

CLASS DESCRIPTIONS—WEEK 4, MATHCAMP 2015

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9:10 Classes**Advanced Linear Algebra.** (🍷🍷, Asilata, Tuesday–Wednesday)

Here’s a puzzle! Suppose you have a set of thirteen weights with the following property: if you remove any one of them, you can divide the rest into two sets of equal weight. What are all the possibilities?

Wait a minute, this was supposed to be a blurb about advanced linear algebra. So, here’s a question about eigenvalues! If someone specifies a (multi-)set of eigenvalues for you, how many essentially different matrices can you come up with that have the given eigenvalues?

In this class, we’ll explore questions like the one above, and discover some beautiful theorems of linear algebra that you might not have seen in a first course. We’ll also see a neat algebra trick that may or may not lead us to a pretty solution to the first puzzle.

Homework: Recommended

Prerequisites: Linear algebra, ring theory recommended

Related to (but not required for): Lie Algebras (W3)

Homotopy Theory. (🍷🍷, Chris, Tuesday–Saturday)

In topology, two spaces are (homotopy) equivalent if they can be “continuously transformed into each other”: a cube is the same as a sphere, but not the same as a torus (donut). Deciding if two spaces are homotopy equivalent is very hard and we often have to turn to invariants. The first such invariants are the homotopy groups of a space X : For $i \in \mathbb{N}_0$ the sets $\pi_i(X)$ are the (homotopy) equivalence classes of maps from the i dimensional sphere S^i to X . If $i \geq 1$ this is a group and if $i \geq 2$ it’s even abelian. It is easy to see that two homotopy equivalent spaces have the same homotopy groups, so calculating those groups is a good way of deciding if spaces are not homotopy equivalent. There also is a huge class of spaces (so called CW-complexes) for which having matching homotopy groups immediately implies homotopy equivalence.

So these homotopy groups are easy to define and tell us a whole lot about the space, but there is one big problem: they are incredibly hard to calculate. So hard that after over 80 years we still have not been able to compute all the homotopy groups of the 2-sphere S^2 ! Calculating those homotopy groups of spheres has been driving modern mathematics for over half a century and these efforts have produced a large amount of new ideas, techniques and theories, and have shaped the way we think about mathematics in many areas such as topology, algebra and geometry. In this course we will introduce the homotopy groups and make our way to the long exact sequence of the homotopy groups of a fibration, the first (and most important) tool used to calculate them.

Homework: Recommended

Prerequisites: Understanding factor groups G/H and direct sums of groups; (some, not necessarily strict) definition of topological spaces and continuity, some idea why the fundamental group is a group (I can explain this to you during TAU if you want to)

Related to (but not required for): Fundamental Group (W2); Classifying Spaces (W3); Category Theory in Sets (W4)

Laurent Phenomenon. (🍷🍷, Kevin, Tuesday–Saturday)

Consider the recurrence

$$a_n = \frac{a_{n-1}a_{n-3} + a_{n-2}^2}{a_{n-4}}.$$

If we begin with $a_0 = a_1 = a_2 = a_3 = 1$, then we find that the sequence always consists of integers, despite dividing by a_{n-4} in the recurrence. Why?? How can we prove this?

It turns out that this recurrence can be viewed as a special example of a process called mutation. In the early 2000s, Sergey Fomin and Andrei Zelevinsky introduced this process and proved that all values yielded by mutation can be written as Laurent polynomials in the initial values. This so-called Laurent phenomenon provides a unified proof for integrality of a large number of rational recurrences. This led directly into their work on cluster algebras, one of the most exciting new fields in modern mathematics.

In this class, I'll give several examples of mutation (from triangulations to rational recurrences and more!) and prove the Laurent phenomenon. Time permitting, I will also sketch out the definition and basic properties of cluster algebras and the myriad places they've been showing up in mathematics in the last decade or so.

Homework: Optional

Prerequisites: None.

Many Facets of Optimization. (☞, Tim!, Tuesday–Saturday)

Odie and Evan play a game. Simultaneously, they each hold out one or two fingers. If the total number of fingers is odd, Odie wins, and if it's even, Evan wins. The winner must pay the loser a number of dollars equal to the total number of fingers up. What's the best strategy for each player?

Games like this will lead us to another kind of problem. Suppose you're starting a delivery service, and you want to buy reindeer and moose to help you with delivery. Each animal has a certain cost, requires a certain amount of food, and can make a certain number of deliveries per day, and you want to maximize the amount you can deliver given your budget.

