

CLASS DESCRIPTIONS—WEEK 3, MATHCAMP 2017

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9:10 CLASSES

Classification of Subgroups of $GL_2(\mathbb{F}_q)$. (🍷🍷🍷, Aaron, Tuesday–Saturday)

What do the platonic solids have to do with 2 by 2 matrices over finite fields? It turns out that they precisely determine the “exceptional” subgroups of $GL_2(\mathbb{F}_\ell)$. Here, $GL_2(\mathbb{F}_\ell)$ denotes the group of invertible two by two matrices over the finite field \mathbb{F}_ℓ . In this inquiry-based learning class, you will discover for yourself the classification of all subgroups of $GL_2(\mathbb{F}_\ell)$. This classification will slickly interweave elegant ideas from linear algebra, group theory, geometry, and combinatorics to determine all the subgroups of $GL_2(\mathbb{F}_\ell)$.

Homework: Recommended

Prerequisites: linear algebra, group theory, finite fields

Related to (but not required for): Group Theory (W1); Finite Fields (W2); Ring Theory (W2); Symmetries of Spaces (W3); Finite Geometries (W4)

Cubic Curves. (🍷, Mark, Tuesday–Saturday)

A curve in the x, y -plane is called a cubic curve if it is given by a polynomial equation $f(x, y) = 0$ of degree 3. Compared to conic sections (which have degree 2), at first sight cubic curves are unpleasantly diverse and complicated; Newton distinguished more than 70 different types of them, and later Plücker made a more refined classification into over 200 types. However, as we’ll see, by using complex numbers and points at infinity we can bring a fair amount of order into the chaos, and cubic curves have many elegant and excellent properties. One of those properties in particular, which is about intersections, will allow us to prove a beautiful theorem of Pascal about hexagons and conic sections, and it will also let us define a group structure on any cubic curve—well, almost. We may have to leave out a singular (“bad”) point first, but a cubic curve has at most one such point (which may be well hidden; for example, $y = x^3$ has one!), and most of them don’t have any. Cubic curves without singular points are known as elliptic curves, and they are important in number theory, for example in the proof of Wiles’ Theorem (a.k.a. “Fermat’s Last Theorem”). However, in this week’s class we probably won’t look at that aspect at all, and no knowledge of number theory (or even groups) is required. With any luck, along the way you’ll pick up some ideas that extend beyond cubic curves, such as how to deal with points at infinity (using “homogeneous coordinates”), what to expect from intersections, and where to look for singular points and for inflection points.

Homework: Recommended

Prerequisites: Mild use of differential calculus, probably including partial derivatives; complex numbers; some use of determinants. Group theory *not* required.

Related to (but not required for): Riemann Surfaces (W4)

Geometry in Motion. (🔪), Zach Abel, Tuesday–Saturday)

Come see how geometry folds and flexes like paper, de- and re-configures itself like transformers, and swings and hinges like a Strandbeest¹. We'll look at some of my favorite geometric topics that involve some form of motion, and we'll frequently detour to look at recently solved or unsolved research questions in these areas (including some from my own research). Topics will include, but will not be limited to:

- **Flexible polyhedra and polyhedral flattening:** Imagine a polyhedron made with rigid metal faces that is only allowed to fold along the edges. Can such a shape be flexible? (*Yes!*) Can one of these be designed that can be folded flat, say for easy transport or storage? (*No!*) What if we allow folds anywhere, not just at the edges?
- **Polygon dissection:** If I give you an equilateral triangle and some scissors, can you cut it into a few polygonal puzzle pieces that can be rearranged into a square? (*Yes!*) (*Try it! Try using just 4 pieces!*) Is the same true in 3D for, say, a tetrahedron and a cube? (*No!*) What if the pieces are required to be hinged to each other as they reconfigure?
- **Mechanical linkages:** Picture a movable mechanical device made with rigid metal rods connected at rotatable hinges. If we affix a pen to one of the bars, what shapes can such a mechanism trace? A circle is easy with just one segment acting as a compass, but is a straight line segment possible? (*Yes!*) If we make the mechanism complicated enough, can we make a sketch of your face? (*Yes!*) What if edges are forbidden from crossing each other during the motion?

Homework: Recommended

Prerequisites: None.

