CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2017

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9:10 Classes

All Things Manifoldy. (🌶🌶, Apurva, Tuesday–Saturday)
Who said that mathematicians are not real doctors, we perform surgeries all the time. In this class we'll take baby steps towards understanding manifolds. We'll learn some of the uber awesome techniques invented by topologists to study manifolds. We will perform surgeries on manifolds and do origami using simplices, and by the end of the class you'll be able to visualize (some) manifolds in higher dimensions.

Incidentally when Einstein tried to combine special relativity with Newton's gravity nothing seemed to work. It took him a decade to finally realize a beautiful solution to the conundrum: our universe is a 4 dimensional manifold and gravity is a measure of how the manifold curves. But what is a manifold?

Homework: Recommended

Prerequisites: None.

Related to (but not required for): The Fundamental Group (W1); Wallpaper Patterns (W1); Coloring the Hyperbolic Plane (W2); Euler Characteristic (W2); Symmetries of Spaces (W3); How to Define the Square Root (W4); Riemann Surfaces (W4)

Generating Functions and Regular Expressions. (🌶🌶, Linus, Tuesday–Saturday)
To cheat at Mathcamp's famed week 4 puzzle hunt, I use regular expressions. For example, if I know a puzzle answer uses the letters d, u, c, and k in that order, I can use the regular expression d.*u.*c.*k to get a list of all English words it could be.

To count anything, e.g. the number of domino tilings of a $4 \times n$ rectangle, I use generating functions, a magic tool of combinatorics.

Learn how regular expressions and generating functions are the same thing, and use them together to instantly solve a bunch of problems like:

- "What's the most chicken nuggets I can't order if they come in 5-piece and 8-piece boxes?"
- "Why do rational numbers have repeating decimals?"

Homework: Recommended

Prerequisites: None! There's no need to have even heard of a regular expression, generating function, or dduckkkkk before

Group Theory. (, Mark, Tuesday–Saturday)
How can you describe the symmetries of geometric figures, or the workings of a Rubik's cube? How do physicists predict the existence of certain elementary particles before setting up expensive experiments to test those predictions? Why can't fifth-degree polynomial equations, such as $x^5 - 3x + 2017 = 0$, be solved using anything like the quadratic formula, although fourth-degree equations can? The answers to these questions are mostly beyond the actual scope of this class, but they all depend on group theory. Knowing some group theory is at least helpful, and often crucial, in other parts of mathematics. So come get your feet wet (we'll consider taking off socks and shoes, but you shouldn't take any of that too literally.) We'll move fairly quickly and with luck, after doing fundamental concepts (and examples), we'll get to permutation groups, Lagrange's theorem, quotient groups, and maybe the First Isomorphism Theorem.

Homework: Recommended

Prerequisites: None beyond the Mathcamp Crash Course

Related to (but not required for): The Fundamental Group (W1); Finite Fields (W2); Classification of Subgroups of $\mathrm{GL}_2(\mathbb{F}_\ell)$ (W3); Riemann Surfaces (W4)

Real Analysis. (, *Nic Ford*, Tuesday–Saturday)
If you've taken a calculus class that was anything like mine, you probably learned about limits and continuity in a way that might have seemed a bit unsatisfying. Something like "when x gets really close to 3, $f(x)$ gets really close to 6" or "f is continuous because when you draw the graph you never have to lift your pencil off the paper." Descriptions like this can be a nice way to understand the general concept that words like "limit" are trying to express, but they're pretty useless for actually proving anything. How close is "really close"? When, exactly, does $f(x)$ have to be close to 6? How would anyone even begin to write a proof that some function is continuous?

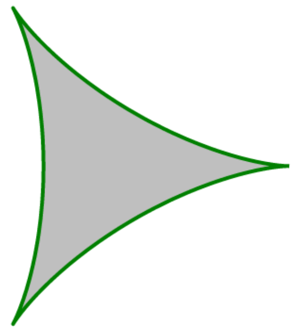
In this class, we'll talk about how to make concepts like these precise, starting with exactly what we mean when we talk about a real number in the first place. We'll start going back through the stuff you learned in calculus class, giving meaning to definitions and proofs to theorems, and when we're done you'll have seen what it really means for a function to be continuous, for a sequence or a series to converge, or for a limit to exist.

