CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2016

Contents

9:10 Classes

Introduction to Graph Theory. (🌶, Marisa, Tuesday–Saturday)
There is a theorem that says that for any map of, say, countries on your favorite continent, you can color the countries so that any two countries that share a border (not just meet at a point, but actually share some boundary) get different colors, and that the number of colors you will need is no more than 4. (Try inventing a complicated political landscape and coloring: no matter how crazy the scene, you'll always be able to color the map with four colors.)

Mathematicians have been pretty convinced about the truth of this Four Color Theorem since the late 1800s, but despite many false starts, no one gave a proof until 1976, when two mathematicians wrote a very good computer program to check 1,936 cases. (To this day, we have no human-checkable proof.)

In this class, we will definitely not prove the Four Color Theorem. You will, however, prove the Five Color Theorem, which is a whole lot shorter (and which was successfully proven by hand in 1890). Along the way, you'll meet many other cool concepts in Graph Theory.

Notice how I said "*you* will prove"? That's because the course will be inquiry-based: I won't be lecturing at all. You'll be working in small groups to discover and prove all of the results yourself!

Homework: Recommended.

Prerequisites: None.

Cluster: Maps, Graphs, Colors, Walks.

Required for: Almost Planar (W2); Graph Minors (W2); King Chicken Theorems (W3); Graph Colorings (W3); The Hadwiger–Nelson Problem (W3); Random Graphs (W3); Harmonic Functions on Graphs (W4); Spectral Graph Theory (W4).

Point-Set Topology. (🌶🌶, Chris, Tuesday–Saturday)
Topology is the study of shapes and deformations "without ripping". But what does "ripping" mean mathematically? And when are two points "close together"? It might be clear how to define these notions inside of the real numbers (you probably can come up with it yourself), but can we generalize them to any set?

These questions of closeness and ripping are so fundamental that they are at the basis of many fields of modern mathematics. The answer to them is—you guessed it—point-set topology.

Homework: Recommended.

Prerequisites: None.

Spin: Numbers as Rotations. (🌶🌶🌶, *J-Lo*, Tuesday–Saturday)
Put a twist on the way you do geometry: every rotation is a number! Once we've let 2D rotations gyrate in our minds for a bit, we'll take our turn with 3D. In this class, we'll whirl through polar form of complex numbers, roots of unity, cyclotomic polynomials, quaternions, and Cayley–Dickson algebras, and wheel have a ball with applications revolving around geometry to round it all out. (I'm on a roll.)

A few words about class structure: We will start working with the arithmetic of complex numbers from scratch (so you're fine even if you've never seen it before), but we will go through it fairly quickly (so you shouldn't be too bored even if you have seen it before). Also, this is a very interactive course. Most of the time will be spent with *you* making conjectures, proving results, and even defining the right concepts, and you may be asked to share your work with the class.

Homework: Required.

Prerequisites: Basic trigonometry, polynomial division.

Statistical Modeling. (, Sam, Tuesday–Saturday)
Does LSD help you do math? Does the Amish population of a state predict whether the state votes for Democrats or Republicans? Do SAT scores predict performance in college? To answer these questions, we need to do data analysis! (Except the first one. LSD does NOT improve your ability to do math.)

Serious blurb: this class is about using statistical techniques to draw meaningful conclusions from data. We'll start with one of the simplest tools, the linear model, and quickly extend it to handle more and more complicated (and entirely non-linear) situations. By the end the course, we'll be set to briefly discuss extensions of the linear model to machine learning. While we'll focus on one set of statistical tools, everything we do will be in terms of a general framework for statistical modeling! Along the way, we'll cover things you might have heard about like p-values and hypothesis testing; we'll highlight how to actually use those things without butchering statistical techniques and garnering the ill-will of the entire statistics community. Oh, and we'll see a few examples of some beautiful statistical theory!

Caveat Emptor: there will be 2 required practical homeworks where you'll get your hands dirty and use R to analyze data. Also, don't do LSD.

Homework: Required.

Prerequisites: Basic familiarity with linear algebra (know what a transpose and inverse are) and probability: ideally, you'd feel good about the sentence "X and Y are independent random variables, both having a normal distribution with mean zero and variance 1." If you're not 100% comfy with it, that's OK: the first lecture might seem more rough, but I'll hand out a probability review sheet on day 1.

