

CLASS DESCRIPTIONS—WEEK 4, MATHCAMP 2016

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9:10 Classes

Asymptotics of Generating Functions. (☞☞), Kevin, Tuesday–Saturday)

Generating functions often allow us to solve combinatorial problems in seemingly miraculous ways. We define a power series whose coefficients are a sequence we're interested in (even if we don't know more than the first few values explicitly!), and then we can pull a closed-form function for the power series out of a hat. If we can then extract the coefficients from this function, we've found a formula for the sequence!

Unfortunately, there are many, many closed-form functions where extracting coefficients exactly is impenetrable. The mathematician seeking an exact formula for the sequence is in this case out of luck. But this is where *we* are just getting started! We will discuss incredibly powerful, incredibly widely applicable strategies to study the asymptotics of generating functions, finding beautiful formulas for the n^{th} term of our sequence that approximate the exact answer arbitrarily closely as $n \rightarrow \infty$. For example, we'll see how to get an arbitrarily close approximation to $n!$, going beyond the usual form of Stirling's formula. The strategies we use will have fancy names like singularity analysis and saddle-point asymptotics, and we'll see how amazingly far a few derivatives and integrals can take us.

If you have no experience with generating functions, that's OK. We'll be starting from the closed-form function (by which point the machinery of generating functions has already done its job), so no prior knowledge is necessary. On the flip side, if you absolutely love generating functions, be warned that this will be a very different, more analytic side of them, without the same sort of algebraic magic found in most generating functions classes.

Homework: Optional.

Prerequisites: Single-variable calculus (derivatives, integrals, and power series).

Cluster: Summing Series.

Does ESP Exist? or, What's Wrong With Statistics? (☞), Mira Bernstein, Tuesday–Saturday)

In 2011, Daryl Bem, a professor of psychology at Cornell, published an unusual paper in the *Journal of Personality and Social Psychology*, titled "Feeling the Future: Experimental Evidence for Anomalous Retroactive Influences on Cognition and Affect". In this paper, Bem presents results from 9 experiments testing for "psi", aka ESP. For instance, in the first experiment, participants are instructed as follows:

On each trial of the experiment, pictures of two curtains will appear on the screen side by side. One of them has a picture behind it; the other has a blank wall behind it. Your task is to click on the curtain that you feel has the picture behind it. The curtain will then open, permitting you to see if you selected the correct curtain.

Bem's analysis of the data shows that participants were able to predict the location of the picture at a rate significantly higher than chance, but only if the picture involved sex or violence. For other pictures, they did no better than chance. The follow-up experiments were designed to determine whether the psi effect was due to clairvoyance, psychokinesis, or retroactive influence in which "the direction of the causal arrow has been reversed".

Note: this is not just an individual loony. This is a highly respected professor of psychology, from one of the top departments in the country, writing for one of the top journals in his field, and being, if anything, more scrupulous in laying out his methodology than is standard practice.

The journal editor wrote a preface to Bem's paper, stating that although he himself does not believe in the existence of psi, neither he nor the other reviewers could find a flaw in Bem's statistical analysis. Therefore, says the editor, scientific honesty compelled him to publish the paper, so that it could be discussed in a wider forum.

If you find all of this shocking, you are not alone. The entire psychology profession cringed in mortification and blamed the editor for not exercising better judgment.

Yet the real culprit here is not the author (who is free to pursue whatever research he wants, and whom no one suspects of violating scientific ethics) nor the editor (who really was just trying to adhere to the standard practice of his field), but the statistical methodology currently employed by most scientists. A famous 2005 paper published in a medical (!) journal states the problem very starkly in its title: “Why most published research findings are false”.

In this course, we’ll discuss the (many) problems with how statistics is currently done (not just in psychology). We’ll talk about the historical reasons for these problems and present an alternative: Bayesian statistics—a methodology that is actually based on math and dates back to Gauss and Laplace. In particular, we’ll apply a Bayesian analysis to Bem’s results. (Spoiler alert: they give no evidence for the existence of ESP. Sorry to disappoint you.)

Homework: Recommended.

Prerequisites: Calculus and basic probability. If you don’t know calculus, you will be able to follow some days but not others. No background in statistics required, though if you do have such a background, you may appreciate the de-brainwashing.

Cluster: Mathematics and Its Applications.