Homework: Required

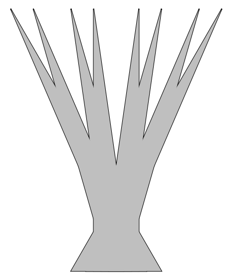
Prerequisites: You should be comfortable with proofs and with the basic language of set theory. In particular, you should know what it means to prove something by contradiction, to prove a statement containing the phrase "if and only if", and what it means to write something like $C = \{x : x \in A \text{ and } x \notin B\}$. It will also help if you know what it means for a set to be "countable". Calculus is not strictly required, but it will be easier to follow if you've been exposed to the ideas of limits and continuity in some form already.

The Kakeya Conjecture. (, Yuval, Tuesday–Saturday)

If you've ever carried a ladder, you know that turning a corner can be very tricky. But in the early 20th century, the Japanese mathematician Sōichi Kakeya tried to understand just *how* tricky this is. In this class, we'll try to answer this question. Along the way, we'll encounter one of the most important open problems in analysis, we'll see how polynomials can magically make our problems disappear, and we'll find out why this nice, simple shape



is much *worse* than this horrifying monstrosity:



Homework: Recommended

Prerequisites: None.

Related to (but not required for): Finite Fields (W2); Fractals and Dimension (W4)

10:10 Classes

Banach-Tarski. (🌶🌶🌶🌶, *Andrew Marks*, Tuesday–Saturday)
The Banach-Tarski paradox is a famous paradox about infinity. It states that a three-dimensional ball can be cut into finitely many pieces which can be reassembled by rotations and translations into two balls each of which has the same size as the original ball! A consequence of the paradox is that not every set in three dimensions can be reasonably assigned a "volume".

We'll start the course by reviewing cardinality, which is the original way Cantor devised for measuring the size of infinite sets. We'll then prove the Banach-Tarski paradox using ideas from linear algebra and set theory. We'll end with some discussion of how the Banach-Tarski paradox is related to Lebesgue measure—the modern way mathematicians have devised for measuring volume.

Homework: Recommended

Prerequisites: Familiarity with matrices.

Intro to Graph Theory. (🌶, Marisa, Tuesday–Saturday)
There is a theorem that says that for any map of, say, countries on your favorite continent, you can color the countries so that any two countries that share a border (not just meet at a point, but actually share some boundary) get different colors, and that the number of colors you will need is no more than 4. (Try inventing a complicated political landscape and coloring: no matter how crazy the scene, you'll always be able to color the map with four colors.)

Mathematicians have been pretty convinced about the truth of this Four Color Theorem since the late 1800s, but despite many false starts, no one gave a proof until 1976, when two mathematicians wrote a very good computer program to check 1,936 cases. (To this day, we have no human-checkable proof.)

In this class, we will definitely not prove the Four Color Theorem. You will, however, prove the *Five* Color Theorem, which is a whole lot shorter (and which was successfully proven by hand in 1890). Along the way, you'll meet many other cool concepts in Graph Theory.

Notice how I said "*you* will prove"? That's because the course will be inquiry-based: I won't be lecturing at all. You'll be working in small groups to discover and prove all of the results yourself!

Homework: Recommended

Prerequisites: None!

Related to (but not required for): Shannon Capacity of Graphs (W2)

Multivariable Calculus Crash Course. (🌶🌶🌶, Mark, Tuesday–Saturday)
In real life, interesting quantities usually depend on several variables (such as the coordinates of a point, the time, the temperature, the number of campers in the room, the real and imaginary parts of a complex number, ...). Because of this, "ordinary" (single-variable) calculus often isn't enough to solve practical problems. In this class, we'll quickly go through the basics of calculus for functions of several variables. As time permits, we'll look at some cool applications, such as: If you're in the desert and you want to cool off as quickly as possible, how do you decide what direction to go in? What is the total area under the bell curve? What force fields are consistent with conservation of energy?