Cluster: Mathematics and Its Applications.

Systems of Polynomial Equations. (, Nic, Tuesday–Saturday)
Suppose someone hands you a system of polynomial equations, that is, a bunch of equations with polynomials of several variables on both sides. There are tons of questions you could ask about it: Are there finitely many solutions? If so, what are they? If not, what does the space of solutions look like? Can you conclude that other polynomial equations also have to be true?

Remarkably, it's possible for many questions of this type to be answered completely systematically—you can even program a computer to do it! In this class we'll develop the main machine (called a Gröbner basis) that makes this go. Along the way, we'll also see some surprising applications to Euclidean geometry.

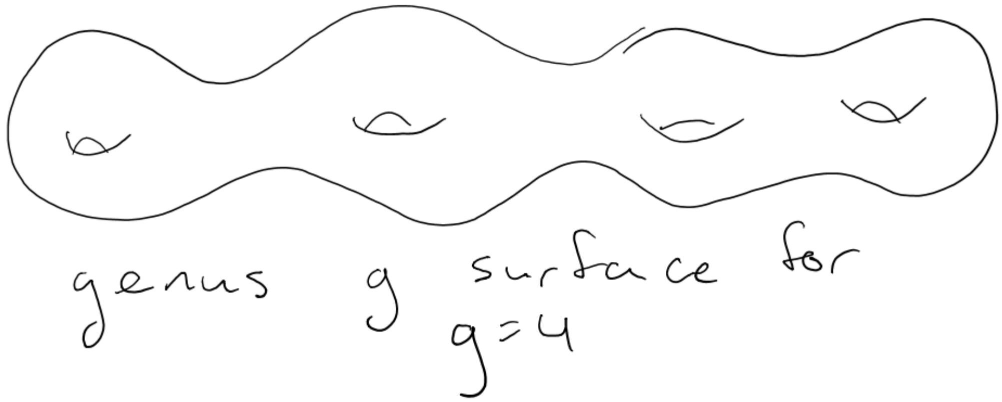
This class is built around a packet of problems that you'll work on with each other during class time and TAU; I plan to do almost no lecturing at all. This means that it'll be more work than most Mathcamp classes, but if you put in the time you'll learn it very well. The packet will be available on Monday night, and if you're planning to attend the first day you should spend a little time looking it over before Tuesday's class starts.

Homework: Required.

Prerequisites: None.

10:10 Classes

Cutting Surfaces into Silly Straws. (, Assaf, Tuesday–Thursday)
Sarah is an inventor, and has invented a remarkable product—the romantic straw. It is a straw with two ends, which forces couples to spend time drinking together as opposed to drinking apart. Sarah builds a factory that manufactures these romantic straws, but one day, the plastic injection machine fails, and instead outputs the genus g surface.



Sarah realizes that in order to recoup her losses, she can cut this surface and stretch the cut parts to get some straws. However, she doesn't want to waste any of the plastic. Her investors would like to know how many straws she can salvage.

In this course, we will define surfaces, and talk about something called the Euler Characteristic, which will help us solve Sarah's problem. If we have time, we will also discuss silly straws: straws with an arbitrary number of holes in them.

Homework: Optional.

Prerequisites: None.

Cluster: The Shape of Things.

Generating Functions and Partitions. (🌶🌶, Mark, Tuesday–Saturday)
Generating functions provide a powerful technique, used by Euler and many later mathematicians, to analyze sequences of numbers; often, they also provide the fun of working with infinite series without having to worry about convergence.

As a "serious" example, we'll use a generating function to get an expression in closed form for a famous and important sequence, that of the Catalan numbers. This sequence starts off $1, 2, 5, 14, 42, \ldots$, and it comes up in the solution of many counting problems, involving, among other things, voting, lattice paths, and polygon dissection.

A partition of a positive integer n is a way to write n as a sum of one or more positive integers, say in nonincreasing order; for example, the seven partitions of 5 are

$$5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, \text{ and } 1+1+1+1+1.$$

The number of such partitions is given by the partition function $p(n)$; for example, $p(5) = 7$. Although an "explicit" formula for $p(n)$ is known and we may even look at it, it's quite complicated. In our class, we'll combine generating functions and a famous combinatorial argument due to Franklin to find a beautiful recurrence relation for the (rapidly growing) partition function. This formula was used by MacMahon to make a table of values for $p(n)$ through $p(200) = 3972999029388$, well before the advent of computers!