Knot Theory. (🌀, Jeff, Tuesday–Saturday)

In the 1860s, Lord Kelvin developed the following theory of matter: that atoms, the indivisible particles that composed the universe, were actually tiny whirlwind vortices in the ether. The shape of these vortices were tiny knots, and you could make compounds out of these knots by linking them together. Inspired by the quest to classify atoms, a mathematician named Tait made a list of all knots up to 10 crossings (no small feat, considering that there are around 250 of them.)

Kelvin’s theory turned out to be bunk (as both the idea of ether and tiny vortices were too crazy), but mathematicians kept on thinking about knots. It took mathematicians nearly a hundred years to realize that Tait’s list was wrong, and we still have a lot to learn about knots. We now study knots not because they represent atoms, but because they are some of the simplest objects a topologist can study: maps from the circle to 3-dimensional space. And despite these objects being so fundamental, a classification of knots eludes mathematicians to this very day.

In this class, we’ll take the first step to classifying knots, by describing invariants of knots and giving a procedure to (non-uniquely) describe every knot.

Homework: Recommended.

Prerequisites: None.

Cluster: The Shape of Things.

Problem Solving: Polynomials. (🌀🌀🌀, Pesto, Tuesday–Saturday)

Polynomials are a frequent topic of Olympiad-style competitions, since there are many and interesting problems using them: for instance, to find the product of the lengths of all the sides and diagonals of a regular n -gon of diameter 2. Don’t see the polynomials? Come to class and find them.

This is a problem-solving class: I’ll present a few techniques, but most of the time will be spent having you present solutions to Olympiad-style problems you will have solved (or at least tried) as homework the previous day.

Disjoint from 2014’s polynomials problem solving class.

Homework: Required.

Prerequisites: None.

Cluster: Problem Solving.

Representation Theory of Finite Groups (Week 2 of 2). (👍👍👍, Mark, Tuesday–Saturday)

This is a continuation of the week 3 class. If you would like to join and you're not sure what has been covered, check with Mark (and/or with someone who has been taking the class and who has good notes).

Homework: Recommended.

Prerequisites: Group theory and linear algebra; comfort with abstraction; week 1 of this class.

Cluster: Groups.

10:10 Classes**From Matrices to Representations.** (👍👍, Noah Snyder, Wednesday–Saturday)

How do you classify matrices up to similarity? What about matrices satisfying some algebraic property, like that their 5th power is the identity? How about pairs of commuting matrices that both have their 5th power the identity? We'll answer these questions and some additional variations, and on the way we'll learn about some representation theory.

Homework: Recommended.

Prerequisites: Some linear algebra is necessary (vector spaces, spans, linear independence, basis). You should not take this class if you've already seen Jordan Normal Form, but the class should be interesting even if you've already learned the character-theoretic approach to representation theory.

Harmonic Functions on Graphs. (👍, Yuval, Wednesday–Saturday)

Let's say you're a camper checked out to Walmart, and you have no idea how to get back to camp. You decide to walk home randomly: at every intersection, you roll a fair die to decide which path to take. Since you want to make sure you get back by the end of sign-in, you wonder: how long, on average, will your walk take?

This sounds like a hard question. However, it turns out that we can solve it—by turning all of the roads into rubber bands. Along the way we'll discover how to draw graphs, tile rectangles, and count trees.

Homework: Optional.

Prerequisites: Introduction to Graph Theory.

Cluster: Maps, Graphs, Colors, Walks.

Ponzi Schemes in Infinite Groups. (👍👍, Fedya Manin, Wednesday–Saturday)

I asked Sachi and Susan to each lend me a dollar, but I don't want to pay them back. So instead I tell them that they should each ask two of their friends for a dollar, and then tell those friends to ask two of their friends for a dollar, and so on. That way everyone ends up with a dollar more than they started with! (Except me, I end up with two dollars.)

Unfortunately for me, in our finite world such a scheme always eventually fails. This is a theorem of Charles Ponzi, who went to prison in the 1920s for attempting a disproof. Things get more interesting when people live on an infinite graph that connects them to their friends. For example, there's no Ponzi scheme on an infinite Euclidean grid, but there is one on a grid in the hyperbolic plane.

My favorite infinite graphs are Cayley graphs of groups. It turns out that groups that admit Ponzi schemes on their Cayley graphs are exactly the “non-amenable” ones. We'll see at least one more of the 15 or so equivalent definitions of what this means, as well as various examples. On the way, we'll develop an extremely powerful and general tool called linear programming duality (nothing to do with programming.)

Homework: Recommended.

Prerequisites: Group presentations and linear algebra (linear transformations, image, kernel).

