CLASS DESCRIPTIONS—WEEK 4, MATHCAMP 2024

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9:10 am classes

Wallpaper patterns (week 2 of 2). (Susan) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes
Continuation of Wallpaper patterns week 1.

Homework: Optional.

Class format: Lecture.

Prerequisites: Week 1 of this class.

Field extensions and Galois theory (week 2 of 2). (Mark) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes
This is a continuation of last week's class. If you'd like to join, talk to someone who took that class and/or to Mark to get an idea of what has been covered so far.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Week 1 of this class, or equivalent knowledge.

Error-correcting codes and sphere packing. (Kailee) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes
Imagine you want to send a message to your friend, but your carrier pigeon is on vacation, so you use a computer instead. If this happens perfectly, well, that's great! However, what if the computer messes up your message, maybe by deleting or corrupting some parts of it? Is there any way for your friend to know what your message really was? This will all be discussed as we learn some basic theory and bounds about error-correcting codes, including the Hamming bound, linear codes, and perfect codes. Spoiler alert for the end of the class: we'll see how our understanding of error-correcting codes gives us bounds about the sphere packing problem!

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: Linear algebra (matrix multiplication, linear independence), Combinatorics (counting rules, binomial coefficients).

Markov triples, continued fractions, and < 3. 🌶🌶🌶 (Misha) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Ask any continuedfractionologist, "Which real number has the worst continued fraction expansion?" I guarantee you that all of them will say that it's the golden ratio: $\frac{1+\sqrt{5}}{2}$. The golden ratio has a "Lagrange number" of $\sqrt{5}$, which is the smallest Lagrange number possible.

Less well-known is the second-worst real number: $1+\sqrt{2}$, with a Lagrange number of $\sqrt{8}$. Practically nobody knows that $\frac{9+\sqrt{221}}{10}$ is the third-worst real number, with a Lagrange number of $\sqrt{221}/5$. It turns out that there is an infinite sequence of bad real numbers whose Lagrange numbers approach, but always remain less than, 3.

To find the numbers in this infinite sequence, we will study Markov triples: solutions to the second-most important quadratic Diophantine equation in three unknowns.[1] That equation is

$$x^2 + y^2 + z^2 = 3xyz,$$

and we will understand its solutions by hanging them up on an infinite binary tree called the Markov tree. To prove the connection between Markov triples and continued fractions, we will hang many other decorations on infinite binary trees: fractions, binary strings, and linear fractional transformations, just to name a few. Along the way, we will catch a few glimpses of the uniqueness conjecture: a problem about Markov triples and irrational numbers that has remained open for 111 years.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Some modular arithmetic is the only thing you'll actually need to come in knowing. If you've seen continued fractions before (say, in Athina's class), or linear algebra, or group theory, or analysis, there will be a few moments at which I'll be able to point out something cool that you'd need that background to appreciate, but you'll be fine without them. (And if you've seen a lot of these topics, you won't be bored, because what we're doing with them is very different.)

Paradoxes in probability and statistics. 🌶 (*Jane Wang*) $\boxed{\text{TW}}\Theta\text{FS}$ 50 minutes

Between 2000 and 2013, the median US wage increased by 1%. But over that same time interval, median incomes for every educational group (e.g. college graduate, some college, etc.) decreased. How was this possible? Probability and statistics are full of examples such as this one that can challenge our thinking and run contrary to our intuition. In this short course, we will explore, discuss, and grapple with some of these paradoxes, ranging from the theoretical to real-world examples. Along the way, we will build tools and intuition for thinking about probability and statistics.

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: Prior experience with probability helpful, but not required.

Fair division using topology. 🌶🌶 (*Jane Wang*) $\text{TW}\boxed{\Theta\text{FS}}$ 50 minutes

How can we fairly divide a cake among multiple people when each person values frosting, edges, etc. differently? We can answer this question using tools from topology, the study of continuous maps and properties that are preserved under continuous deformation. It turns out that topology has many surprising applications to fields ranging from economics to combinatorics to data science. In this course, we will explore some tools from topology and then survey some applications to problems of fair division (of cake, necklaces, rent, and more!).

Homework: Optional.

[1] Pythagorean triples, of course, are solutions to the most important such equation: $x^2 + y^2 = z^2$.

Class format: Interactive lecture.

Prerequisites: Prior experience with topology might be helpful, but is not necessary.

Totally positive, dude. 🌶🌶🌶 (*Mia Smith*) $\boxed{\text{TW}}\Theta$FS 50 minutes

If the 80's were all "totally radical" and "totally wicked", then the 2020's are "totally positive" (at least for algebraic combinatorialists, that is). So what IS total positivity and why do we care?? Consider the following two questions:

How many tests are needed to determine if a $2 \times n$ matrix is totally positive?

How many tests are needed to determine if an $n \times n$ matrix is flag-totally positive[2]?

If we decide to naively check all minors, then $\binom{n}{2}$ and $(n-1)(n+2)/2$ tests are required. However, we can actually do much better than that. And the solution lies in combinatorics! By cleverly recasting the problems in terms of triangulations and wiring diagrams, we can quickly argue that only a linear number of tests are needed. And intriguingly, we'll see that the arguments for both examples are strikingly similar...