Luckily, computers can solve this sort of problem very efficiently, as long as the number of animals doesn't need to be an integer. When we've set up the problem, what was that about an integer number of animals? That seems kind of important. . .

As soon as we want all our variables to be integers, the problem becomes way harder! But there's a strategy that often works well: Think of your set of possible solutions as a polygon, lift it up to a higher dimension, cut off some of its corners, then project it back down. We'll see what this means and how to do things with it.

This class is a tour through zero-sum matrix games, linear programming, and lift-and-project hierarchies.

Homework: Recommended

Prerequisites: Linear algebra

Normal Numbers. (☞, Steve + Susan, Thursday–Saturday)

A real number between 0 and 1 is *normal* if, whenever we consider its base- b representation, every finite pattern of digits occurs with the expected frequency. So, for example, if x is normal, then when we write x in binary, “half” the digits are “0.”

It is not at all clear that normal numbers exist. Perhaps surprisingly, it turns out that *most* numbers are normal! In fact, it's conjectured that lots of interesting real numbers — π , e , $\sqrt{2}$ — are all normal. However, no individual natural examples are known, and for a long time nobody knew a nice way to produce an example at all.

Come let us introduce you to these marvelous objects! We'll prove they exist, and show you how to build them. Time permitting, we'll discuss some recent work by one of Steve's professors (and others) on quickly computing normal numbers.

Homework: Recommended

Prerequisites: None.

Shortest Distance. (☞☞☞, Jeff, Tuesday–Saturday)

How do we find the shortest distance between two points? Here’s a strategy that Euler used (with a bit of help from calculus):

Let’s pick two points $p, q \in \mathbb{R}^2$. If r is any third point, then the distance from p to r plus the distance from r to q is longer than the distance from p to q . Therefore, the line is the shortest distance. If one would like to show the same thing about curves, approximate the curve by line segments and use the tools of calculus to show that the line minimizes distances.

See any problems?

We defined the distance between two points by saying that it *is the length of the line between them!* It seems that we’ve somehow used real world intuition¹ that the line minimizes distances to come up with our definition of distance. . . probably not the best idea for our proof!

Here is a different approach. Let \mathcal{P}_{ab} be the space of all paths between a and b . There is a function from \mathcal{P}_{ab} to \mathbb{R} , called the length. Now, use calculus to find the minimum of this function. That was easy!

Surprisingly, this method (basically) works, and will allow us to find geodesics without referring to straight lines. We will use these tools to find length minimizing curves on hyperbolic and spherical spaces (which have no straight lines), look at brachistochrone curves, and minimize other quantities over spaces of paths.

Homework: Recommended

Prerequisites: Can you define what a vector field is? Can you define what $\frac{\partial f}{\partial x}$ is?

Related to (but not required for): Metric Spaces (W1); Special Relativity (W1); Fundamental Theorem of Calculus in Dimension n (W3)

10:10 Classes**P vs NP.** (☞☞☞, Pesto + Jalex, Wednesday–Saturday)

Are there problems whose answers we can check easily (in NP) but not find easily (in P)?

It certainly seems so. We can check whether a filled-in Sudoku grid is correct easily, but none of the millions of people who do Sudokus because they’re nontrivial knows how to fill one in as easily. We can check whether a claimed proof of a theorem is correct much more easily than we can find proofs (otherwise, mathematicians’d be out of their jobs!).

But those could be false, because no one can prove that $P \neq NP$. In fact, we’ll prove that none of the methods that people have tried to use to prove that $P \neq NP$ can possibly work, and talk about other results related to this most famous open problem in computer science.

Homework: Recommended

Prerequisites: Understand the statement (not necessarily the proof) “NPSpace \subseteq PSPACE”. Have seen a “proof by diagonalization” (e.g. of the time hierarchy theorem). (Intro complexity theory suffices.)

Related to (but not required for): Cryptography (W3)

Trail Mix. (☞ → ☞☞☞, Mark, Wednesday–Saturday)

Is your mathematical hike getting a little too hardcore? Would you like to relax a bit with a class that offers an unrelated topic every day, so you can pick and choose which days to attend, and that does

¹In fact, we have not used real world intuition, as straight lines are not geodesics in the theory of general relativity.

not expect you to do homework? If so, how about some Trail Mix? Individual descriptions of the four topics follow. There is basically no homework, although there will certainly be some things you could look into if you like.