Quantum Mechanics. (🍷🍷, Nic, Wednesday–Saturday)

Here are some things you might have heard about quantum mechanics:

- The thing that makes it so strange is the presence of probability.
- The thing that makes it so strange is that you can't measure something without changing the result.
- The thing that makes it so strange is that things can be two things at the same time, and that's what makes quantum computers work.
- Like, Schrödinger's cat, man: is it alive or dead?

In this class we'll talk about how probability actually behaves in quantum systems—I hope to convince you that none of these descriptions is anywhere near weird enough to get to the point of quantum mechanics! We'll start from an axiomatic perspective for the first couple days, focusing on the way in quantum states and observables behave in the abstract; we probably won't talk about position and momentum directly until near the end.

Homework: Recommended.

Prerequisites: Ideally you should know the material from both weeks of Linear Algebra. You should know the basic of basis, independence, and dimension, how inner products on finite-dimensional vector spaces work, and what eigenvalues and eigenvectors are. We'll be using linear algebra over the complex numbers in this course, but it's fine if you don't know much about that yet. Come talk to me if you have questions about this.

Some knowledge of derivatives will be necessary for the second half.

Cluster: Mathematics and Its Applications.

Spectral Graph Theory. (🍷🍷, Sachi, Wednesday–Saturday)

Here's something we know how to do really well: linear algebra. In comparison with many other areas of mathematics, matrices are really well understood.

On the other hand, I don't really particularly care about matrices for their own sake. One subject that I do find interesting to study in its own right is graph theory. So: here's what I propose. Let's apply linear algebra to graph theory, and see where it goes.

To each graph, we can associate a matrix, which is called its adjacency matrix. The construction works like this: put a 1 in position (i, j) if and only if vertex i is adjacent to vertex j , and otherwise fill it with a 0. Since adjacency is a symmetric relation, this yields a real symmetric matrix. So, an n vertex graph has a collection of n eigenvalues and n eigenvectors. In this class, we'll discover what the matrix, its eigenvectors, and eigenvalues can tell us about our graph.

This class will be about working on problem sets and learning through discovery. During the 2 hour superclass expect to be immersed in problem solving, discussing problems with a team of other mathcampers, and writing up solutions. There will be no homework to be done outside of class.

Homework: Recommended.

Prerequisites: None.

Cluster: Techniques in Graph Theory.

11:10 Classes**Burnside's Lemma.** (🍷, Alfonso Gracia-Saz, Tuesday–Saturday)

- (1) In how many different ways (up to rotation) can you colour a cube if each face can be coloured in blue, red, or green?
- (2) How many different necklaces can you make with 6 pearls if you have unlimited black and white pearls?

- (3) How many different necklaces can you make with $2N$ pearls which use exactly 2 pearls of each of N colours?

Burnside's Lemma is a cute, little gem that will allow you to solve the first two problems quickly and elegantly, without having to brute-force things or count lots of cases.

To answer the third question, though, you need the Redfield–Pólya Theorem, which is like Burnside's Lemma on steroids.

You will learn both results in this class.

Homework: Recommended.

Prerequisites: Basic group theory.

Cluster: Algebraic Novelties.

Harmonic Analysis on Abelian Groups. (🍷🍷🍷, Michael Orrison, Tuesday–Saturday)

In this course, we will focus on how and why you might want to rewrite a complex-valued function defined on a finite abelian group as a linear combination of surprisingly simple functions called “characters”. As we will see, doing so will quickly lead us to discussions of far-reaching algorithms and ideas in mathematics such as discrete Fourier transforms (DFTs), fast Fourier transforms (FFTs), random walks, and the Uncertainty Principle.

Homework: Recommended.

Prerequisites: Linear Algebra (bases, invertible matrices, eigenvalues, inner products, orthogonality), Group Theory (examples of finite abelian groups and their subgroups, cosets, homomorphisms), and the ability to add and multiply complex numbers.

Math Campers Show Presentations. (🍷, Sam + Chris, Tuesday–Saturday)

Once upon a time, a mathcamper had to give a technical talk. She was surprisingly self-aware, and realized that giving a good technical talk takes lots of practice and effort. If you've ever been in—or hope to be in—the position of teaching someone math, presenting to a math circle, or sharing research, this class is for you. If you're not as self-aware as that camper, this class is definitely for you!

Parts of the class will involve discussion and activities to facilitate giving good talks, but the main point of this class is for every camper to give a clear and compelling five minute technical talk to the class on Friday or Saturday. You'll also get to hear lots of awesome short talks by other campers!