Note: If you were at Mathcamp 2015 and took a class with a similar name, this is indeed the same class, so you should probably look for something different.

Homework: Recommended.

Prerequisites: Summation notation; geometric series. Some experience with more general power series may help, but is not really needed. A bit of calculus may be useful.

Cluster: Summing Series.

Introduction to Group Theory. (🌶🌶, Kevin, Tuesday–Saturday)
Groups are a type of mathematical object which generalize the notions of operations like addition

and multiplication to things that aren't numbers in a way that allows us to analyze symmetries and transformations of other objects. They are essential to the study of geometry, linear algebra, number theory, real and complex analysis, topology, and many, many other fields.

This class is an introduction to the theory of groups. We'll define groups and discuss several related constructions and examples, like cyclic and dihedral groups, homomorphisms, subgroups, cosets, quotients, and, time permitting, group actions.

The point of this class is to be a first exposure to the definition and basic theory of groups to prepare students for a number of classes coming up later in camp. If you've studied groups before and have some experience working with them, this is probably not the class for you. On the other hand, if you're new to the topic, you'll probably get a lot out of it, and it should be great preparation for a wide variety of other areas of math.

Homework: Required.

Prerequisites: None.

Required for: Geometric Group Theory (W2); Group Actions (W2); Field Extensions and Galois Theory (W2–W3); The Banach–Tarski Paradox (W2); Algebraic Groups (W3); A Tale of Combs and Hedgehogs (W3); What Can We Exponentiate? (W3); Representation Theory of Finite Groups (W3–W4); Random Groups (W4); The Word Problem for Hyperbolic Groups (W4); Universal Properties (W4); Algebraic Number Theory (W4); Ponzi Schemes in Infinite Groups (W4); Burnside's Lemma (W4); Harmonic Analysis on Abelian Groups (W4); The Fundamental Group (W4).

Problem Solving: Triangle Geometry. (🌶🌶🌶, Zach, Tuesday–Saturday)
Come explore the rich, diverse, and endlessly surprising world of triangle geometry! Triangles have loads of named "center" points, and we'll venture well beyond the classical centroid and orthocenter into some lesser-known yet unreasonably beautiful ones. Why has the symmedian point been called "one of the crown jewels of modern geometry"? Why is the existence of Feuerbach's point even reasonable (I'm still not convinced...), and how might we approach its construction synthetically (i.e., without inversion)? What are the (literally!) more than $10,000$ triangle centers listed in the Encyclopedia of Triangle Centers, and how can this encyclopedia be interpreted?

This class is largely problem based: there will be some lecturing, but much of the time you will present your solutions to the previous day's olympiad-style homework problems.

Homework: Required.

Prerequisites: Some familiarity with synthetic geometry (similar triangles, cyclic quadrilaterals, etc.).

Cluster: Problem Solving.

The Democracy of Number Systems. (🌶🌶🌶, *Clifton Cunningham*, Tuesday–Saturday)
This course begins by asking how 'real' the real numbers are. To explore this, we'll consider some famous divergent series and see how to interpret them by leaving the realm of real numbers entirely for the amazing p-adic numbers. After spending time time getting to know the p-adic numbers, we'll consider the democracy of number systems obtained by putting the real numbers and the p-adic numbers all in the same room. Then we'll bring this all back down to earth by using the democracy of number systems to answer a deceptively simple problem: Find all functions f from the rational numbers to complex units such that $f(x + y) = f(x)f(y)$.

Homework: Recommended.

Prerequisites: None.

Cluster: Rings and Fields.

Board Game Theory. (🌶, Assaf, Friday–Saturday)
Have you ever wanted to win a board game so much that you resorted to making probability calculations on a piece of paper under the desk? In this short course, we will discuss how this can be done. We will talk about probabilities in the context of board games, with specific examples. We will also show how some classical combinatorial game theory scenarios can be found in board games, and discuss the differences between combinatorial game theory and board game theory.

Note: This course will continue as an evening activity where we will analyze board games. There will also be a project in which we will design a board game.

