also a mathematical artist, transforming everyday objects like binder clips or playing cards into intricate works of art (look out for his activities during camp!). Zach is a former Mathcamper ('03 and '04) and Mentor ('13).

Ari Nieh. Ari has been at every Mathcamp in some form (student, JC, mentor, faculty, or visitor) since 1996. He teaches mathematics for AoPS and The Math Circle. He is also a professional singer, specializing in Renaissance and baroque music. His favorite webcomic is Bad Machinery, and his favorite cooking technique is braising.

Dave Savitt. Originally from Vancouver, Canada, was the first-ever counselor at Mathcamp in 1996. David received his PhD at Harvard University in 2001 (where his work focused on an extension of the results which led to the proof of Fermat's Last Theorem) and did his postdoctoral research at McGill University (Montreal) and Institut des Hautes Etudes Scientifiques (France). He is now an Associate Professor at the University of Arizona and Deputy Director of Mathcamp.