Warning: You'll be expected to prepare—and practice at least 3 times—a technical talk in the course of the week. This takes time and effort!

Homework: Required.

Prerequisites: None.

The Fundamental Group. (🍷🍷, Jane, Tuesday–Saturday)

Topologists can't tell a donut and a coffee cup apart because they have the same topology. That is, we can deform the donut into a coffee cup without ripping it. This process of deformation is called a homotopy. But suppose that we wanted a way to prove that two spaces, like the sphere and the torus, were topologically different. In enters the fundamental group, a super important tool in algebraic topology! The fundamental group is a topological invariant that records what loops we have in our space, up to homotopy.

In this class, we'll learn some methods of computing the fundamental groups of spaces, talk about covering spaces, and also see some applications of the fundamental group. As a preview, consider the following question: given a ham sandwich consisting of two possibly strangely shaped pieces of bread and one piece of ham, can we always split the sandwich evenly between two people with one cut (i.e. a plane) so that both people get equal amounts of each piece of bread and ham?

Homework: Recommended.

Prerequisites: Point-Set Topology, Group Theory.

Cluster: Algebraic Topology.

1:10 Classes

Cut That Out! (🔪, Zach, Tuesday–Saturday)

I hand you a square of paper and a pair of scissors, and your goal is to cut the square into some number of “puzzle pieces” that can be rearranged precisely into an equilateral triangle. Can you do it? Perhaps surprisingly this *is* possible, and even more surprisingly, you can do it with just 4 pieces. Try it! More generally, is it always possible to dissect any polygon into any other (assuming they have the same area)? What about a square and a circle? What about a cube and a tetrahedron? What if we allow infinitely many cuts, or “fractal” cuts, or...? In addition to discussing all of these questions and more, this class will feature a plethora of perplexing puzzles and a panoply of pretty pictures.

Homework: Optional.

Prerequisites: None.

Random Groups. (🔪🔪, Assaf + Misha, Friday–Saturday)

In the first half of this class, we will be talking about Gromov's model for random groups. Such a random model is useful for showing that there exist groups with certain properties without explicitly constructing them. We will present the model, prove that random groups of density $\geq \frac{1}{2}$ are trivial, and state some recent results about random groups at other densities.

In the second half, we'll talk about subgroups of S_n , the symmetric group, obtained by choosing permutations at random to serve as generators.

Homework: Recommended.

Prerequisites: None.

Cluster: Groups.

Stupid Games on Infinite Sets. (🔪, Susan, Tuesday–Saturday)

Let's play a game. You name a number. Then I name a bigger number. Then you name a number that's even bigger. We keep doing this forever, and then when we're done we check to see who wins.

This may sound like the worst game ever, but actually we can use this game to learn some cool things about the ordinal numbers. By the end of this class you'll be able to show that there exists a game in which neither player has a winning strategy. In fact, until the very end of the game, we will have no information about who is going to win.

No previous knowledge necessary—this class will include a nice introduction to the ordinal numbers.

Homework: Recommended.

Prerequisites: None.

Cluster: Games Mathematicians Play.

Probably Combinatorics. (☺☺☺, *Po-Shen Loh*, Friday–Saturday)

Some of the most interesting developments in the world come from the fusion of different ways of thinking. This applies in the mathematical world as well. In the middle of the last century, several incredibly creative researchers discovered surprising applications of Probability in Combinatorics, now known as the Probabilistic Method, popularized by Paul Erdős. In this class, we'll discuss a few applications of this technique, starting with a discussion of crossing numbers which were introduced earlier at Mathcamp.

Homework: Optional.

Prerequisites: A solid understanding of probability is probably required.

The Lonely Torus—A Lie Group Opera. (☺☺☺☺, *Assaf*, Tuesday–Thursday)

A Lie group G is a group that is also a manifold M , and the group operations are continuous. The group structure induces a nonvanishing vector field on M by push-forward, and since Lie groups are orientable, by the Hopf–Poincaré Index Theorem, $\chi(M) = 0$. This proves that the torus is the only compact 2-dimensional Lie group.

If you want to understand and prove the above, come to this class.

Homework: Recommended.

Prerequisites: Group Theory, Linear Algebra (week 1), differential calculus, continuous maps and local homeomorphisms.

Cluster: Groups.

Trail Mix. (☺ → ☺☺☺, *Mark*, Tuesday–Thursday)

Is your mathematical hike getting a little too strenuous? Would you like to relax a bit with a class that offers an unrelated topic every day, so you can pick and choose which days to attend, and that does not expect you to do homework? If so, some Trail Mix may be just what you need to regain energy. Individual descriptions of the three topics follow.