Homework: Recommended.

Prerequisites: Played at least one game of Love Letter, Hanabi, Sushi Go!, and Dominion.

Cluster: Games Mathematicians Play.

11:10 Classes

How Not to Prove the Continuum Hypothesis. (🌶🌶🌶🌶, Susan, Tuesday–Saturday)
We know that the natural numbers are countable, and that the real numbers are uncountable, but what happens in between? What does it mean for a subset of the reals to be "small?" Is a small set necessarily countable? In this class we will explore different notions of smallness and largeness, touching on ideas from topology, measure theory, and logic. Especially logic! We'll learn the rules to several games played on infinite sets that can give us valuable information about how sets behave. One of these games will allow us to almost-but-not-quite prove the Continuum Hypothesis.

Homework: Required.

Prerequisites: Understand why $\mathbb{Q}$ is countable, and why $\mathbb{R}$ is not.

Huuuge Primes. (🌶🌶, David, Tuesday–Saturday)
If you want to check whether a 100-digit number is prime, trial division is infeasible: it would take trillions of years on our fastest supercomputer. But we'll see how to test such numbers for primality in milliseconds, and look at applications of such large primes to modern cryptography. Along the way we'll build up foundational ideas of number theory: modular arithmetic, the Euclidean algorithm, Fermat's theorem, Euler's theorem and the Chinese remainder theorem.

This class will be "inquiry-based": you'll spend class time working in small groups at the board on problems exploring these ideas.

Homework: Recommended.

Prerequisites: None.

Linear Algebra (Week 1 of 2). (🌶🌶🌶, Mark, Tuesday–Saturday)
You may have heard that linear algebra involves computations with matrices and vectors, and there is some truth to that. But this point of view makes it seem much less interesting than the subject really is; what's exciting about linear algebra is not those computations themselves, but

(1) the conceptual ideas behind them, which are elegant and which crop up throughout mathematics, and
(2) the many applications, both inside and outside mathematics.

In this class we'll deal with questions such as: What is the real reason for the addition formulas for sin and cos? What happens to geometric concepts (such as lengths and angles) if you're not in the plane or 3-space, but in higher dimensions? What does "dimension" even mean, and if you're inside a space, how can you tell what its dimension is? What does rotating a vector, say around the origin, have in common with taking the derivative of a function? What happens to areas (in the plane), volumes (in 3-space), etc. when we carry out a linear change of coordinates? If after a sunny day the next day has an 80% probability of being sunny and a 20% probability of being rainy, while after a rainy day the next day has a 60% probability of being sunny and a 40% probability of being rainy, and if today is sunny, how can you (without taking 365 increasingly painful steps of computation) find the probability that it will be sunny exactly one year from now? If you are given the equation $8x^2 + 6xy + y^2 = 19$, how can you quickly tell whether this represents an ellipse, a hyperbola, or a parabola, and how can you then (without technology) get an accurate sketch of the curve? How do astronomers know the chemical composition of distant stars?

Note: Although there was a linear algebra course at Mathcamp last year which I taught also, this won't be the same class: It runs for two weeks instead of one, and it has an extra chili so the pace should be brisker—and we should get to cover a lot more material.

Homework: Recommended.

Prerequisites: Although the blurb refers to taking a derivative, you'll be able to get by if you don't know what that means. If you have no previous exposure to abstract concepts, you should at least take the Mathcamp Crash Course at the same time.

Required for: Field Extensions and Galois Theory (W2–W3); Extending Inclusion-Exclusion (W2); Multilinear Algebra (W2); The Banach–Tarski Paradox (W2); Algebraic Groups (W3); What Can We Exponentiate? (W3); Representation Theory of Finite Groups (W3–W4); Problem Solving: Polynomials (W4); From Matrices to Representations (W4); Ponzi Schemes in Infinite Groups (W4); Quantum Mechanics (W4); Spectral Graph Theory (W4); Harmonic Analysis on Abelian Groups (W4).

Models of Computation Simpler than Programming. (, Pesto, Tuesday–Saturday)
Almost all programming languages are equally powerful—anything one of them can do, they all can.

We'll talk about less powerful models of computation—ones that can't even, say, tell whether two numbers are equal. They'll nevertheless save the day if you have to search through 200MB of emails looking for something formatted like an address.[1]

This is a math class, not a programming one—we'll talk about clever proofs for what those models of computation can and can't do.