Homework: Recommended

Prerequisites: Knowledge of single-variable calculus (differentiation and integration)

The Fundamental Group. (🌶🌶🌶, Aaron, Tuesday–Saturday)
The fundamental group is an algebraic invariant we can attach to a geometric space which tells us about the holes in that space. For example, the real line has trivial fundamental group because it has no holes, while the circle has nontrivial fundamental group. As an application, we will prove the Borsuk-Ulam theorem, which implies that at any time there are always some two points on exact opposite sides of the earth with the same temperature and barometric pressure. We will use this to show you can always slice any ham sandwich, however lopsided, with a single cut, so that there is the same amount of both pieces of bread and ham on each side of the slice.

Homework: Recommended

Prerequisites: An understanding of open and closed subsets of $\mathbb{R}^n$. Group theory is not a prerequisite though it may be helpful to take simultaneously if you have never seen it before.

Related to (but not required for): Group Theory (W1); Euler Characteristic (W2); Riemann Surfaces (W4)

Wallpaper Patterns. (🌶🌶, Susan, Tuesday–Saturday)
Your wallpaper is a fascinating mathematical object! Well, maybe not your wallpaper in particular—you may not even have wallpaper. However, any repeating pattern that we use to decorate a wall is an example of a mathematical object called a "wallpaper pattern."

In this class we will be discussing the classification of wallpaper patterns. We will explore a beautiful topological argument that shows that there are exactly seventeen distinct types of wallpaper pattern. Expect lots of drawing, cutting, pasting, folding and smershing—this is a hands-on class!

Homework: Recommended

Prerequisites: None.

11:10 Classes

Linear Algebra. (🌶🌶🌶, Don, Tuesday–Saturday)
Linear algebra is the area of math that deals with vectors and matrices. It is one of the most useful methods in mathematics, both within pure math and in its applications to the real world. One could argue that most of what mathematicians (and physicists, and engineers, and economists) do with their time is try to reduce hopelessly complicated non-linear problems to linear ones that can actually be solved. Thus for many applied fields, the most important math to know is not calculus, but linear algebra. Obviously we can't cover all of linear algebra in one week, but this class will give you a basic background, as well as a preview of some of the most important results. We're going to start out on the plane, where linear algebra springs out of geometry. We'll define linear maps and give an intuitive preview of the central themes of linear algebra. Then we'll leave our two-dimensional pictures behind and introduce the more general concepts of vector spaces, linear independence, dimension, kernels, images, eigenvectors, eigenvalues, and diagonalization. (If you don't know what any of these words mean, that's great: come to the class! If you know all of them, then you probably don't need this class.)

Homework: Required

Prerequisites: None

Related to (but not required for): Finite Fields (W2)

Mathcamp Crash Course. (🌶🌶, Tim!, Tuesday–Saturday)
This course covers fundamental mathematical concepts and tools that all other Mathcamp courses assume you already know: basic logic, basic set theory, notation, some proof techniques, how to define and write carefully and rigorously, and a few other tidbits. If you are new to advanced mathematics or just want to make sure that you have a firm foundation for the rest of your Mathcamp courses, then this course is *highly* recommended.

Here are some problems to test your knowledge:

(1) Negate the following sentence without using any negative words ("no", "not", etc.): *"If a book in my library has a page with fewer than 30 words, then every word on that page starts with a vowel."*
(2) Given two sets of real numbers A and B, we say that A *dominates* B when for every $a \in A$ there exists $b \in B$ such that $a < b$. Find two, disjoint, non-empty sets A and B such that A dominates B and B dominates A.
(3) Prove that there are infinitely many prime numbers.
(4) Let $f : A \to B$ and $g : B \to C$ be maps of sets. Prove that if $g \circ f$ is injective then f is injective. (This may be obvious, but do you know how to write down the proof concisely and rigorously?)
(5) Define rigorously what it means for a function to be increasing.
(6) Prove that addition modulo 2017 is well-defined.
(7) What is wrong with the following argument (aside from the fact that the claim is false)?