Day 1 (☺, Tuesday): **Perfect Numbers.** Do you love 6 and 28? The ancient Greeks did, because each of these numbers is the sum of its own divisors, not counting itself. Such integers are called *perfect*, and while a lot is known about them, other things are not: Are there infinitely many? Are there any odd ones? Come hear about what is known, and what perfect numbers have to do with the ongoing search for primes of a particular form, called Mersenne primes—a search that has largely been carried out, with considerable success, by a far-flung cooperative of individual “volunteer” computers.

Day 2 (☺☺, Wednesday): **Intersection Madness.** When you intersect two ellipses, you can get four points, right? So why can't you get four points when you intersect two circles? Well, actually you can, and what's more, two of the four points *are always in the same places!* If this seems paradoxical (and, I hope, interesting), wait until we start intersecting two cubic curves (given by polynomial equations of degree 3). There's a “paradox” there too, first pointed out by the Swiss mathematician Cramer in a letter to Euler, and the resolution of that paradox leads to a “magic” property of the nine intersection points. If time permits, we'll see how that property (known as the Cayley–Bacharach theorem) gives elegant proofs of Pascal's hexagon theorem and of the existence of a group law on a cubic curve.

Day 3 (☺☺☺, Thursday): **The Jacobian Determinant and $\sum_{n=1}^{\infty} \frac{1}{n^2}$.** How do you change variables in a multiple integral? For example, when you change to polar coordinates, a somewhat mysterious factor r is needed. This is a special case of an important general fact involving a determinant of partial derivatives. We'll see how and roughly why this works; then we'll use it to evaluate the famous sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (You may well know the answer, but do you know a proof? If so, do you know a proof that doesn't require Fourier series or complex analysis?)

Prerequisites:

(Day 1) None.

(Day 2) A little bit of linear algebra would help; having attended Nick's colloquium would help, too. And we'll use complex numbers, but very lightly. There are no "serious" prerequisites.

(Day 3) Some multivariable calculus, and a bit of experience with determinants - what was covered in week 2 of linear algebra this year should do.

Homework: None.

Universal Properties. (☺☺☺, Don, Tuesday–Saturday)

The Cartesian product of sets, direct product of groups, the product topology on the cartesian product of topological spaces. If you've seen some of these before, you may have noticed some similarities between the different definitions—and that's because they're all objects with the same universal property.

Category theory is a language used to describe these kinds of properties. It's a framework that almost any kind of math fits into, and often by putting a subject into the language of category theory, interesting questions arise. Additionally, you can use category theory to prove many theorems at once, by proving them in the general case. In this class, we'll learn about categories, universal properties that can exist across categories, and functors, which let you compare different categories.

Homework: Recommended.

Prerequisites: Group theory, or ring theory, or linear algebra, or topology. The more, the better, though.

Marathons

Algebraic Number Theory. (☺☺☺☺, David, Tuesday–Saturday)

We will explore number fields: their Galois groups, class groups, unit groups, and factorization into prime ideals. We will then proceed to a discussion of local and global class field theory.

You should have already talked to me if you want to take this class.

Homework: Required.

Prerequisites: Field Extensions and Galois Theory, Domains and Factorization: When Everything Goes Wrong.

Cluster: Rings and Fields.

Colloquia

Should We Vote on How We Vote? (*Michael Orrison*, Tuesday)

Voting is something we do in a variety of settings, but how we vote is seldom questioned. In this talk, we'll explore a few different voting procedures from a mathematical perspective as we try to make sense of the paradoxical results that can occur when we vote in more than one way.

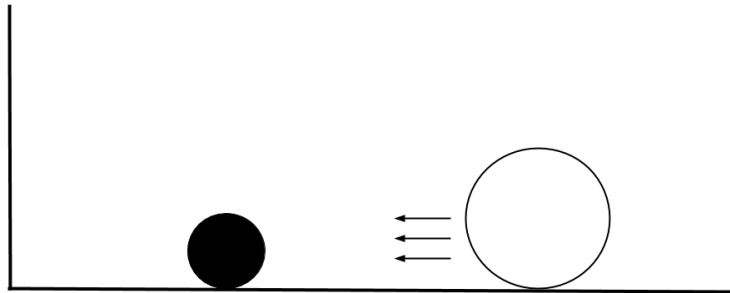
Algebraic Proof of Heron's Formula. (*Noah Snyder*, Wednesday)

Heron's formula for triangular area says that the area of a triangle is given in terms of the three side lengths by the formula

$$\frac{1}{4}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}.$$

I'll give a largely algebraic proof of this formula, which uses very little geometry. Roughly the idea of the proof is that this formula is the simplest possible formula that gives the right answer for triangles of zero area.

