CLASS DESCRIPTIONS—WEEK 4, MATHCAMP 2017

Contents

9:10 Classes

Bernoulli Numbers. (🌶🌶🌶, Lara, Tuesday–Saturday)
The Bernoulli numbers $\{B_m\}$ are a mysterious sequence of rational numbers that arise naturally in many places. For example, in the Taylor series for trig functions and when studying Pascal's triangle. Once we know what Bernoulli numbers are, we'll use their generating function to better study them and to gain detailed information about series and insight into understanding the Riemann zeta function. We'll prove the von Staudt-Clausen theorem which will tell us for each prime, how pB_m behaves (mod p) and see the beautiful consequences this theorem has.

Homework: Recommended

Prerequisites: Integration and series. To understand everything in the class, you'll need to know a bit of complex analysis, but you should follow most of the class without this.

Elliptic functions. (🌶🌶🌶 → 🌶🌶🌶🌶, Mark, Tuesday–Saturday)
Complex analysis, meet elliptic curves! Actually, you don't need to know anything about elliptic curves to take this class, but they will show up along the way. Meanwhile, if you like periodic functions, such as cos and sin, then you should like elliptic functions even better: They have two independent (complex) periods, as well as a variety of nice properties that are relatively easy to prove using some complex analysis. Despite the name, which is a kind of historical accident (it all started with arc length along an ellipse, which comes up in the study of planetary motion; this led to so-called elliptic integrals, and elliptic functions were first encountered as inverse functions of those integrals), elliptic functions don't have much to do with ellipses. Instead, they are closely related to cubic curves, and also to modular forms. If time permits, we'll use some of this material to prove the remarkable fact that

$$\sigma_7(n) = \sigma_3(n) + 120\sum_{k=1}^{n-1}\sigma_3(k)\sigma_3(n-k),$$

where $\sigma_i(k)$ is the sum of the i-th powers of the divisors of k. (For example, for $n = 5$ this comes down to

$$1 + 5^7 = 1 + 5^3 + 120[1(1^3 + 2^3 + 4^3) + (1^3 + 2^3)(1^3 + 3^3) + (1^3 + 3^3)(1^3 + 2^3) + (1^3 + 2^3 + 4^3)1],$$

which you are welcome to check if you run out of things to do.)

Homework: Recommended

Prerequisites: Functions of a Complex Variable (in particular, Liouville's theorem and the residue theorem)
Related to (but not required for): Advanced Complex Analysis

How To Get Rich Quick By Playing The Lottery. (🌶, Aaron, Friday)
In 2004, Massachusetts wanted to make more money from the state lottery, so they devised a new game called cash winfall. In this game, instead of having only one large prize, there were many smaller prizes. When more people bought lottery tickets, the big prize "rolled down" into the smaller prizes, making these smaller prizes more valuable. In fact, so much money rolled down that on certain days, it was possible to buy lottery tickets for $2 with an expected return of $5. We'll tell the story of how three groups of people exploited this lottery, and explain the math behind it, including a surprising appearance of the Fano plane.
Homework: None
Prerequisites: None.

The Range of a Rotor Walk. (🌶🌶🌶, *Yuval Peres*, Saturday)
Pólya (1920) showed that a simple random walk on a lattice is recurrent (returns to the starting point with probability 1) in one and two dimensions, but is transient (not recurrent) in higher dimensions. It is an open question whether the same holds for a rotor walk, where successive exits from each vertex follow a counter-clockwise sequence. We show that any rotor walk on the two dimensional lattice will visit at least order $t^{2/3}$ sites in the first t steps. This is conjectured to be sharp when the initial directions of the rotors are uniformly random. We prove a shape theorem for the uniform rotor walk on the comb graph, showing that the size of the range in that case is of order $t^{2/3}$ and the asymptotic shape of the range is a diamond. (Joint work with Laura Florescu and Lionel Levine).
Homework: None
Prerequisites: None.

The Traveling Salesman Problem: Recent Breakthroughs. (🌶🌶🌶🌶, Sam, Tuesday–Saturday)
Suppose you wanted to visit all of the 559 Starbucks in Washington in as short a time as possible; this problem can be phrased as finding a Hamiltonian tour of minimum cost on a graph (i.e. finding the shortest giant cycle visiting each of the 559 Starbucks exactly once). This example is one of the most fundamental applications of the Traveling Salesman Problem[1] (TSP). Starbucks aside, the TSP is an important problem in mathematics and computer science that has taunted researchers for decades. It is hard (in a formal sense) to come up with an algorithm that exactly solves the TSP, and so instead one searches for approximation algorithms: clever ways of finding approximate solutions with theorems bounding those approximate solutions as "not too bad."