Prerequisites: See individual mini-blurbs below.

Trail Mix Day 1: Perfect Numbers. (♫, Mark, Wednesday) Do you love 6 and 28? The ancient Greeks did, because each of these numbers is the sum of its own divisors, not counting itself. Such integers are called perfect, and while a lot is known about them, other things are not: Are there infinitely many? Are there any odd ones? Come hear about what is known, and what perfect numbers have to do with the ongoing search for primes of a particular form, the so-called Mersenne primes — a search that has largely been carried out, with considerable success, by a far-flung network of individual “volunteer” computers.

Trail Mix Day 2: Integration by Parts and the Wallis Product. (♫♫, Mark, Thursday) Integration by parts is one of only two truly general techniques known for finding antiderivatives (the other is integration by substitution). In this class you’ll see (or review) this method, and two of its applications: How to extend the factorial function, so that there is actually something like $(1/2)!$ (although the commonly used notation and terminology is a bit different), and how to derive the famous product formula

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots,$$

which was first stated by John Wallis in 1655.

Prerequisites: Basic single-variable calculus

Trail Mix Day 3: Intersection Madness. (♫♫, Mark, Friday) When you intersect two ellipses, you can get four points, right? So why can’t you get four points when you intersect two circles? Well, you can, and two of the four points are always *in the same place!* If this seems interesting and/or paradoxical, wait until we start intersecting two cubic curves (given by degree 3 polynomial equations). The configuration of their intersection points has a “magic” property (known as the Cayley-Bacharach theorem) that leads to proofs of various other cool results, such as Pascal’s hexagon theorem and the existence of a group law on a cubic curve. I can’t promise that we’ll have time for all these things, but we’ll do some of them.

Prerequisites: None, although a little bit of linear algebra might show up.

Trail Mix Day 4: The Jacobian Determinant and $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (♫♫♫, Mark, Saturday) How do you change variables in a multiple integral? In the “crash course” in week 1 we saw that when you change to polar coordinates, a somewhat mysterious factor r is needed. This is a special case of an important general fact involving a determinant of partial derivatives. We’ll see how and roughly why this works; then we’ll use it to evaluate the famous sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (You may well know the answer, but do you know a proof? If so, do you know a proof that doesn’t require Fourier series or complex analysis?)

Prerequisites: Multivariable calculus (the crash course is plenty); some experience with determinants.

Homework: None

Prerequisites: See individual mini-blurbs (above)

Ultrafilters. (♫♫♫♫, Steve, Wednesday–Saturday)

You might have played the game “20 questions” before, where I think of some object and you get 20 yes/no questions to figure out what it is. Well, there’s a related game called “infinity questions” — I think of a natural number x , and you get to ask me infinitely many yes-no questions about x : e.g.,

“Is x prime?” or “Is x the sum of two cubes?” Note that each question can be phrased as “Is $x \in A$?” for some set A .

Now, certainly you can win this game — just have your n th question be “Is $x = n$?” But what if I cheat, and don’t actually have a number in mind? Well, as long as my pattern of answers is consistent, it will be as if I’m describing something that’s like a number, but not quite: it’s not a natural number, but it is either even or odd, either prime or composite, etc.

These “generalized numbers” are *ultrafilters* on the set of natural numbers. In this class, we’ll talk about what ultrafilters are and what you can do with them. We’ll see that there is a natural “space” of ultrafilters, and that this space has properties which make it useful for combinatorics(!); and we’ll show how ultrafilters can let us take the “average” of an infinite collection of mathematical structures (groups, rings, etc.).

Homework: Required

Prerequisites: None

Related to (but not required for): Measure and Martin’s Axiom (W1); Infinitesimals (W1)

Voting Theory. (🍷, Alfonso, Wednesday–Saturday)

When a large group of people have to make a decision together, bad things can happen. For example, suppose that a group of 10 campers is trying to decide which game they want to play tonight. Suppose further that 3 of them want to play Dominion, and the remaining 7 would prefer to play any game they can possibly think of other than Dominion. If the remaining 7 are divided between 5 or 6 different games, a strict plurality election system will force them to play Dominion, even though a majority of the 10 campers would be thoroughly unsatisfied. It seems, then, that the plurality election system is unfair. What could we do to make it fair? Which election system is the most fair? What does fair mean, anyway?