Problem Solving: Probabilistic Method. (🔪), Tim!, Tuesday–Saturday)

“When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth.” — Sherlock Holmes. This is a good way to way to solve crimes, and a good way to solve math problems. If you need to prove that some Mathcamp staff is a spy, calculate the probability that a randomly-chosen staff is a spy. If the probability is greater than 0, then you can safely conclude that a traitor walks among us (even though you might not know who it is).

Perhaps the most surprising thing about this method is that it is actually useful! In fact, the principle above is all you need to solve this problem:

- Prove that there exists a graph with 1,000,000 vertices such that every set of 40 vertices has a pair of adjacent vertices and a pair of nonadjacent vertices.

One might be worried that a probability-based proof to this problem might not be air-tight because it leaves things to chance, but fear not — even though the proof uses probability, the final result is true with absolute certainty.

In addition to this strategy, we'll see more probability-based approaches to solving problems (even problems whose statements often don't reference probability at all!). Part of the class time will consist of campers working on problems in groups and presenting solutions.

Homework: Required

Prerequisites: None.

¹If you don't know what this is, do yourself a favor: https://www.youtube.com/watch?v=LewVEF2B_pM

The Logistics of Zombies: Cobwebs and Chaos. (☺☺☺, Beatriz, Tuesday–Saturday)

A zombie outbreak has been detected at Math Camp. Fearing for their lives, Math campers are trying to predict how the zombie population will evolve in time. After doing some research, they chose the logistic map as their model. The logistic difference equation is given by

$$x_{n+1} = rx_n(1 - x_n)$$

When $r = 3.57$, chaotic behaviour arises, but there are also what we call “windows of stability”, in which for some values of $r > 3.57$ some stable orbits occur, giving rise to oscillating populations.

Homework: Recommended

Prerequisites: Mathematica Workshop

Related to (but not required for): From the Intermediate Value Theorem to Chaos (W1)

10:10 CLASSES

A Crash Course in Axiomatic Probability. (☺☺☺, Sam, Tuesday–Saturday)

This class will be a brisk walk through some of the most fundamental topics in probability. We’ll start from Kolmogorov’s axioms and build our way through formal notions of independence, conditional probability, and random variables. By the end of the course, we’ll have sufficient tools to prove some high-level theory and asymptotic results! We will mostly focus on discrete probability—so if you haven’t seen much calculus that’s fine—but we will touch briefly on continuous random variables. Throughout the class, we’ll look at a few fun “applications” of probability. Typically, these will be applications to other areas of mathematics (like Graph Theory!).

Homework: Required

Prerequisites: You should know the following words and corresponding symbols: (finite and countable) union, intersection, complement, and partition. Calculus will help for 20 minutes of this course. We’ll also do one or two examples from graph theory, where knowing very basic terminology will be helpful (edge, vertex, clique, independent set, complete graph), but these are just fun asides and can be safely ignored.

How to Pronounce “Lucas”. (☺☺☺, Misha, Tuesday–Saturday)

The French mathematician Édouard Lucas is kind of awesome. In between studying Fibonacci numbers, finding patterns in Pascal’s triangle, and inventing an algorithm to look for Mersenne primes and perfect numbers, he invented the paper-and-pencil game Dots and Boxes, as well as the Tower of Hanoi puzzle (which he marketed under an anagram name, like Voltaire or Voldemort: N. Claus de Siam).

He’s also probably tied with Euler for being the mathematician with the most mispronounced name. To pronounce his last name, say “Lu” to rhyme with “flew” and “cas” as though you’re saying “car” in an exaggerated British accent.

This class will be an overview of many of the cool things Lucas did. Each day will be a separate, mostly independent topic in combinatorics, number theory, or the gray area in between.

Homework: Optional

Prerequisites: None.

Hyperreal Numbers. (☺☺, Don & Tim!, Tuesday–Saturday)

In the beginning, there were integers, \mathbb{Z} , and they were good. We could add and subtract and multiply them, and for a time, that was enough.

Then, came the rational numbers, \mathbb{Q} , and the big spaces between the integers were filled in, and it was good. Now we could also divide by numbers other than zero, and between any two numbers, there were infinitely more. And for a time, that was enough.

Then, came the real numbers, \mathbb{R} , and even the tiniest spaces between the rational numbers were filled in, and it was good. Now, finally, our number line was complete, and for a time, that was enough.