The state of the art approximation algorithm dates back to 1976. In the 40+ years since, considerable effort has gone into trying to improve on that approach. In this course, we'll discuss some of the newest and most important breakthroughs. Doing so will involve involve a hodgepodge of ideas from graph theory, combinatorics, and CS theory. We'll also see how exciting

$$1.4999999999999999999999999999999999999999999999999996$$

is!

Finally: the goal of this class is to read a recent, exciting paper about the TSP. In class we'll work through the motivation, background, and key ideas in the paper. By the end of the course, however, you should be set to read and internalize a pretty awesome research paper!
Homework: Recommended

[1] OK, not really.

Prerequisites: You should feel fairly good about graph theory, including the phrases: complete graph, Eulerian graph, Hamiltonian graph, tree, and matching. You should also know, be willing to learn on the fly, or take on faith that a graph is Eulerian if and only if it is connected and every vertex has even degree.

Topological Tic Tac Toe. (🌶, Beatriz, Thursday)
The game of Tic-Tac-Toe is famously boring: it has a simple perfect strategy, and if two players play this strategy, the game is guaranteed to end in a draw every time. The problem is that the topology of the game board – a flat square – is too simple to allow for sufficiently many possible moves to make the game interesting. So let's allow the game board to have a more interesting topology. For example, what happens when we play Tic-Tac-Toe on a torus? A Klein bottle? A Möbius band? Or some other 2-dimensional surface? Is there still a perfect strategy? How many different first moves are there? Can two Tic-Tac-Toe games on different surfaces be equivalent?

Homework: None

Prerequisites: None

Trig Functions by Hand. (🌶🌶, Misha, Wednesday)
When you learn about trig functions, you typically memorize a few of their values (for 30° or 45°, say) and if you want to know any of the other values, you get pointed to a calculator.

Has that ever seemed unsatisfying to you? If so, take this class, in which we'll see that finding some of these values is as easy as solving polynomials, and approximating all of them is as easy as multiplication. If time allows, we'll learn how to compute inverse trig functions, and also how to quickly find lots of digits of π.

Homework: None

Prerequisites: Be familiar with the formula $e^{ix} = \cos x + i \sin x$.

Unique Factorization Domains. (🌶🌶, *Alfonso & Kevin*, Tuesday–Saturday)
You know that every integer can be written as a product of primes in a unique way. But, are you sure this is true? It turns out that proving the uniqueness part is not easy at all, even though we all take this fact for granted since kindergarden!

In this class you learn how to prove this rigorously, and you will also study other "number systems" where the same result is true, and where it fails. Sometimes the uniqueness part fails. Sometimes some numbers cannot be written as product of primes at all! Pathological examples are delicious.

This is an IBL class, where you will be doing most of the work yourselves, while we help you. There will be a daily handout, and we will expect that you finish some problems during TAU if you could not do them during class (since otherwise you will get lost on the next day).

Homework: Required

Prerequisites: The class will be easier if you know what a ring is and what an ideal is. You can still take the class if you do not know this, as long as you are willing to work hard (perhaps with our help during TAU).

What's It Like to Live in a Hyperbolic World? (🌶, Linus, Tuesday)
The video game HyperRogue takes place in a hyperbolic plane. I learned what the hyperbolic plane is like from playing this game. Come to the computer lab and play this game for one hour.

Homework: None

Prerequisites: None.

Related to (but not required for): Coloring the Hyperbolic Plane (W2)

10:10 Classes

Combinatorial Gems. (🌶🌶, *Alfonso & Kevin*, Tuesday–Saturday)
Come join us in a discovery journey of some of combinatorics' most precious gems. Each day we will propose a different problem and we will guide your exploration of it. We promise that every problem has an unexpected, mind-blowing, beautiful pattern at the end. If you are already familiar with some of the topics, ask Kevin or Alfonso if they are right for you. Do not look these problems up before class, or your risk spoiling the pleasure of discovering the answers yourself.