In this class we will try to formalize the question of what is a fair voting system mathematically, and we will analyze actual voting systems used in the world accordingly. We will use real data from recent elections in various countries!

Warning: Your faith in democracy may vanish.

Homework: Optional

Prerequisites: None

pNumber Theory. (🍷🍷, Noah Snyder, Wednesday–Saturday)

Number theory is the study of integers, while pnumber theory is the study of polynomials. pNumber theory has a lot in common with number theory, for example pnumbers also have unique factorization into prime pnumbers. Fortunately, pnumbers are easier to study than numbers for several reasons, most notably that you can take derivatives of polynomials and that polynomials are related to geometry. In this class we will prove the pABC conjecture, pFermat’s Last Theorem, and the pRiemann hypothesis.

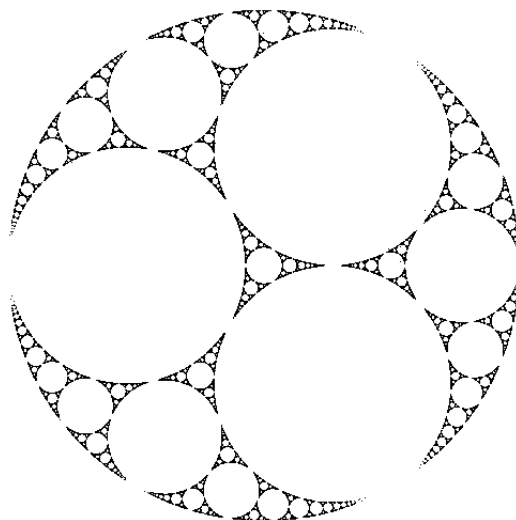
Homework: None

Prerequisites: elementary number theory, the derivative rules for polynomials

Related to (but not required for): Algorithms in Number Theory (W1); Ring Theory (W1); Continued Fractions (Week 1 of 2) (W2)

11:10 Classes

Apollonian Circle Packings. (🍷, Sunny Xiao, Tuesday–Saturday)



Given any three touching circles, do there exist other circles that simultaneously touch all three? The answer was discovered by a renowned Greek geometer, Apollonius of Perga, around 200 BC. It turns out that there are, and there are always exactly two choices!

Grab a pen and try it now. Can you find both of them? Awesome! Now you have five circles. Pick another set of three touching ones and iterate the process. What kind of picture do you get in the end? What if you change the radii of the initial three circles? Do you get a different packing? How many possible configurations are there? Can we classify them?

Come to this class to find out! The family of beautiful, intricate fractals generated by this procedure is known as the Apollonian circle packing (ACP), an object with fascinating properties—for example, the amount of space filled up by an ACP has a “fractional dimension” of about 1.3057, meaning that it is “fuller” than a line but “skinnier” than the whole plane! In this class, we will first derive some classical geometric results about the ACP. Then we will study more of its interesting properties in connection with other fields of modern mathematics (number theory, group theory, dynamics, and topology). Time permitting, we will look at some open questions in this area and give them a try ourselves.

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Non-classical Constructions (W1)

Category Theory in Sets. (🌀🌀, Don, Tuesday–Saturday)

Category Theory is a relatively young field of mathematics, founded in the mid-1900’s. Originally, it was used to work with intimidating topics like cohomology functors and homological algebra. Fortunately, in this class, we won’t need anything that complicated. When doing category theory in the category of sets, the thing we get out is much easier: arithmetic.

The arithmetic we’ll be doing will have an unusual character to it; instead of taking our numbers and operations as given, we’ll describe what they ought to look like, and then define them using categorical language. From this, we can prove the ordinary rules of arithmetic; but having defined things categorically, we’ll be able to apply it not just to arithmetic, but to any kind of mathematical object you might encounter.

Homework: Required

Prerequisites: None

Related to (but not required for): Linear Algebra (W1); Introduction to Groups (W1); Classifying Symmetry (W1); Homotopy Theory (W4)

DIY Hyperbolic Geometry. (🍷), Katie Mann, Tuesday–Saturday)

If you’ve ever tried to do geometry on the surface of a sphere, you’ll know that it’s a wacky place: what should be “parallel” lines always eventually intersect, the area of a disc is less than πr^2 , and when you try to flatten a piece of sphere (perhaps you’ve tried this with a piece of orange peel), you end up tearing it.