> **Claim:** On a certain island, there are $n \geq 2$ cities, some of which are connected by roads. If each city is connected by a road to at least one other city, then you can travel from any city to any other city along the roads.
>
> **Proof:** We proceed by induction on n. The claim is clearly true for $n = 1$. Now suppose the claim is true for an island with $n = k$ cities. To prove that it's also true for $n = k+1$, we add another city to this island. This new city is connected by a road to at least one of the old cities, from which you can get to any other old city by the inductive hypothesis. Thus you can travel from the new city to any other city, as well as between any two of

the old cities. This proves that the claim holds for $n = k + 1$, so by induction it holds for all n. QED.

(8) Explain what it means to say that the real numbers are uncountable. Then prove it.

If you are not 100% comfortable with most of these questions, then you can probably benefit from this crash course. If you found this list of questions intimidating, then you should *definitely* take this class. It will make the rest of your Mathcamp experience much more enjoyable and productive. And the class itself will be fun too!

Homework: Required

Prerequisites: None.

Metric Space of Metric Spaces. (🌶🌶🌶🌶, Steve, Tuesday–Saturday)

A *metric space* is just that–a set of *points* together with a notion of *distance*. We demand that this distance notion not be too silly (e.g. the distance from a to b should be the same as the distance from b to a), but otherwise anything goes.

This means we can have very weird metric spaces. Some of these are just oddly shaped; others have "points" that, strictly speaking, aren't points. For instance, we can have metric spaces whose points are *lines* in other, more familiar spaces. And, because metric spaces are fun, we can put metric spaces in metric spaces—there is a nice metric space whose "points" are themselves metric spaces!

This class will begin by studying metric spaces in general, and some properties and constructions of metric spaces—compactness, completions, isometries and homeomorphisms, etc. From there we will move on to the metric space of metric spaces, and have fun playing around with such a strange object. Finally, we'll talk a bit about how this space is used in mathematics—in particular, how it gets used in *algebra* (because when you have a geometry of geometries, obviously that's all about algebra).

Time permitting, we'll look at some other fun metric spaces—for instance, the *pseudoarc*, a "line" that can't be cut into two shorter "lines," or the Urysohn space, a metric space which is "as big as possible" in a precise sense.

Homework: Recommended

Prerequisites: Comfort with proofs

Not Your Grandparents' Algorithms Class. (🌶🌶, Sam, Tuesday–Saturday)

Have you ever found yourself in a panic, wondering what to do if you ended up as a contestant on the not-totally-made-up hit game show *Wait, wait. Who'd I just marry?* This class will study algorithms for making progressively more and more complicated decisions; by the end of the week, we'll have just the right set of ideas to placate any fears about that game show!

It turns out that, in *Wait, wait. Who'd I just marry?*, the decisions you make are complicated. You get new information in various rounds, and the "matchmakers" employed by the show provide less than truthful insight (to add drama and boost ratings, naturally). These decisions are tedious, so we'll start by figuring out how to solve optimization problems that are about as mathematically nice as possible: linear programs. To solve linear programs we'll take an atypical path and study the ellipsoid algorithm: an algorithm with beautiful geometric intuition that's important theoretically, and kind-of sort-of useful practically! Unlike a traditional algorithms class, we'll spend approximately zero time focusing on the concept of "running time" beyond vague notions about whether or not an algorithm is efficient in a formal sense. Instead, we'll emphasize the ideas behind a whole bunch of cool algorithms!

Homework: Optional

Prerequisites: Basic comfort with working in $\mathbb{R}^n$, including set notation. We may briefly use modular arithmetic on day 5.

Weak Separation. (🌶🌶🌶, Kevin, Tuesday–Saturday)
Here's a fairly unremarkable-looking combinatorial definition. Label n points around a circle in order from 1 to n, and let S and T be k element subsets of the points. We say S and T are *weakly separated* if we cannot find a chord with endpoints in $S-T$ and a chord with endpoints in $T-S$ that cross. Symbolically, if S and T are k element subsets of $1, 2, \ldots, n$, then they are weakly separated if there do not exist $a, c \in S-T$ and $b, d \in T-S$ so that $a < b < c < d$ or $b < c < d < a$.

It turns out this harmless definition hides many secrets. In 1998, it was conjectured that if you try to collect as many k element subsets of $1, 2, \ldots, n$ as you can so that any two are weakly separated from each other, you'll always end up with exactly $k(n-k)+1$ of them. This so-called "purity conjecture" took over a decade to prove, and along the way this innocent idea of weak separation made a crucial appearance in the nascent study of cluster algebras, among several other modern topics.