And then, came the hyperreal numbers, ${}^*\mathbb{R}$, AND THEY WERE TOTALLY BANANAPANTS. Numbers got shoved in between real numbers where they had no business going, numbers started getting unboundedly infinite, and the number line basically exploded. And finally, we said, “enough is enough.”

In this class, we’ll learn about the crazy explosion of numbers known as “The Hyperreals.” We’ll learn how to build them, what their properties are, what they look like, and even what they can tell us about math that isn’t totally bananapants.

Homework: Recommended

Prerequisites: None.

Pseudo-Telepathy via Representation Theory of Finite Groups. (🍷🍷🍷, Jalex, Tuesday–Saturday)

Your friends Alice and Bob are experimental quantum physicists. They claim that they have established high-fidelity quantum entanglement between their labs. This is a valuable resource: it can be used for things like teleporting quantum states and secure distribution of cryptographic keys. You’d like to verify or falsify their claim. If you were an experimentalist, this would be straightforward: you could just take the particles into your lab and do some experiments—these would be only as complicated as the ones Alice and Bob have already done.

However, you’re a mathematician; you’d prefer not to get your hands dirty. Instead, you’ll call them up on the telephone to ask them some questions about systems of linear equations over finite fields. If they answer all of your questions correctly, you can be pretty sure that their claim is correct. How is this possible? Answering this question will lead us to talk about the foundations of quantum mechanics, a planar algebra for equations in finitely generated groups, and the representation theory of the extraspecial groups of order p^5 .

(Based on joint work with Andrea Coladangelo. If we have time at the end of the class, we’ll discuss some low-hanging open questions!)

Homework: Recommended

Prerequisites: Linear Algebra (be able to prove that a vector space is classified up to isomorphism by its dimension) Group Theory (be able to define specific groups using generators and relations)

Related to (but not required for): Group Theory (W1); Linear Algebra (W1); Planar Algebras (W2); Cryptography, and How to Attack It, Week 2 (W3)

Ramsey Theory. (🍷, Lynn Scow, Tuesday–Saturday)

If you color each integer between 1 and 100 either red or blue, it may not be surprising that you are able to find a sequence of the form

$$a, \quad a + d, \quad a + d + d$$

where all three of these numbers are red, or all three are blue. Our next question is: did we need all one hundred numbers for this result? Results like these fall within the realm of Ramsey theory. In this course we will become familiar with different Ramsey-type problems and the tools with which to build solutions. We will leave plenty of room to experiment with small examples, and paper and colored pencils will be provided! Ramsey theory can be thought of as the theory of existence of patterns/order/regularity in large and complex structures. The cool thing about the theory is that it doesn’t just say that you that these patterns probably exist, it guarantees that you could find them!

And you are guaranteed to have fun doing so! Actually, that is not guaranteed by the theory, but it is probably true. Bring your love of counting!

Homework: Optional

Prerequisites: addition/multiplication mod n , definitions of vertices and edges from graph theory, some ideas from counting like binomial

11:10 CLASSES

Functions of a Complex Variable, Week 2. (☞☞, Mark, Wednesday–Saturday)

This is a continuation of last week's class. If you would like to join the class now, check first whether that would be realistic, by talking to Mark and/or by looking at last week's problem sets.

Homework: Recommended

Prerequisites: Functions of a Complex Variable week 1, or equivalent.

Related to (but not required for): Riemann Surfaces (W4)

Quadratic Field Extensions. (☞☞, Lara, Wednesday–Saturday)

In this class we'll figure out why fields of the form $\mathbb{Q}\sqrt{d}$ are important and what we can say about them. We'll see what it means to be an integer in such a field and work with rings of such integers. We'll explore primes and unique factorisation, but instead of ring elements, we'll see that we should—and will—play with ideals instead. One question we'll answer is: which integers can be written as the sum of two squares?

Homework: Recommended

Prerequisites: Ring theory. The material from Susan's week 2 class is definitely sufficient. Talk to me if you're unsure.

Related to (but not required for): Finite Fields (W2)

Symmetries of Spaces. (☞☞, Apurva, Wednesday–Saturday)

This course is about Lie (pronounced Lee) groups in low dimensions. Lie groups (or matrix groups) are groups which arise as symmetries of Euclidean spaces with extra structures.