Homework: Recommended.

Prerequisites: None.

Sum and Product Puzzles. (, Don, Tuesday–Saturday)

Sasha and Polina are both Mathcampers (and thus perfect logicians). I think of two numbers, X and Y, such that $1 < X < Y$, and $X + Y < 100$. I tell Sasha their sum, $X + Y$, and tell Polina their product, $X \cdot Y$. They then have the following exchange:

Polina: "I do not know the value of X."

Sasha: "I knew that you did not know X."

Polina: "I now know X."

Sasha: "Now I do too."

Not only is such an exchange possible, it is sufficient to inform an impartial observer (like you!) of the values of X and Y!

In class we'll look at this puzzle, and others like it (the recently popular Cheryl's Birthday puzzle is among them), where it is crucial that knowledge propagate at just the right speed. We'll work

[1] `http://www.xkcd.com/208`

together to develop a general method for solving them, prove results about when solutions exist, and glimpse some tantalizing open conjectures.

Homework: Recommended.

Prerequisites: None.

1:10 Classes

Analytic Number Theory. (🌶🌶🌶🌶, Sachi, Tuesday–Saturday)
Suppose I pick a very large integer n. What is the expected value of the number of divisors of n?

How quickly does the sum

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverge?

Are there infinitely many primes congruent to 1 mod 4?

Given two random integers a and b, what is the probability that a and b are relatively prime?

To answer these questions, we have to develop tools that can predict the average behavior of number-theoretic functions. We will study functions like the number of divisors function, the Euler totient function, and the prime counting function. Surprisingly, we can say a lot about the average behavior of these functions using techniques in analysis that estimate sums using integrals and by manipulating the order of summation.

Homework: Optional.

Prerequisites: You should be comfortable with integral calculus and series.

Cluster: Summing Series.

Card Shuffling. (🌶🌶, Zach, Tuesday–Wednesday)
The two days of this class do not depend on each other, so feel free to come to either (or both!).

Day 1: Seven Shuffles Aren't Enough! Conventional wisdom (namely, renowned mathematician Persi Diaconis) says that 7 shuffles are enough to thoroughly randomize a deck. Why, then, is there a simple experiment that works only 50% of the time on a truly randomized deck, but succeeds more than 80% percent of the time when only 7 shuffles are used?! We'll run this experiment in a few different incarnations (and lots of decks of cards), build a strong intuition for why it is so skewed, and discuss what this says about the "7 shuffles" wisdom.

Day 2: Perfect Shuffling. How do I make sure a deck of cards is perfectly randomized? This is not a practical question—I don't just want it "close enough" to random. I want a mathematically precise uniformly random deck, where each possible ordering has exactly a 1/52! chance of arising. Worse, what if all I have is a coin? How many flips will it take? What if I have a *weighted* coin with an unknown bias? We'll discover methods for all of these questions

Homework: None.

Prerequisites: None.

Combinatorial Games. (🌶🌶, Jane, Tuesday–Friday)
Let's play a game. We'll start out with two piles of pebbles, one with 8 pebbles and one with 11. We'll take turns making moves. On my turn, I'll remove all of the pebbles from one pile and then divide the other pile into two piles with at least one pebble each. Then, you'll move and do the same. The last person to make a move wins. If we both play optimally, who will win in this game? Does the result change if we change the number of pebbles in the starting piles?

Combinatorial game theory is the study of games like the above empty and divide game, back and forth games of perfect information and no chance. In this class, we'll play and study various combinatorial games, and in the process learn some ways of analyzing the strategy of these games.

Homework: Recommended.

Prerequisites: None.

Cluster: Games Mathematicians Play.

Guarding an Art Gallery. (🌶🌶, Jane, Saturday)
Suppose that you're in charge of security for an art gallery. The gallery is a polygonal room, and you would like to station guards around the room. Each guard will stand in place, but can see infinitely far in all directions from where he is standing. What is the least number of guards that you need to place so that every spot in the gallery can be seen by at least one guard? In this short class, we'll examine this problem and some variations of it.

Homework: Optional.

Prerequisites: None.