In this class, we'll get a glimpse of what cluster algebras are and how weak separation gets involved in one particularly beautiful cluster algebra. We'll also develop the combinatorics of wonderful objects called plabic graphs to start shedding some light on the purity conjecture.

Homework: Recommended

Prerequisites: None.

1:10 Classes

From the Intermediate Value Theorem to Chaos. (🌶, Beatriz, Tuesday–Saturday)
The Intermediate Value Theorem is a theorem that—despite looking very simple or even obvious—has amazing consequences. In particular, it allows us to prove one of my favourite theorems, called Sharkovsky's Theorem. Sharkovsky's Theorem gives us very valuable information about the periodic points of continuous functions on the real line. In particular, we will show that period 3 implies chaos!

Homework: Recommended

Prerequisites: None.

Related to (but not required for): The Logistics of Zombies: Cobwebs and Chaos (W3)

Prime Numbers. (🌶🌶🌶🌶, Lara, Tuesday–Saturday)
How far out do we have to go to ensure we've found the nth prime? What can we say about the gaps between consecutive primes? What happens if we sum the reciprocals of all the primes?

In this class we'll try to gain insight into these questions and into the mysterious prime counting function $\pi(x)$. Starting right at the beginning with Euclid's proof that there are infinitely many primes, we'll use the technique in this proof to find an upper bound for the nth prime that will shed some initial light on the behaviour of $\pi(x)$. We'll then develop more machinery that will give us sharper bounds on $\pi(x)$ and see the connections it has with sums like $\sum\limits_{p \le x} \log p$. Finally we'll come full circle and give another proof there are infinitely many primes, this time with a better understanding of what 'infinitely many' means here.

Homework: Recommended

Prerequisites: Some understanding about how to play with integrals and series.

Related to (but not required for): Finite Fields (W2)

Problem Solving Discussion. (🌶🌶, Misha, Saturday)
So how do you actually solve olympiad problems?

This is the first day of a discussion class that will be held on several days of camp. We'll pick apart a competition problem, discuss different solutions to it, and try to answer one question: how would you come up with those solutions?

This Saturday, we'll meet to discuss a problem many of you have seen already: problem 6 from this year's Qualifying Quiz.

(See `http://www.mathcamp.org/quiz` for the text of the problem if you haven't seen it.)

Homework: None

Prerequisites: Think a little bit about the problem we're going to discuss.

Problem Solving: Geometric Transformations. (🌶🌶, Misha, Tuesday–Friday)
In this class, we will learn how to use geometric transformations to solve math competition problems. The following topics will be covered:

(1) Translation and central symmetry (Tuesday)
(2) Rotation and reflection (Wednesday)
(3) Similarity and spiral similarity (Thursday)
(4) Inversion (Friday)

In class, we will learn about how to use these transformations, and how to spot when they can be used, by solving problems together. There will be problems left to solve on your own. You won't need to solve these to keep up with the class, but you should, because solving problems on your own is critical to learning problem-solving.

Homework: Recommended

Prerequisites: The equivalent of a high school geometry class.

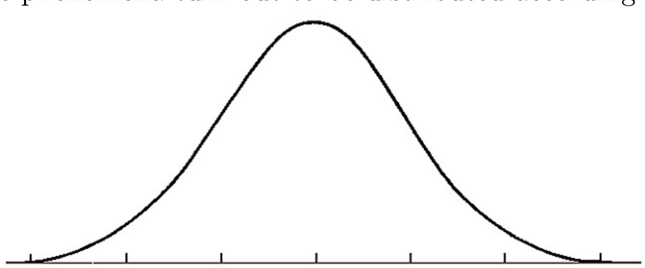
The Bell Curve. (🌶🌶🌶, Mira, Tuesday–Saturday)

> I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the "Law of Frequency of Error." The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason.

Sir Francis Galton, 1889

Human heights; SAT scores; errors in scientific measurements; the number of heads you get when you toss a million coins; the number of people per year who forget to write the address on a letter they mail. . . . what do all of these (and numerous other phenomena) have in common?