- *Tuesday:* **Wythoff's Game.** We have a plate with blueberries and strawberries in front of us. We take turns eating them. In your turn, you may eat as many berries as you want as long as they are all of the same kind (at least one) or exactly the same number of berries of both kinds (at least one). Then it is my turn. The player who eats the last berry wins. Will you beat me?
- *Wednesday:* **Treacherous Chords.** Draw a circle. Draw N points on it. Join every pair of points with a line segment. In how many regions is the circle divided? If you compute this for N from 1 to 5 and then make a guess for $N = 6$, your guess will almost certainly be wrong.
- *Thursday:* **Pascal Parity.** Which numbers in Pascal's triangle are odd?
- *Friday:* **Nim.** Let's play a game. We have various plates with chocolates. We take turns eating them. In your turn, you may eat as many chocolates as you want as long as they are all on the same plate. Then it is my turn. The player who eats the last chocolate wins. Will you beat me?
- *Saturday:* **Error-correcting Codes.** You place a coin on each square of a chessboard, some face up and some face down, any way you like. Then you tell Alfonso what your favourite square is. Alfonso will then flip one single coin of his choice. Then Kevin enters the room, and looking only at the chessboard, he can guess what your favourite square was. What was their trick? It is the same strategy that allows them to play 20 questions with a liar.

Homework: Optional
Prerequisites: None.

How to Define the Square Root. (🌶🌶🌶, Apurva, Tuesday–Saturday)
If a function on real numbers $f : \mathbb{R} \to \mathbb{R}$ is not injective it is usually not possible to define an inverse, no matter how well behaved the function is. On the other hand, if a *complex differentiable* function on complex numbers $f : \mathbb{C} \to \mathbb{C}$ is not injective in many cases it is possible to define an inverse function by creating a new Riemann surface by gluing pieces of the complex plane together.

In this course we'll learn how to do this and in the process see how Riemann surfaces naturally come up and learn about their ramified coverings.
Homework: Recommended
Prerequisites: Complex Variables: you should be comfortable with the definition of a complex differentiable function.
Related to (but not required for): Euler Characteristic (W2)

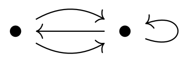
Math Writing Workshop. (🌶, Beatriz, Tuesday–Saturday)
This class is suitable for all levels. What you learn from this class will be tailored to your needs. Whether you need help with the general basics of mathematical writing or writing your Mathcamp project, you will benefit from this class.

Writing a solution to a problem set is very different from writing a report or an article to be published in a journal or a blog. Good writing is an essential part of mathematics that is very often overlooked and very difficult to master. We will talk about what constitutes good mathematical writing and do some hands-on activities. The goal of this class is to help improve your ability to write rigorous and elegant proofs. The emphasis will be not on how to figure out the answer but on how to turn your solution into a rigorous argument that is clear, concise, and effective.

Homework: Required

Prerequisites: None

Quivers. (🌶🌶🌶, *Asilata Bapat*, Tuesday–Saturday)
In algebra and representation theory, directed graphs like to go by the fancy name of *quivers*. Here is an example.



Given a quiver, we play the following game: we put a vector space at each vertex, and a corresponding linear map at each arrow, and then try to classify how many fundamentally different such configurations we can construct.

This simple idea leads us surprisingly quickly to some classic (solved and unsolved!) problems in algebra, as well as some topics of current research. In this class we'll see how to do algebra on quivers, and explore lots of concrete examples. Along the way we'll invoke some tools from ring theory, and get a sneak preview into the world of homological algebra.

Homework: Recommended

Prerequisites: Linear algebra (you should know about kernels and cokernels of linear maps, and direct sums of vector spaces). Familiarity with rings would be useful but not necessary.

Related to (but not required for): Linear Algebra (W1); Ring Theory (W2)

Riemann Surfaces. (🌶🌶🌶🌶, Aaron, Tuesday–Saturday)
Riemann surfaces form a beautiful breeding ground for ideas from many fields of math such as algebraic geometry, number theory, symplectic geometry, dynamics, and complex analysis. A Riemann surface is a surface which looks like the complex numbers if you zoom in around any point. For example, the sphere and the torus are Riemann surfaces. In the first couple days, we will use Riemann surfaces to give slick proofs of theorems from complex analysis, like Liouville's theorem and the open mapping theorem. By the end of the course, we will obtain a bound on the number of maps from any compact Riemann surface (other than the sphere or a torus) to itself.

Homework: Required

Prerequisites: complex analysis, some familiarity with point-set topology may be helpful but is not required.