Introduction to Ring Theory. (🌶🌶🌶, *Ari Nieh*, Tuesday–Saturday)
The idea of a ring is simple: it's just an algebraic structure with two operations, addition and multiplication, made compatible by the distributive law. This framework, however, provides a way to talk about so many useful mathematical objects: not only numbers of many sorts, but also polynomials, matrices, and different flavors of functions. In fact, it's difficult to find an area of pure mathematics that doesn't have rings in it somewhere!

In this class, we'll look at basic notions of rings and subrings. We'll talk about how to construct new rings out of old ones. Then, we'll discuss maps of rings, ideals, quotient rings, and the relationship between these concepts. As time allows, we'll also discuss fields of fractions, localization, modules, and other assorted topics.

Homework: Required.

Prerequisites: None.

Cluster: Rings and Fields.

Required for: Field Extensions and Galois Theory (W2–W3); K-Theory (W2); Bad Domains, Bad Factorization (W3); Algebraic Groups (W3); Algebraic Number Theory (W4).

Mathcamp Crash Course. (🌶, *Nina White*, Tuesday–Saturday)
This course covers fundamental mathematical concepts and tools that all other Mathcamp courses assume you already know: basic logic, basic set theory, notation, some proof techniques, how to define and write carefully and rigorously, and a few other tidbits. If you are new to advanced mathematics or just want to make sure that you have a firm foundation for the rest of your Mathcamp courses, then this course is *highly* recommended.

Here are some problems to test your knowledge:

(1) Negate the following sentence without using any negative words ("no", "not", etc.): *"If a book in my library has a page with fewer than 30 words, then every word on that page starts with a vowel."*
(2) Given two sets of real numbers A and B, we say that A *dominates* B when for every $a \in A$ there exists $b \in B$ such that $a < b$. Find two disjoint, non-empty sets A and B such that A dominates B and B dominates A.
(3) Prove that there are infinitely many prime numbers.

(4) Let $f : A \to B$ and $g : B \to C$ be maps of sets. Prove that if $g \circ f$ is injective then f is injective. (This may be obvious, but do you know how to write down the proof concisely and rigorously?)
(5) Define rigorously what it means for a function to be increasing.
(6) Prove that addition modulo 2013 is well-defined.
(7) What is wrong with the following argument (aside from the fact that the claim is false)?

    Claim: On a certain island, there are $n \geq 2$ cities, some of which are connected by roads. If each city is connected by a road to at least one other city, then you can travel from any city to any other city along the roads.

    Proof: We proceed by induction on n. The claim is clearly true for $n = 1$. Now suppose the claim is true for an island with $n = k$ cities. To prove that it's also true for $n = k+1$, we add another city to this island. This new city is connected by a road to at least one of the old cities, from which you can get to any other old city by the inductive hypothesis. Thus you can travel from the new city to any other city, as well as between any two of the old cities. This proves that the claim holds for $n = k + 1$, so by induction it holds for all n. QED.
(8) Explain what it means to say that the real numbers are uncountable. Then prove it.

If you are not 100% comfortable with most of these questions, then you can probably benefit from this crash course. If you found this list of questions intimidating, then you should *definitely* take this class. It will make the rest of your Mathcamp experience much more enjoyable and productive. And the class itself will be fun too: we'll be spending most of our time working in small groups on problems.

Note: This class will extend into TAU for one hour.

Homework: Required.

Prerequisites: None.

Required for: Everything!

de Bruijn Sequences. (🌶🌶, Pesto, Thursday–Saturday)
de Bruijn sequences are sequences of 0s and 1s containing all possible subsequences of a given length exactly once: for instance, 0001110100 contains every possible sequence of 3 0s and 1s.

Days 1 and 2: For what lengths do such sequences exist, and how easily can we find them if they do?

Day 3: How were they used as an early error-correction code in Sanskrit poetry, 2500 years before error-correction codes even existed?

(Day 3 is 🌶 and doesn't depend on Day 2 or on any knowledge of Sanskrit.)

Homework: Optional.

Prerequisites: None.

Colloquia

Spin! (*Clifton Cunningham*, Tuesday)

Rotation by 360 brings you back to your starting point. But if your limbs are tied to the walls, you'll get tangled, with no way to untwist the ropes. Amazingly, if you continue your rotation to a full 720 degrees, there is a way to untangle yourself. We'll see a demo of this phenomenon in action, explain it in terms of stereographic projection, and describe the connection with spin $\frac{1}{2}$ from quantum mechanics. Come get tangled up in the fundamental group of SO(3)!

