

# CLASS CLUSTERS FOR MATHCAMP 2017

## 0. ABOUT CLUSTERS

The Mathcamp schedule provides a plethora of options, and we give you the freedom to choose among them as you will.<sup>1</sup> However, you may find the academics this summer more rewarding if you're able to look back in five weeks and reflect that you've learned *a lot* about something. To this end, we've constructed *clusters*: sets of classes with a common theme.

It is completely fine to only take part of a cluster. Note that fewer than half of the classes this summer are in a cluster; just because a class doesn't live in a cluster doesn't mean that it's not awesome!

## 1. ANALYSIS

Analysis is the study of limits, especially in the context of real and complex analysis; it can be thought of as the foundation of calculus, although it goes beyond that and is a fundamental subject in its own right. The classes in this cluster develop the theory of limits in various aspects. If you are interested in the behavior of functions in the real and complex numbers, you may like the classes in this cluster.

*Classes in this cluster:*

- **Real Analysis.** (👉👉, Nic Ford, Week 1)
- **The Kakeya Conjecture.** (👉👉👉, Yuval, Week 1)
- **Systems of Differential Equations.** (👉👉, Mark, Week 2)
- **Functions of a Complex Variable.** (👉👉, Mark, Weeks 2, 3)
- **Axiomatic Probability.** (👉👉, Sam, Week 3)
- **Riemann Surfaces.** (👉👉👉, Aaron, Week 4)
- **Advanced Complex Analysis.** (👉👉, Yuval, Week 4)

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<sup>1</sup>Unfortunately, we don't have any time turners to hand out.

## 2. FIELDS AND RINGS

Fields and rings generalize our notion of number: their elements can be added, subtracted, multiplied, and (sometimes) divided. In this cluster, you'll see how factorization into primes breaks, how to measure how badly it breaks, and how to fix it. You'll learn how to build rings that do absolutely anything you want. If you want to expand your understanding of what a number is, check out these classes!

*Classes in this cluster:*

- **Finite Fields.** (☺☺☺, Aaron, Week 2)
- **Ring Theory.** (☺☺☺, Susan, Week 2)
- **Quadratic Field Extensions.** (☺☺☺, Lara, Week 3)
- **Division Rings.** (☺☺☺☺, Susan, Weeks 3, 4)
- **Factorization Domains.** (☺☺, Alfonso and Kevin, Week 4)

## 3. GEOMETRY

The classes in this cluster develop mathematical techniques for studying shapes, as well as specific interesting shapes or classes of shapes. Sometimes these are shapes we are familiar with. Other times, we study stranger things, like triangles with fewer than 180 degrees or shapes with a fractional number of dimensions, or even stranger properties. If you enjoy playing with shapes, you might like the classes in this cluster.

*Classes in this cluster:*

- **Metric Space of Metric Spaces.** (☺☺☺☺, Steve, Week 1)
- **Vertex Transitive Polytopes.** (☺☺, Viv, Week 2)
- **Coloring the Hyperbolic Plane.** (☺☺, Ina, Week 2)
- **Geometry!.** (☺☺, Zach Abel, Week 3)
- **Ancient Greek Mathematics.** (☺☺, Yuval, Week 3)
- **Finite Geometries.** (☺☺☺, J-Lo, Week 4)
- **Fractals and Dimension** (☺☺, Steve, Week 4)

## 4. LOGIC

Logic is the study of reasoning. Sometimes this means learning to reason carefully in strange situations to arrive at surprising conclusions; sometimes this means using “reasonable” axioms to build weird, fascinating objects that shouldn’t exist; and sometimes it means studying the way we reason mathematically itself. The classes in this cluster take what you already know about proofs in mathematics and do weird things with it.

*Classes in this cluster:*

- **Banach-Tarski.** (👉👉👉👉, Andrew Marks, Week 1)
- **Set Theory.** (👉👉👉, Steve, Week 2)
- **Constructive Logic.** (👉👉, Anti Shulman, Week 3)
- **Logic Puzzles.** (👉, Don, Week 4)

## 5. MATH IN THE REAL WORLD

Math is all around us, and just about any math you meet can, hypothetically, be applied to something out in the real world. If hypothetically isn’t good enough for you, check out the classes in this cluster, where you’ll apply mathematical thinking to understanding cities, brains, and democracy!

*Classes in this cluster:*

- **Urban Planning.** (👉, Luke Joyner, Week 1)
- **Math and Brains.** (👉👉, Nora Youngs, Week 2)
- **Voting Systems.** (👉, Ari Nieh and Mira, Week 3, Superclass)

## 6. NUMBER THEORY

Which are rarer, prime numbers or perfect squares? Which numbers can be written as the sum of four cubes? Is there a prime number which yields a remainder of 4 when divided by 7, 9, and 11?

Questions like these, about the *arithmetic properties* of natural numbers, fall under the umbrella of “number theory.” Despite its specific focus, number theory interacts with practically every area of mathematics, and ranges from the concrete to the incredibly abstract. Classes in this cluster will teach both number theory on its own, and its interactions with other fields—in particular, analysis (both real and complex) and combinatorics.

*Classes in this cluster:*

- **Prime Numbers.** (👉👉👉👉, Lara, Week 1)
- **Analytic Number Theory.** (👉👉👉👉, Djordje Milicevic, Week 2)
- **Riemann and Series.** (👉👉, Lara, Week 2)
- **How to Pronounce “Lucas”.** (👉👉, Misha, Week 3)
- **Bernoulli Numbers.** (👉👉, Lara, Week 4)

## 7. PROBLEM SOLVING

Classes in this cluster will teach you problem-solving techniques used in math competitions. Expect to solve many math problems along the way!

*Classes in this cluster:*

- Problem Solving: Geometric Transformations (👉, Misha, Week 1)
- Problem Solving: Probabilistic Method (👉, Tim, Week 3)
- Problem Solving: Diophantine Equations (👉👉, Misha, Week 4)
- Problem Solving Discussion (👉👉, Misha, Saturdays)

## 8. VISUAL ALGEBRA

Much of algebra involves manipulating strings of equations, turning one equation into another until you've reached your desired result. In these classes, you'll get a chance to actually see your algebra, dealing with patterns, strings, and loops. If you want to see (really see!) some surprising results, check these classes out!

*Classes in this cluster:*

- **Wallpaper Patterns.** (👉, Susan, Week 1)
- **Planar Algebras.** (👉👉, Noah Snyder, Week 2)
- **Quivers.** (👉👉, Asilata Bapat, Week 4)