Related to (but not required for): Group Theory (W1); The Fundamental Group (W1); Functions of a Complex Variable, Week 1 (W2); Euler Characteristic (W2); Cubic Curves (W3); Functions of a Complex Variable, Week 2 (W3)

11:10 Classes

Advanced Complex Analysis. (🌶🌶🌶, Yuval, Wednesday–Saturday)
This class is mostly a continuation of Mark's Functions of a Complex Variables class, in which we'll be talking about some of my favorite theorems in Complex Analysis. The main goal is to understand various geometric properties of analytic functions: in particular, it will turn out that analytic functions are "conformal", meaning that they preserve all local geometric structure. This means that we expect analytic functions to be extremely rare and special, and we will prove that indeed they are: in fact, we will write down a list of all analytic functions from the unit disk to itself. Finally, we will see the

Riemann Mapping Theorem, which tells you that everything I said above is wrong; analytic functions are actually ubiquitous, and we can find them *everywhere*.

Along the way, we'll see many other useful and beautiful theorems, which will allow us to count zeroes, compute integrals, and find maxima.

Homework: Recommended

Prerequisites: Functions of a Complex Variable (both weeks)

Related to (but not required for): Elliptic Functions

Discrete Derivatives. (🌶🌶, Tim!, Wednesday–Saturday)
Usually, we define the derivative of f to be the limit of $\frac{f(x+h)-f(x)}{h}$ as h goes to 0. But suppose we're feeling lazy, and instead of taking a limit we just plug in $h = 1$ and call it a day. The thing we get is kind of a janky derivative: it's definitely not a derivative, but it acts sort of like one. It has its own version of the power rule, the product rule, and integration by parts, and it even prefers a different value of e. We'll take an expedition into this bizarre parallel universe. Then we'll apply what we find to problems in our own universe. We'll talk about Stirling numbers and solve difference equations and other problems involving sequences.

Homework: Recommended

Prerequisites: Calculus (derivatives)

Finite Geometries. (🌶🌶🌶, *J-Lo*, Wednesday–Saturday)
There are infinitely many points in the Euclidean plane. Just think, all those points that no human being will ever use—what a waste! Suppose instead that we limit our geometrical landscape to having finitely many points—how much geometry could we reproduce? Can we meaningfully talk about distance? Lines? Circles? Angles? Trigonometry? Come to play around in these surprisingly small worlds and rewire your intuition for what various geometric concepts "look like"—and along the way, discover Elliptic Curve Cryptography, the state of the art in NSA-certified internet security protocols.

Homework: Required

Prerequisites: None.

Related to (but not required for): Finite Fields (W2); Classification of Subgroups of $\mathrm{GL}_2(\mathbb{F}_\ell)$ (W3)

Fractals and Dimension. (🌶, Steve, Wednesday–Saturday)
The usual three dimensions are fun and all, but they get kind of boring after a while. One way to liven things up is to add more dimensions; billion-dimensional shapes are probably super cool! But you know what I like even more than big numbers? *Wrong numbers.* I want a two-and-a-half-dimensional shape. Or a π-dimensional shape. Or a shape with a decent number of dimensions, but for terrible reasons.

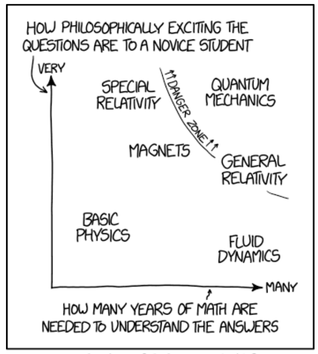
It turns out we can make this happen! The answer is *fractals*, a particularly weird and beautiful kind of shape. Fractals crop up throughout mathematics in all sorts of weird ways, and have lots of fascinating properties *besides* just being dimensionally weird. This class will be about what dimensions are, why fractals have silly numbers of them, and how awesome fractals are.

Homework: Optional

Prerequisites: None

Related to (but not required for): The Kakeya Conjecture (W1)

Special Relativity. (🌶🌶, Lotta, Wednesday–Saturday)
According to xkcd, Special Relativity is very philosophically exciting and doesn't require that much mathematical background.



WHY SO MANY PEOPLE HAVE WEIRD IDEAS ABOUT QUANTUM MECHANICS

Special relativity is about things that go really really fast. Almost as fast as the speed of light! When things go fast, classical physics breaks down and unintuitive things start happening. Things appear to become shorter and they experience time differently. Even stranger, two people moving at different speeds relative to each other may disagree on the order that events happened. In this class, we will work from the two postulates of special relativity and derive all these strange effects.