In this course we'll study matrix groups $O(2)$ = isometries of \mathbb{R}^2 , $O(3)$ = isometries of \mathbb{R}^3 , $SL_2(\mathbb{R})$, $SL_2(\mathbb{C})$ = symmetries of \mathbb{C}^2 , $O(1,3)$ = symmetries of the space-time \mathbb{R}^{1+3} , $Spin(3)$ = symmetries that give rise to electron spin = unit quaternions, etc.

We'll try to see how these correspond to observables and transformations in physics like spin, time dilation and space contraction.

Homework: Recommended

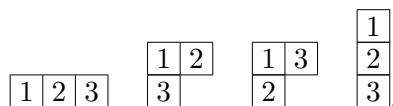
Prerequisites: *Groups:* you should know what a group is and what homomorphisms are.

Linear algebra: you should know the definitions/statements of the following terms: linear transformations, eigenvalues, eigenvectors and the Cayley-Hamilton theorem.

Related to (but not required for): Wallpaper Patterns (W1); Using quaternions to describe symmetries of Platonic solids (W1); Ring Theory (W2); Classification of Subgroups of $GL_2(\mathbb{F}_q)$ (W3); Special Relativity (W4)

Young Tableaux. (☞☞, Kevin, Wednesday–Saturday)

A *standard Young tableau* (SYT) is a way to fill a portion of a grid of boxes with the first n positive integers so that rows and columns are increasing. For example, here are all the SYT with three boxes:



Notice that the shape in the middle corresponds to two different SYT. A natural question to ask, then, is how many different SYT there are of a given shape.

In this class, we'll study the wonderful world of combinatorics associated with counting the number of SYT. We'll see a beautiful bijection between pairs of SYT of the same shape and permutations. We'll also meet the celebrated hook length formula of Frame, Robinson, and Thrall, which was the subject of recent controversy for trivializing a USAMO problem from 2016.

Homework: Recommended

Prerequisites: None.

1:10 CLASSES

Ancient Greek Mathematics. (♫), Yuval, Tuesday–Saturday)

You probably know that Pythagoras proved the Pythagorean Theorem, and you may have heard that Euclid proved the infinitude of the primes. But did you know that Eudoxus basically invented the Dedekind Cut definition of the real numbers 2500 years before Dedekind, or that Archimedes was basically doing calculus 1500 years before Newton and Leibniz invented it? You may have heard that Archimedes yelled “Eureka” in the bathtub when he discovered the law of buoyancy that controls when things float and when they sink, but did you know that he subsequently used this idea to prove arguably the most intricate theorem before the 17th century? And you may have heard about the controversy around Euclid’s famous “Parallel Postulate,” but what does this postulate actually say?

In this class, we'll be learning about Ancient Greek math, from two separate perspectives: what they knew, and how they thought about it. In both cases, we'll see some surprises: they knew much more than you probably think they did, and they thought about math in a fundamentally different way from how we think about it today.

Note: This class has required homework, since I will be assigning you a (fairly small) amount of reading each day, which we will discuss the following day. In all cases, we will be reading bona fide Ancient Greek math; if you don't put in the time to carefully read and think about these, you will get very little out of the class the following day.

Homework: Required

Prerequisites: None.

Constructive Logic. (♫♫♫), Anti Shulman, Tuesday–Saturday)

In the early 20th century, a group of mathematicians staged a revolt against the prevailing orthodoxy of mathematical practice, and in particular against the unrestricted use of the law of excluded middle (“everything is either true or false”). Known as “constructivists”, they insisted that any *proof* in mathematics should be a *construction*, and that non-constructive “existence proofs” should not be considered proofs at all. At the time, the revolt failed; but the late 20th and early 21st centuries have experienced a revival of constructive mathematics, based no longer on dogmatic arguments but on pluralist and pragmatic grounds, such as the increasing importance of computability and the need for flexibility to describe many different kinds of mathematics.

In this class we'll learn the basics of constructive logic, and what you can and can't do while avoiding the law of excluded middle. We'll see that some parts of mathematics (like elementary number theory) look just about the same constructively, some require small modifications here and there (like calculus), and others look completely different (like set theory). At the end we'll explore a few of the magical things that constructive logic makes possible, such as all functions being continuous, all existence being computable, and the use of true nilpotent infinitesimals instead of epsilon-delta limits.

Homework: Recommended

Prerequisites: None.