The Music of Zeta. (*J-Lo*, Wednesday)

The Riemann Zeta function is defined quite simply as a sum over the natural numbers, all raised to a given exponent. But what exactly does it measure?

On an unrelated note, ever wondered why a musical scale (CDEFGAB) has seven notes, or why the chromatic scale has 12?

Oh wait, my bad, these aren't unrelated at all. Find out why, and you'll also discover how Pythagoras wrote the score for his own worst nightmare, why you sometimes need to listen to a function before you can define it (or more precisely, its so-called "analytic continuation"), and how you can earn a million dollars for transcribing a song played by the world's strangest keyboard.

Intersecting Curves in the Plane. (Nic, Thursday)

From any polynomial f in two variables, you can form a curve by looking at all the points (x, y) in the plane for which $f(x, y) = 0$. These are called "algebraic plane curves"—lines, ellipses, hyperbolas, and graphs of one-variable polynomials are all examples. In this colloquium, we're going to study Bézout's Theorem, a result from the eighteenth century which says that the number of intersection points of two algebraic plane curves is given by the product of their degrees.

We won't be able to prove this theorem in an hour, and for a very good reason: the way I've stated it, it's not true! But by looking more closely at the cases where it fails, I hope to convince you that a true statement is hiding underneath the surface; in our attempt to uncover it, we'll get a tour of some of the most beautiful foundational ideas in twentieth-century algebraic geometry.

Tree Trotting. (Zach, Friday)

As a criminal mathtermind on the run from the law (of large numbers?), you have constructed a complicated network of hideouts to navigate via underground tunnels. To throw the authorities off your (not-necessarily acyclic) path, you sporadically move from each hideout to one of its randomly chosen neighbors in your trusty van. As you do this, every time you visit a new hideout for the first (I mean zeroth) time, you mark the corridor you just traversed as that hideout's "escape route" to use in case of a surprise raid. Once you have visited all hideouts, your choices of escape routes form a smaller network for quickly driving to freedom—an emergency vanning tree spanning your hideouts. What is this tree likely to look like? Which kinds of trees are more likely to arise? Do answers to these questions in any way facilitate your daring escape?

Visitor Bios

J-Lo. J-Lo (known to some as Jonathan Love) was a Mathcamper in MC'09 and MC'10 and a JC in MC'14 and MC'15. He is currently a Masters student studying algebraic number theory at University of Toronto (where he also did his undergrad), and is starting a Ph.D. program at Stanford University in the fall. Other interests include making music (and oh man the connections between math and music ajskdhfwoieuhfh), cycling, speaking Japlish, acting (Bacon number = 3), puns, wandering (in cities, on mountains, in Wikipedia, in my and/or others' minds, etc.), and trying to explain complicated things.

Clifton Cunningham. Clifton loves algebraic geometry, group representation theory and number theory, so he found a home in the Langlands program, mainly using perverse sheaves to study admissible representations of p-adic groups. Right now he's also using Hilbert modular forms to manufacture interesting automorphic representations of spin groups. Once a Science Busker for Beakerhead, Clifton was the Calgary Director of the Pacific Institute for the Mathematical Sciences from 2008 to 2015, but he's recovering nicely now.

Ari Nieh. Ari is a freelance bass-baritone singer, specializing in Medieval, Renaissance, and Baroque music. He has attended Mathcamp as a student, JC, mentor, co-pear, or visitor every year since 1996. When not singing, he teaches for AoPS, writes puzzles, cooks, and photographs flowers. This fall, he'll begin a new job as a lecturer in Writing, Rhetoric, and Professional Communication at MIT.

Nina White. Nina White turned her background in mathematics into a specialty in teacher education. She teaches mathematics to future and current elementary, middle, and high school teachers. Teaching these courses provides her with enjoyable opportunities to think about simple things deeply. Her education research focuses on instruction at the college level—how do we describe instruction? measure instruction? measure instructional efficacy? train effective college instructors? She still enjoys her first mathematical loves: geometry and topology, with a touch of algebra.