Homework: Optional

Prerequisites: None, but being familiar with classical mechanics will help you appreciate the awesomeness.

1:10 Classes

Decomposing and Factoring. (🌶🌶, Marisa, Tuesday–Saturday)
Some math friends of mine once called up a professional football league (the American kind of football) and offered to design them a better schedule, and the league took them up on it.[2] This is how I imagine that conversation going.

> Mathematicians: Your schedule is a mess. We'd like to make it better.
> League: ??
> Mathematicians: Trust us. We have design theory.
> League: Fine. We have eight teams, and we want four games per week, for seven weeks, and no team can play two games on the same day, and every team will play every other team.
> Mathematicians: Here is a 1-factorization of K_8. Boom.
> League: Neat. Here is $1,000.

In this class, we'll be talking about design theory: from Kirkman's discovery in 1847 of what would (over Kirkman's strong objection) later be called Steiner triple systems, through perfect matchings in 1891 (when Petersen published his paper "The Theory of Regular Graphs"), to systems of distinct representatives and Hall's Theorem in 1935, and all the way up to 2012 when Alpach's Conjecture about cycle decompositions of K_n was finally proved.

Homework: Recommended

Prerequisites: Intro Graph Theory or equivalent; really, all you need to know is what K_n and C_n mean.

[2] http://www.nytimes.com/2001/02/03/arts/what-good-is-math-an-answer-for-jocks.html

Division Rings, Week 2. (🌶🌶🌶🌶, Susan, Tuesday–Saturday)
Continuation of Week 1!

Homework: Required

Prerequisites: Division Rings (Week 1)

Logic Puzzles. (🌶, dun dun dun DON, Tuesday–Saturday)
In a world where all people are either liars or truthtellers, suppose Armond says, "Burt is a liar," and Burt says, "Armond and Charlie are the same type of person." What can you tell me about Charlie?

A group of 4 campers and 4 staff need to cross a river, via a boat that can take 1 or 2 people. If the campers ever outnumber staff at a location, those staff will get covered in magic marker. Can they all make it across the river without anyone getting drawn on?

The ""Schedule" Making "Committee"" decides to meet at a particular day during the first four weeks of camp; it's Wednesday, Thursday, or Sunday of Week 1, Friday or Saturday of Week 2, Tuesday or Thursday of Week 3, or Tuesday, Wednesday, or Friday of Week 4. Two "members" of the "committee," identified as M and N, remember this list of dates. Further, M knows the week for the meeting, and N knows the day of the week for the meeting. They then make the following statements:

- M: "I know that N doesn't know the date."
- N: "I didn't know it before, but now I do."
- M: "Now I too know the date."

When is the "meeting"?

In this class, we'll look at puzzles like these, figure out how to solve them, and then go even further, studying the underlying logic behind the puzzles, and ultimately figuring out how to write them.

Homework: Recommended

Prerequisites: None

Problem Solving Discussion. (🌶🌶🌶, Misha, Saturday)
So how do you actually solve olympiad problems?

This is day 2 of a class in which we'll pick apart a competition problem, discuss different solutions to it, and try to answer one question: how would you come up with those solutions?

You don't need to have been to day 1 of this class (which happened in Week 1), or to remember what happened if you were there. You *do* need to think about the following math problem, which comes from the 2012 IMO:

Let k and n be fixed positive integers. In the liar's guessing game, Amy chooses nonnegative integers x and N with $0 \leq x \leq N$. She tells Ben what N is, but not what x is. Ben may then repeatedly ask Amy whether $x \in S$ for arbitrary sets S of integers. Amy will always answer with yes or no, but she might lie. The only restriction is that she can lie at most k times in a row. After he has asked as many questions as he wants, Ben must specify a set of at most n positive integers. If x is in this set he wins; otherwise, he loses.

Prove that:

(a) If $n \geq 2k$ then Ben can always win.
(b) For sufficiently large k there exist $n \geq 1.99k$ such that Ben cannot guarantee a win.

Homework: Required

Prerequisites: Think about the problem in this blurb. (This is also what I mean by the required homework: this is a one-day class, so the required homework must be done by the time you get to class.)

Problem Solving: Diophantine Equations. (🌶🌶🌶, Misha, Tuesday–Friday)
This is a class about solving equations which end with words such as "where x, y, and z are integers". Sometimes this involves reasonable techniques like "consider the prime factors of both sides". Sometimes this involves bizarre techniques like "take the equation modulo 19".