Cryptography, and How to Attack It, Week 2. (☞☞), Linus, Tuesday–Saturday)

If you didn't take Crypto Week 1 and want to take this, talk to Linus.

A random selection of more advanced topics in cryptography. The focus shifts somewhat from attacking bad crypto to “things Linus thinks are cool.” Topics that might show up:

- Why the NSA has been encouraging everyone to use one specific prime p for Diffie-Hellman
- Alice and Bob might want to date each other. Or maybe not. Alice is shy: if she loves Bob, then she *cannot* let Bob find out unless Bob loves her too. Bob is similarly shy. How can they find out whether the love is mutual?
- What does it mean for a sequence to “look random”?

Homework: Recommended

Prerequisites: Introductory group theory – enough to say “ $(\mathbb{Z}/5\mathbb{Z}) \times (\mathbb{Z}/12\mathbb{Z})$ is an abelian group isomorphic to $\mathbb{Z}/60\mathbb{Z}$ ”

Related to (but not required for): Finite Fields (W2); Pseudo-Telepathy via Representation Theory of Finite Groups (W3)

Division Rings, Week 1. (☞☞☞), Susan, Tuesday–Saturday)

We know that rings have addition, subtraction, and multiplication. But where does division fit into this picture? Dividing, in the context of a ring, is essentially multiplying by a multiplicative inverse. But these inverses don't always exist. In the commutative setting, the field of fractions construction allows us to add inverses to a ring that did not previously have them.

In the noncommutative setting things are...not so nice. In this class, we'll see an example of a noncommutative domain that cannot be embedded into any division ring. We'll look at Ore domains, the closest noncommutative analogue of an integral domain, and see how we can expand it into an Ore Division Ring of Fractions. We'll see twisted Hilbert polynomials, which we can transform into Laurent division rings. We'll also explore the idea of inversion height, and encounter the mystery of what happens to the number three!

Homework: Required

Prerequisites: Ring Theory

The Mathematics of Voting. (☞), Mira & Ari, Tuesday–Saturday)

Note: this class is a Superclass! It meets for two periods a day, plus up to one hour of TAU (though we may not always need that full hour). On the other hand, this class does not assign homework: you will spend a large part of your time in class solving (fun) problems, but you won't need to do any work outside of class.

Everyone knows that elections involve choices. But it turns out that the most important choice is one that most voters don't even think about: it is the choice of *voting system*, including what information gets collected from the voters and how this information is used to determine the winner (or set of winners). For instance, do the voters get to list only their first choice of candidate or do they get to rank all the candidates? Do we split the country into geographic districts each of which elects a single representative, or should everyone in the country have a say in the composition of the legislature as a whole? And of course, once the votes are in, what algorithm do we use to select the winner(s)?

Voting theory is the study of voting systems and their properties, which are often completely un-intuitive and pathological. The choice of voting system can have a huge effect on the outcome of an

election, so this topic is obviously important from a political point of view. But it is also really fun and cool math!

On Tuesday and Wednesday in class, we will introduce the basics of voting theory, explore different single-winner systems, and prove some depressing theorems showing that no voting system can have all the nice properties you want it to have. If you've seen some voting theory before, much of this will be review.

After that, we will move on to less standard topics: gerrymandering (Thursday), apportionment (Friday), and partisan symmetry (Saturday). During some of the classes, you will be doing computer simulations and/or working with real US data using a “geographic information system” (i.e. software for manipulating and analyzing maps). Even if you've taken voting theory at Mathcamp (or elsewhere) in the past, these topics are likely to be new to you.

Wednesday depends on Tuesday, but otherwise the topics are more or less independent, so you can pick and choose which days of the class you want to attend. However, the schedule given above is subject to change, so if you really want to see a particular topic, talk to Mira or Ari.

Homework: None

Prerequisites: None.

COLLOQUIA

The Party Problem and Ramsey. (*Lynn Scow*, Tuesday)

The Party problem states that at any party with at least 6 people, there will be 3 among these 6 who all know one another, or 3 among these 6 who are mutually strangers. Actually, it's not a problem! It's fun to meet new people, and it's fun to catch up with old friends! What is the minimal number of people (n) that need to be at the party to find 4 who all know one another, or 4 who are mutually strangers? In this talk I'll give an introduction to what are called Ramsey numbers (the numbers n above) and mention some extensions of this idea to other situations.