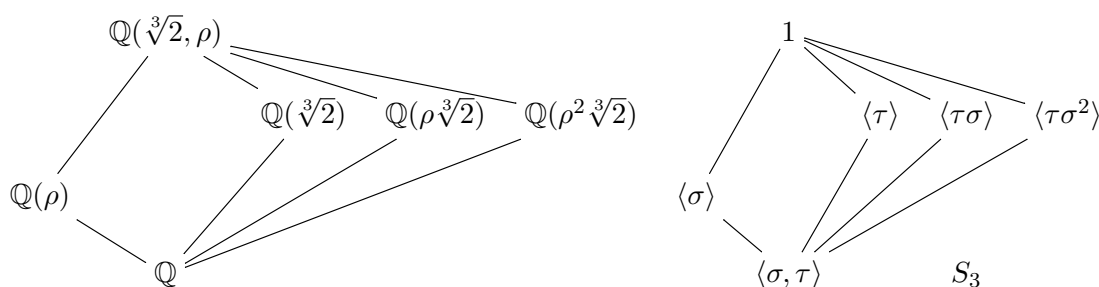
In this class we’ll explore an even wilder place to do geometry, the hyperbolic plane. This space is the opposite of the sphere: you can draw both parallel and non-parallel lines that never intersect, the area of a disc is way larger than you expect, and when you try to flatten a piece of it, you are forced to wrinkle it up.

This class is called “DIY” for a reason: expect to make and experiment with paper models, attempt to build a hyperbolic soccer ball, discover the laws of the hyperbolic universe for yourself, and re-invent some of M. C. Escher’s tessellations. (Artistic talent is definitely not a requirement though!)

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Introduction to Groups (W1)

Galois Theory. (🍷🍷), Nancy, Tuesday–Saturday)

Do you think it is a coincidence that the subgroup lattice of S_3 looks so similar to the subfield lattice of $\mathbb{Q}(\sqrt[3]{2}, \rho)$, where ρ is a primitive cube root of unity? It’s not!! This beautiful relationship is called the Galois Correspondence.

Here is roughly how it works:

- (1) Start with a field F and a polynomial $f(x)$ that has no roots in F
- (2) Adjoin all the roots of f to F to get a new field K
- (3) Hey look! When we permute all the roots of f in K we get a group! Lets call it the Galois group of K over F
- (4) Hey look again! The subgroups of the Galois group look a lot like the subfields of K but just flipped upside-down! Ta-da, the Galois correspondence.

Galois theory is considered one of the most beautiful subjects in mathematics. Everything that you want to to work just fantastically works!

Let’s climb Mt. Évariste!

Homework: Recommended

Prerequisites: Group theory, ring theory. Understand the statement “A ring mod out by a maximal ideal is a field”

Related to (but not required for): Abel’s Theorem (Week 1 of 5) (W1); Non-classical Constructions (W1); Ring Theory (W1)

Unlikely Maths. (🍷), Misha, Tuesday–Saturday)

A popular way to construct a combinatorial object we want is to choose it at random. This works great

if the properties we desire hold for almost any object we could choose. But sometimes we are greedy and want so much from our construction that a randomly chosen object has, at best, an exponentially small chance of making us happy.

This class is about the Lovász Local Lemma: one of the ways to prove that this exponentially small chance is still positive. We will use the LLL to show the existence of a few unlikely objects, with applications to graph theory and computer science. We will also see a few approaches to taking the final step: actually finding such an unlikely object.

Homework: Recommended

Prerequisites: Graph theory

1:10 Classes

Abel’s Theorem (Week 4 of 5). (🍷🍷, Alfonso + Mira, Tuesday–Saturday)

We continue our explorations this week, looking at either Riemann surfaces or more advanced group theory. If you want to join the class this week, please come to speak with us.

Homework: Required

Prerequisites: None.

Development of Probability. (🍷 → 🍷🍷, Sam, Tuesday–Saturday)

In the middle of the 17th century, the mathematical field of probability emerged almost entirely out of the blue. People had been gambling for millennia, so you might wonder why it could have taken so long for a mathematical study of games of chance to emerge. You might also wonder what it even means for a field of mathematics to emerge, or how I can feel even remotely confident about when it actually emerged. These are some of the questions we’ll explore on day 1.