A simple question with a very curious answer. (*Mira Bernstein*, Thursday)



In the diagram above, a white ball of mass $M \geq 1$ is rolling toward a stationary black ball of mass 1. Once they collide, the black ball will roll toward the wall, bounce off, and collide again with the white one. (Small print for the physicists: we assume all collisions are perfectly elastic.)

What happens next? If $M = 1$, the black ball stops moving and the white ball rolls away—so there are 3 collisions total. If $M > 1$, the black ball will bounce some more between the white ball and the wall, until eventually both balls roll away.

Our question is: how many collisions will there be, as a function of M ? Come to colloquium to find out; I promise the answer will blow your mind!

(Note: this talk requires no physics background and only very elementary math. Everyone should be able to follow.)

Elo × Education. (*Po-Shen Loh*, Friday)

When new mathematics enters an industry, it has the potential to transform the status quo. We'll introduce and motivate the Elo rating system (originally used for competitive Chess), and then talk about how to use this breakthrough to solve personalized Education on a global scale: using math to teach math, science, and more.

Visitor Bios

Fedya Manin. Fedya was a camper in '04 and '05 and a mentor in '10. These days he's a postdoc at the University of Toronto who believes in Misha Gromov's dictum that we can learn more about what topological invariants Really Mean by studying how they affect the geometry of a space. When not doing math, he enjoys bouldering, playing klezmer music on the clarinet, vegan cooking, and learning weird things, especially about language, history, and biology.

Jesse Geneson. Jesse Geneson is a mathematician and data scientist. He is a lead mentor for Crowd-Math, an open project that gives all high school and college students the opportunity to collaborate online on a large research project. He graduated from Harvard University in 2010 and received a Ph.D. in applied mathematics from MIT in 2015. He has mentored nine math research projects at MIT including the winner of the 2014 Siemens individual competition, the fifth prize in the 2012 Siemens team competition, and the third prize in the 2011 Siemens individual competition.

See www.artofproblemsolving.com/polymath/mitprimes2016 for more information on Crowd-Math.

Michael Orrison. Michael wanted to be a fighter pilot until the tenth grade. He then switched to mathematics and he hasn't looked back since. (Okay, he looks back every now and then, but just for fun.) His teaching interests include linear algebra, abstract algebra, discrete mathematics, and representation theory. His research interests include voting theory and harmonic analysis on finite groups. He particularly enjoys finding, exploring, and describing novel applications of the representation theory of finite groups with the help of his talented and energetic research students.

Mira Bernstein. Mira has been at Mathcamp since 1997, and was largely responsible for the 1997 Mathcamp Junta. After that, she was Ze Top Banana of Ze Mathcamp Power Structure until 2012, and is now Ze Top Banana Emerita. She used to be a pure mathematician doing algebraic geometry, but in the last 15 years has been doing all sorts of other things instead—dabbling in population genetics and health economics, using data science to fight slavery (did you know that there are still about 30 million slaves in the world?), and, most recently, moving to San Francisco for a year to help start Proof School, the first math school in the US. This summer she is missing most of Mathcamp to teach at BEAM (ask her what that is...), but she is thrilled to be back for a week. Mathcamp is home.

Noah Snyder. Noah Snyder is an Assistant Professor and Indiana University in Bloomington. He was a mentor in 2006–2008, AC in 2009, and faculty in 2012. He works on planar and 3-dimensional algebra and their relationships to representation theory, topology, and operator algebras. He enjoys reading Wikipedia, but does not enjoy figuring out how to stop skunks from living under his house.

Po-Shen Loh. Po-Shen Loh studies questions that lie at the intersection of two branches of mathematics: combinatorics (the study of discrete systems) and probability theory. Randomness can manifest itself in the construction of a combinatorial system, as in the case of a so-called “random graph,” but may also be artificially introduced as a proof technique to solve problems about purely deterministic systems, as was pioneered by Paul Erdős in what is now known as the Probabilistic Method. He is also passionate about developing mathematical talent at all levels, from coaching the USA International Math Olympiad team, to speaking at math enthusiast gatherings around the world, to crowd-sourcing interactive math/science lessons by all, for all, at expii.com.