The Icosian Game. (*Misha*, Wednesday)

Can a knight visit all 64 squares of a chessboard in 63 jumps, then come back to the start? What if we ask the same question for a 4×4 board? What if we're instead walking around the vertices of a dodecahedron?

In this colloquium, we will figure out when the answer to such a question is definitely “yes”, and when it is definitely “no”. In between, there will be a disturbingly large range of cases where we can only say “I don't know”. But that's okay, because I'll also explain why, if you could always solve this problem easily, then you'd be able to win a million dollars, steal billions of dollars, and break all of mathematics as we know it.

Down the Rabbit Hole. (*Anti Shulman*, Thursday)

Come enter a world where everything you think you know about mathematics is in doubt: where not every subset of a finite set has to be finite; where a real number need not be either positive, negative, or zero; and where the Intermediate Value Theorem and Extreme Value Theorem can fail to hold. This is the land of constructive mathematics, where we deny the law of excluded middle (“everything is either true or false”) and forbid proof by contradiction (“if something isn't false, it must be true”).

Why would we do such a thing? (Other than to annoy your calculus teacher, I mean.) It turns out that like Alice's rabbit hole, ours is inhabited not only by weird creatures, but also by magic.

In constructive mathematics, every function is continuous, everything that exists can be found by a computer, and we can do calculus with true infinitesimals rather than epsilon-delta limits. Arguably, constructive mathematics reflects the “real world” even better than classical mathematics does!

(This colloquium will be just a taste of constructive mathematics; to learn more about it, come to my class this week.)

Opinionated Primes. (*Zach Abel*, Friday)

VISITOR BIOS

Adina Gamse. Adina was a camper in 2005, 2006 and 2007 and a JC in 2009. She is currently a postdoc studying symplectic geometry and topology at the University of Toronto. When not doing math(s) she can sometimes be found on the flying trapeze.

Anti Shulman. Anti discovered his love of mathematics at Mathcamp 1996 as a camper, and met his future spouse at Mathcamp 2001 as a JC. Subsequently he spent over a year of his life at Mathcamp, as a JC, a mentor, an Academic Coordinator, and a sysadmin; this year he is excited to be an invited visitor for the first time, as well as a parent of two potential future Mathcampers. When not visiting Mathcamp, he works on revolutionizing the foundations of mathematics and physics using formalized constructive homotopy type theory. Set theory, pencil and paper, and classical logic are so 20th century; nowadays the basic objects of mathematics are ∞ -categories, we can formalize it using a computer, and we can design new kinds of logic whenever we need them.

Ari Nieh. Ari started coming to Mathcamp in 1996 and can't seem to stop. He teaches writing and rhetoric for mathematicians. He is also a freelance bass-baritone singer, specializing in Medieval, Renaissance, and Baroque music. He does not know very much about tapirs, and would very much appreciate it if you approached him with facts about tapirs during his visit.

Jeff Hicks. Hi! I'm Jeff—I was a camper in '08-'09, and have JC'd and Mentor'd at Mathcamp in '10, '11, '13-'16. I'm currently a graduate student at UC Berkeley studying symplectic geometry, which is a geometry naturally related to physics. I also enjoy thinking about knots, topological spaces, music and salsa dancing!

Lynn Scow. Lynn is interested in problems in the intersection of Ramsey theory and model theory. In model theory, we ask what a mathematical object can “say” in a given language. In Ramsey theory, we pose a certain combinatorial problem and ask if a solution is guaranteed to exist. Some problems have solutions depending on who (which object) is posing the problem. Lynn has also been involved in groups for women and underrepresented minorities in math and likes to encourage young people to see themselves in mathematics. She is learning to play the French horn.

Zach Abel. Zach's research lies somewhere between the intersection and union of discrete geometry and theoretical computer science. He often thinks about the ancient and beautiful art of origami from an algorithmic perspective, exploring just how powerful (or useless) computers can be at folding-related tasks relating to robotics, nanomanufacturing, architecture, and (of course) recreational paper folding. He is also a mathematical artist, transforming everyday objects like binder clips or playing cards into intricate works of art (look out for his activities during camp!). Zach is a former Mathcamper ('03 and '04) and Mentor ('13 and '16).⁴

⁴Zach writes the worst footnotes.