From then on, the course will focus on the mathematical development of probability during the first 50–100 years of the field’s existence. We’ll look at the early problems that motivated the field, the clever (and sometimes convoluted) methods that were used to solve these problems, and the cultural context that drove these problems/methods. We’ll see brilliant mathematicians propose “solutions” that just sound crazy (like distinguishing between moral and mathematical probabilities to resolve the St. Petersburg Paradox), and we’ll see the conflation of ideas required for a new field of mathematics to emerge.

Bonus: Along the way, we’ll see how society and mathematics influence each other’s development. And we’ll see lots of cool and brilliant mathematics. Sadly, still no free set of steak knives. . .

Homework: Recommended

Prerequisites: None! If you know lots of probability, you’ll get to see the context in which it developed; if you don’t know too much, then you’ll get to better empathize with the people who developed the field!

Related to (but not required for): History of Math (W3)

Representation Theory (Week 2 of 2). (🍷🍷🍷, Mark, Tuesday–Saturday)

Continuation of the week 3 class. If you want to join now, talk to me about what you know and what we’ve done.

Homework: Recommended

Prerequisites: Week 1 of the same class, or equivalent.

The Hidden Dance of Partial Differential Equations. (🍷🍷, Adam Larios + Jared Whitehead, Tuesday–Saturday)

At the heart of almost all science lie partial differential equations. They govern diverse areas, such as the flow of blood in the heart, the turbulent patterns of the weather, the spread of diseases, the shape of sound waves and light waves, the flickering motion of flames, and even the formation of traffic jams. Moreover, the mathematics involved is strikingly beautiful, full of chaos, order, and mystery.

We will begin by learning one of the key ingredients to understanding partial differential equations: Fourier series. Fourier series are wonderful mathematical tools in their own right. To get a feel for them, we'll see how we can pull apart images using Fourier transforms. We will then discuss some strange and remarkable partial differential equations, and use the machinery of Fourier series to begin to untangle them and see the hidden dance that makes them work. With these ideas in hand, we will be able to discuss one of the most difficult problems in all of mathematics: the \$1,000,000 Millennium Prize Problem for the Navier-Stokes equations, posed by the Clay Mathematics Institute in the year 2000.

Homework: Recommended

Prerequisites: None.

Tiling Problems. (♁, Sachi, Tuesday–Saturday)

If you take a chessboard, and remove two opposite corners, then there is no tiling of the chessboard by dominos (each of which covers two adjacent chess squares.) Every domino covers a black and a white square, but two opposite corners have the same color.

Suppose instead you take a tiling of the plane by equilateral triangles. Then, if someone outlines a region on this triangular lattice, is it possible to tell whether one can tile it with tiles which cover two adjacent triangles at a time (a shape which is called a lozenge)? It would take a long time to check all the possible tilings, and such a brute force attack would be hard to keep track of and replicate. Instead, we will borrow some tools from group theory: we will see that groups have graphs associated to them, and how words in these graphs actually constitute tiles which we can use to simplify elements of our group.

Homework: Recommended

Prerequisites: Group theory

Colloquia

Computers: Friends or foes of the modern mathematician? (*Adam Larios*, Tuesday)

As computers gain a stronger hold in mathematics, should mathematicians start to worry? Computers obviously depend on mathematics, but more and more areas of mathematics are starting to depend on computers. Does this mixture violate the purity of mathematics? Will the beauty of mathematics be forgotten, reduced to cold hard computation? Fortunately, the situation may be not so scary. We will see many examples of mathematics and computers living happily together. Moreover, pairing these disciplines can be beneficial to both sides. I will argue that computers do not spell doom for mathematics; indeed, they make mathematics more exciting than ever.

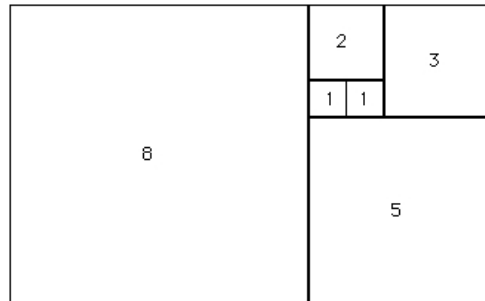
Cauchy's functional equation. (*Katie Mann*, Wednesday)

In 1820 or so, Augustin-Louis Cauchy wondered: "Which real-valued functions satisfy the property $f(x + y) = f(x) + f(y)$?" (for all real numbers x and y), and probably thought they were all pretty nice functions.