We'll solve problems in class together, and then I'll leave you more problems to solve for homework, and hopefully you will end up walking away with an answer to the question "How do you know to use 19?"

Homework: Recommended

Prerequisites: You should be comfortable with modular arithmetic.

Trail Mix. (🌶 → 🌶🌶🌶, Mark, Tuesday–Saturday)
Is your mathematical hike getting to be a bit much? Would you like a break with a class that offers a different topic each day, so you can pick and choose which days to attend, and that does not carry any expectation of your doing homework? If so, why not come have some Trail Mix? Individual descriptions of the topics for the five days can be found below.

- Exploring the Catalan Numbers (Tuesday)

  What's the next number in the sequence $1, 2, 5, 14, \ldots$? If this were an "intelligence test" for middle or high schoolers, the answer might be 41; that's the number that continues the pattern in which every number is one less than three times the previous number. If the sequence gives the answer to some combinatorial question, though, the answer is more likely to be 42. We'll look at a few questions that do give rise to this sequence (with 42), and we'll see that the sequence is given by an elegant formula, for which we'll see a lovely combinatorial proof. If time permits, we may also look at an alternate proof using generating functions.

  Prerequisites: None, but at the very end generating functions and some calculus may be used.
- Integration by Parts and the Wallis Product (Wednesday)

  Integration by parts is one of the only two truly general techniques for finding antiderivatives that are known (the other is integration by substitution). In this class you'll see (or review) this method, and encounter two of its applications: How to extend the factorial function, so that $(\frac{1}{2})!$ ends up making sense (although the standard terminology used for it is a bit different), and how to derive the famous product formula

  $$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots$$

  which was first stated by John Wallis in 1655.

  Prerequisites: Basic single-variable calculus.
- The Prüfer Correspondence (Thursday)

  Suppose you have n points around a circle, with every pair of points connected by a line segment. (If you like, you have the complete graph K_n). Now you're going to erase some of those line segments so you end up with a tree, that is, so that you can still get from each point to each other point along the remaining line segments, but in only one way. (This tree will be a spanning tree for K_n). How many different trees can you end up with? The answer is a surprisingly simple expression in n, and we'll go through a combinatorial proof that is especially cool.

  Prerequisites: None
- The Jacobian Determinant and $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ Friday)

  How do you change variables in a multiple integral? In the "crash course" in week 1 we saw that when you change to polar coordinates, a somewhat mysterious factor r is needed. This is a special case of an important general fact involving a determinant of partial derivatives. We'll see how and roughly why this works; then we'll use it to evaluate the famous sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (You may well know the answer, but do you know a proof? If so, do you know a proof that doesn't require Fourier series or complex analysis?)

Prerequisites: Multivariable calculus (the crash course is plenty); some experience with determinants.

- A Tour of Hensel's World (Saturday)

  In one of Euler's less celebrated papers, he started with the formula for the sum of a geometric series:

  $$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

  and substituted 2 for x to arrive at the apparently nonsensical formula

  $$1 + 2 + 4 + 8 + \cdots = -1\,.$$

  More than a hundred years later, Hensel described a number system in which this formula is perfectly correct. That system and its relatives (for each of which 2 is replaced by a different prime number p), the p-adic numbers, are important in modern mathematics; we'll have a quick look around this strange "world".

  Prerequisites: Some experience with the idea of convergent series.

Homework: None

Prerequisites: Vary by day - see individual "mini-blurbs"

Colloquia

Many Campers Split Pizza. (*Asilata Bapat*, Tuesday)

How can we split a circular pizza among n campers and make sure everyone gets an equal share? The usual way is to slice it by diameters at equal angles, so that the number of pieces is a multiple of n.

But this is not the only easy solution! In this talk we will discover some other surprising ways to solve this problem, with the help of some Euclidean geometry, some calculus, and some pictures.

Rational Tangles. (Tim!, Wednesday)

Imagine four people standing in a circle holding the ends of two ropes. A caller gives them instructions to move around, twisting the ropes into a tangle. After some time, a magician enters the room. The caller tells the magician a single rational number: with just that information, and without looking at the ropes, the magician gives instructions for how to undo the knot.

This magic trick is just one of the things we'll learn about rational tangles, which are a systematic approach, proposed by Conway, to describing knots. In this colloquium, we'll see how to build complicated knots from simple pieces, and how to use group theory to untangle the secrets of their hidden structure.