Empirically, all of these phenomena turn out to be distributed according to "the bell curve":



The bell curve, known in the 19th century as the "Law of Error", is now usually called the *normal* or *Gaussian* distribution. It is the graph of the function $e^{x^2/2}/\sqrt{2\pi}$ (scaled and translated appropriately). We will see how Gauss derived this function from a completely backward argument – a brilliant leap of intuition, but pretty sketchy math. We'll see how the great probabilist Laplace explained its ubiquity through the Central Limit Theorem. (Maybe you've learned about CLT in your statistics class . . . but do you know the proof?) We'll talk about how the normal distribution challenged the nineteenth century concept of free will. Finally, we'll look at some other mathematical contexts in which the normal distribution arises – it really is everywhere!

Homework: Required

Prerequisites: Integral calculus. (There will be a lot of integrals!)

Cities and Grids. (🌶, *Luke Joyner*, Tuesday–Saturday)
Cities, both real and imaginary, are as complex as anything humans have ever created. On the one hand, they are consummate examples of structure and planning, as profound and intricate as a good fugue. On the other hand, they are the messy artifacts of time and people doing unpredictable things, artifacts of politics and culture and power and resistance and greed. . . they can become characters in our lives, friends even. How can we look at cities to understand them, appreciate them. . . imagine them? This class will be a project-based introduction to cities and urban design, including a dive into the geometry and topology of city grids and networks. It will include some interesting math (mostly in problems to work on, rather than lecture), but will not be exclusively math, because while math bubbles up everywhere when you think about cities and places, it's necessary to look at them in other ways too. So we will walk all those places, sometimes randomly, sometimes with intent, and use what we've learned, and our own life experience, to try and redesign Tacoma's streets by the end of the week. Cause we're here, and we're crazy like that.

Homework: Required (Note: homework for this class will be a mix of math, reading, group activities, and a final project in small groups. This is a one chili class, but will include some open-ended math you can run with, and it will be challenging in other ways.)

Prerequisites: Basic understanding of Graph Theory (Note: drawing skills are not required, but drawing by hand or on the computer will be an important part of our final project; I will lead an optional seminar on drawing techniques for anyone interested on Wednesday or Thursday.)

Colloquia

Using Math to Protect Democracy. (Mira, Tuesday)
Every 10 years, the US has a census to determine the number of representatives each state should get in Congress. Then the legislature in each state comes up with a *districting plan*: a way of splitting the state into the correct number of regions (districts) of equal population, each of which will elect one representative to Congress.

The problem is, the specific choice of districting plan can have a huge effect on the results. For instance, suppose a state of 400,000 people is allotted four representatives. There are two political parties, A and B, and Party B happens to be in power at the time of the census. It does some polling and adopts the following districting plan:

District	A supporters	B supporters
1	85,000	15,000
2	45,000	55,000
3	45,000	55,000
4	45,000	55,000

Now B can count on winning 3 out of 4 districts in the next election, even though 55% of the the voters in the state support A!

This kind of thing happens all the time; both Republicans and Democrats do it. It seems so obviously wrong that you wonder how it can be legal. But it turns out that determining whether a districting plan is “fair” or “neutral” is much more complicated than you might think. Right now, many people, including lawyers, political scientists, computer scientists, and mathematicians are working hard to figure out a solution to the gerrymandering problem before the next census in 2020. You can help! Come to this talk to learn how.

Squaring the circle. (*Andrew Marks*, Wednesday)
The idea of dissecting a set and rearranging its pieces to form another set dates back to the ancient Greeks. One application of this idea is finding formulas for areas of polygons. For example, we can dissect a parallelogram into a triangle and trapezoid and then rearrange them to form a rectangle. This idea can be used to show that the area of a parallelogram is its base multiplied by its height. Building on these ideas, mathematicians have been investigating the general problem of when we can show that two shapes have the same volume by cutting them into congruent pieces.

In two dimensions, it turns out that this always works for polygons—a famous theorem of Wallace-Bolyai-Gerwien states that any two polygons of the same area can be chopped into smaller congruent polygons. This theorem realizes the dream of the ancient Greeks, and has a beautiful proof using pictures.