We'll see what the nice ones look like, then discover some terrible monsters. Expect to meet:

- A function whose graph you can't possibly draw
- A vector space of shocking proportions, and

- A proof that (unlike the famous example below) *most* rectangles can't possibly be subdivided into little squares.



(Amazingly, Cauchy's question and its friends have made an appearance recently in my research. . . we might not get to this in the colloquium, but if you're curious, ask!)

Putting the Algebra in Algebraic Topology. (*Noah Snyder*, Thursday)

Algebra is the study of operations like multiplication, addition, and composition. Topology is the study of rubbery bendy spaces. Algebraic topology is usually thought of as a way to use algebraic structures to study topology, but there's another way to think about it which is that topological spaces themselves can be thought of as algebraic structures. For example, paths can be "multiplied" by concatenating them end-to-end. The goal of this talk will be to introduce paths, loops, 2-paths, and 2-loops, and to explain why 1-loops can only be multiplied in one way, but 2-loops can be multiplied in a tremendous number of ways. No background in topology will be assumed, but some familiarity with the definition of a group would be helpful.

TBA. (*Dan Zaharopol*, Friday)

Visitor Bios

Adam Larios. Adam does research in fluid dynamics, computational science, and partial differential equations. He is interested in turbulence, ocean dynamics, the complex interaction between fluids and solids, and the properties of magnetic fluids, such as the plasmas in stars. Much of his work is related to the \$1,000,000 Clay Millennium prize for the equations behind the chaotic motion of fluids. This work is highly theoretical, but also involves performing large-scale massively-parallel simulations on supercomputers to test mathematical and scientific hypotheses computationally. He is inspired by a deep curiosity for nature and a fascination with its inner workings. He loves sharing his passion for mathematics and science with students.

Jared Whitehead. Jared works in the field of partial differential equations, using complicated mathematical analysis in combination with numerical simulations, asymptotics, and insight gain from physical scientists, to make progress in understanding how fluids (ranging from the earth's mantle, oceans and atmosphere to solar plasmas) dynamically interact. In his free time, he very much enjoys the outdoors including: hiking, biking, but mostly trail running and spending time with his wife Samantha and five children.

Katie Mann. Katie combines techniques from geometry, topology and dynamics in her research, studying groups of homeomorphisms of manifolds. She also has a fondness for hyperbolic geometry and explaining math with pictures and models and shapes, which you'll surely see some of at Mathcamp. Katie also enjoys cycling — she's been known to bike 70 miles to get to a math conference.

Sunny Xiao. Sunny is a PhD student at Brown University. She's interested in geometry, topology, and dynamics. She spends most of her time thinking “what do things look like?” and “where do the points go?” Outside of math, she enjoys running, Karate, puzzles and board games, folk music, reading novels, writing letters, travelling, eating ice cream and chocolate and Greek yogurt (not all at once though). She always gets the bad guy card in various versions of Mafia but can manage to win most of the times.

Dan Zaharopol. Dan used to be a topologist who was frequently mistaken for a computer scientist, until a strange incident involving James K. Polk, the fifth homology group of the torus, and a third of a parsnip made him turn his dastardly mind to saving the world instead. When he woke up, he started a math (summer) program for low-income students in NYC whose disaster/student ratio appears to be about 2.5 times Mathcamp's for no good reason. In his free time, he enjoys at least four out of the following five things: playing board games, writing ridiculously ambitious LARPs, going to the theatre, developing new methods of camper mind control, and running math study groups for lapsed mathematicians. He does vaguely remember sleeping back in 1995, but he's somewhat hazy on the details. Oh, and he's been at Mathcamp for a pretty long time.

Noah Snyder. Noah is an Assistant Professor at Indiana University in Bloomington. He was a mentor in 2006-2008, AC in 2009, and faculty in 2012. He's sad to be breaking his streak of being full-time staff for 3 straight Tacoma Mathcamps. He works on planar and 3-dimensional algebra and its relationships to representation theory, topology, and operator algebras.

Greg Burnham. Greg was a camper '04-'06 and a JC '07-'08, '10. Some things he did this past week that generally exemplify things he's interested in: ate a bagel, worked on a software platform for facilitating confederated scientific research, read an essay about machine comprehension of text.