Covering Spaces and Square Dancing. (*Alfonso Gracia-Saz*, Thursday)

A covering space of a topological space (for example a surface or a curve) is what you get when you "unfold" it. For instance, you can unfold a circle entirely and get a line, or unfold it partially and get ... another circle. You could also unfold a torus, and get another torus, a cylinder, or a plane.

You can unfold almost anything, like a Klein bottle or $GL(n)$, but you cannot unfold a Hawaiian ring. Interestingly, when we unfold a topological space, paths that started and ended at the same point end up wandering in space, creating something called monodromy.

Covering spaces have many applications in daily life, such as Lie groups, quantum field theory, or square dancing. What does square dancing have to do with covering spaces? Usual square dances have 8 dancers, but there is a 12-dancer variant called "hexagon dancing". SD callers often go through a lot of trouble to explain the rules for hexagon squares, and are usually at a loss to figure out when a choreography that resolves in regular squares will resolve in hexagon squares. Their lives would be so much simpler if they simply said "hexagon squares are a triple cover of the quotient of regular square dancing by a $\mathbb{Z}/2\mathbb{Z}$ symmetry and a choreography that resolves in regular squares also resolves in hexagons if and only the path of every boy composed with the inverse of the path of his girl has winding number around the center congruent to 0 mod 3." In other words, this is a real-life application: a question posed by dancers that algebraists managed to solve.

This talk will be illustrated with shiny animations, courtesy of Ryan Hendrickson. No topology or dancing knowledge will be assumed.

Hydras. (Susan, Friday)

The Lernean Hydra was a legendary monster with many heads, poisonous breath, and an all-around bad attitude. The hero Heracles was sent to kill the beast, but found that whenever he cut off one of its heads, two would grow back in its place. What's a hero to do? We will attempt to slay a different kind of Hydra. In the Hydra game, we start with a rooted tree (our Hydra), and in each turn, we remove a "head". On the n^{th} turn, n new Hydra heads will grow back in its place. Heracles's story has a happy ending—he was able to kill the Lernean Hydra with an extremely clever plan of attack. What sort of cleverness do we need to kill our Hydra? Come and find out!

Visitor Bios

Alfonso Gracia-Saz. Alfonso is an Assistant Teaching Professor at the University of Toronto. He has taught math in many places, including San Quentin State Prison in California, but Mathcamp remains his favourite. He enjoys using IBL and guiding students to discover theorems and prove them by themselves. After all, nobody should be deprived of the fun part.

His latest favourite toy is an Instant Pot, where he can make both a perfect risotto and a perfect paella. He enjoyed Arrival more than La La Land. He is a fan of Percy Jackson, he sees algebraic topology in square dancing, and his favourite board game is Castles of Burgundy.

Asilata Bapat. Asilata is a math postdoc who studies representation theory and algebraic geometry. She is very excited to be back at Mathcamp, where she has been a mentor several times. She loves to travel, solve puzzles with friends, cook foods from around the world, and practice on her very-recently-bought guitar. She will be accompanied by her husband Anand, who is also a mathematician.

J-Lo Love. J-Lo (Mathcamper in '09 and '10, JC in '14 and '15) is a Canadian who was born and raised in Tokyo, and is currently a second-year PhD student at Stanford University studying algebraic number theory. He's a huge music geek who plays violin, French horn, guitar, and bass guitar, participated in Contrapositones and Kernel in all of his years at Mathcamp, and gets unreasonably excited by the interplay of mathematics and music. He also dabbles in applied physiotopology and is often attributed with the original proof the the camper number of the trefoil knot is 1.[3]

[3][Love 2010]. There is some controversy over this attribution, as an alternate proof, [Smith 2010], was discovered

Yuval Peres. Yuval Peres is a Principal Researcher at Microsoft Research near Seattle. Previously he was a Professor at the Hebrew University, Jerusalem and at the University of California, Berkeley. He has co-authored books on Brownian motion, Markov chains, Probability on Networks, Fractals, and Game Theory. He will deliver a plenary lecture at the Mathematics Congress of the Americas in 2017. He has advised 21 Ph.D. students. In 2016 he was elected to the U.S. National Academy of Sciences. His favorite quote is from his son Alon, who at age 5 asked a friend: "Leo, do you have a religion? You know, a religion, like Jewish, or Christian, or Mathematics?"

independently and contemporaneously.