The analogous problem in three dimensions was one of Hilbert’s famous problems. Breakthrough work of Dehn from 1901 showed that there are two polytopes of the same area which are not dissection congruent. Dehn proved this by introducing an important geometrical invariant called the Dehn invariant.

More recently, mathematicians have been thinking about similar problems for shapes that are not polygons. In 1925 Tarski asked if a disk can be partitioned into finitely many sets which can be reassembled to form a square of the same area. This question became known as Tarski’s circle squaring problem. It remained open until Laczkovich gave a positive solution in 1990. At the end of the talk, we’ll say a little about Laczkovich’s solution, and our recent result joint with Spencer Unger that gives an explicit way to square the circle.

Twenty-Seven Lines. (*David Morrison*, Thursday)
Let $f(x, y, z)$ be a polynomial of degree 3 in 3 variables, and suppose that at every point of the corresponding cubic surface $S = \{f(x, y, z) = 0\}$ the tangent plane to the surface S is well-defined. Suppose we seek straight lines in space which lie completely within S. Under suitable conditions, there are exactly 27 such lines! We will see why this is true, and investigate the intricate combinatorial structure which the lines posses. (For example, each line meets exactly 10 other lines.)

The “suitable conditions” include: (1) thinking carefully about the behavior of S “at infinity” and the possibility that some of the lines may be located “at infinity,” and (2) solving equations over the complex numbers rather than over the real numbers. (Over the real numbers, the conclusion is “at most 27 lines.”)

In spite of these caveats, there exist polynomials with real coefficients having exactly 27 real lines, and we will see an example.

Games People (Don’t) Play. (Steve, Friday)
Let’s play a game! *Hackenbush* is the best game for people who don’t like art—we start with a pretty drawing, and then get rid of it, and the first person who can’t make less art loses.

Hackenbush is a really mathematically interesting game. One easy thing we can say about a game is whether a given player has a winning strategy. But there’s more we can do: it turns out we can assign *numbers* to games, measuring how *much* a given player wins the game, and do arithmetic with these numbers by combining the games in certain ways. Hackenbush is a particularly good example of this. So Hackenbush is *also* the best game for people who like adding numbers.

But we can only get *some* numbers this way. I want more numbers! It turns out the right thing to do here is consider *infinitely long* Hackenbush games. Actually playing an infinite game isn't really something we can do, but they turn out to be very mathematically interesting and useful, and Hackenbush is one of the easiest impossible games to play. So infinite Hackenbush is the best game for people who don't exist.

In this colloquium I'm going to pretend not to exist—come pretend not to exist with me!

Visitor Bios

Andrew Marks. Andrew's research is in descriptive set theory, which studies the line between explicitly definable objects in mathematics and those we can only obtain nonconstructively using tools like the axiom of choice. This field has interesting connections to computability theory, which organizes mathematical structures by the relative difficulty of computing them, and ergodic theory, which studies measure preserving dynamical systems. Recently he has been working on the definability of geometrical paradoxes, like the Banach-Tarski paradox.

Luke Joyner. Luke is most interested in cities and places, both real and imaginary. At Mathcamp, he will teach about cities and ways of looking at cities, including some geometry and topology related to city grids. His research is split between social and historical observations of real places, especially over time, and imaginations of places that do not exist, but could. One time, he walked from Chicago to Pittsburgh.

Nic Ford. Nic has taught at Mathcamp off and on since 2010. His first love is algebraic geometry, but in addition to math research he's spent time working in finance at Jane Street and is about to start doing machine learning research at Google. He also loves hiking and has been working on writing a video game for several years that might get finished someday.

Dave Morrison. As a rising high school junior, Dave Morrison attended a math summer program at San Diego State which helped put him on the path to a lifetime of mathematics. His original research interests in algebraic geometry broadened considerably when string theorists started applying algebraic geometry to the study of the universe at its smallest scales, and particularly when those string theorists made unexpected discoveries about some of Dave's favorite mathematical objects, using reasoning from physics. Dave now works alongside theoretical physicists bringing abstract mathematical tools to bear upon some of their problems, and finding new mathematical problems inspired by their work.