As it turns out, both examples can be encapsulated by the same combinatorial object, a cluster algebra. Introduced in 2002 by Fomin and Zelevinsky, cluster algebras revolutionized algebraic combinatorics, unifying numerous questions about positivity with one shared structure. Now that's totally ~~wicked~~ positive!

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: Linear algebra (familiarity with bases, matrices, and determinants).

10:10 am classes

Commutative algebra/algebraic geometry (week 2 of 2). 🌶🌶🌶🌶 (Mark) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

This is a continuation of last week's class. If you'd like to join, talk to someone who took that class and/or to Mark to get an idea of what has been covered so far.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Week 1 of the class, or equivalent knowledge.

What is diagonalization? 🌶🌶 (Della) T$\boxed{\text{W}\Theta}$FS 50 minutes

You may know that there are uncountably many real numbers, there can't be a set of all sets that don't contain themselves, the axioms of arithmetic are incomplete, it's impossible to define truth, there's no algorithm to decide whether a program halts, and the Y combinator finds fixed points of functions. But you probably don't know that these are secretly all the same theorem! We'll explore what they have in common, and then use the language of category theory to express a generalization of all of them.

Homework: Optional.

Class format: Lecture.

Prerequisites: You should have seen at least two of the results listed above.

[2] A matrix is totally positive if all minors are positive and flag-totally positive if particular minors are.

Hyperreal numbers. (Krishan) TWΘ$\boxed{\text{FS}}$ 50 minutes

Intuitively, the derivative $f'(x)$ should be the slope of the line between $f(x)$ and $f(x+\varepsilon)$ where ε is an "infinitely small" number. Unfortunately this ε is not a real number. But what if we could extend the real numbers to add an "infinitely small" ε as well as infinite numbers? Even better, what if we could do this while preserving the fundamental characteristics of $\mathbb{R}$? We can do all this and more! In this class, we will be constructing the hyperreal numbers from scratch. Along the way we'll learn about ultrafilters as well as some neat theorems and ideas from logic.

Homework: None.

Class format: Mix of lecture and group work.

Prerequisites: None.

The first black hole: Schwarzschild spacetime. (Laithy) T$\boxed{\text{WΘFS}}$ 50 minutes

We will explore the fascinating world of black holes through the lens of General Relativity, Einstein's theory of gravity. In 1915, Einstein formulated a theory of spacetime and gravity in which he described gravity as an intrinsic geometric property of spacetime called curvature that is caused by mass and energy present in spacetime. He asserted what's now called the Einstein's field equations which give a precise relation between the curvature of spacetime and the matter present, hence describing precisely how massive objects cause a distortion in the fabric of spacetime, which we perceive as gravity.

Einstein's field equations are very complex and highly nonlinear; understanding the solutions to these equations and their behaviour is still an active area of research in both mathematics and physics. Nonetheless, a particular solution to Einstein's equations, called the Schwarzschild spacetime, was discovered soon after the appearance of General Relativity. It's the simplest model of a universe that describes what we now know as a black hole. More specifically, this spacetime represents a universe that is a vacuum except for a single static non-rotating black hole.

The goals of this class is to study the Schwarzschild spacetime and its many properties. Assuming the spacetime is static and spherically symmetric, we will reduce Einstein's vacuum equations to a manageable system of ordinary differential equations and derive the geometry of the Schwarzschild spacetime. We will explore Birkhoff's theorem, which states that the Schwarzschild spacetime is the unique spherially symmetric solution containing a black hole, reinforcing the importance of the Schwarzschild solution in the study of black holes.

Homework: Recommended.

Class format: Lecture.

Prerequisites: Linear algebra (specifically, inner products and bilinear forms on vector spaces) and basic calculus. Special relativity is recommended but not required.

Algebraic complexity. (*Yuval Wigderson*) T$\boxed{\text{WΘFS}}$ 50 minutes

I originally considered calling this class "How to prove P $\neq$ NP using middle-school algebra". But eventually I decided that "Algebraic complexity" is a better name.

Algebraic complexity is a branch of theoretical computer science that studies how difficult it is to compute polynomials. For example, the polynomial $P(x,y) = x^3 + 4x^2y + xy^2 + 2y^3$ naively requires 13 arithmetic operations to compute, since there are 13 $+$ and $\cdot$ signs in the expression

$$P(x,y) = x \cdot x \cdot x + 4 \cdot x \cdot x \cdot y + x \cdot y \cdot y + 2 \cdot y \cdot y \cdot y.$$

However, if we note that

$$P(x,y) = (x+y)^3 + y(x-y)^2 = (x+y)\cdot(x+y)\cdot(x+y) + y\cdot(x-y)\cdot(x-y),$$

we can reduce the number of arithmetic operations to 10. How much further can we go with this example? More interestingly, can we understand *which* polynomials can be efficiently represented, and which ones cannot? This question—which is at the heart of algebraic complexity—can be studied from the perspectives of computer science, algebra, algebraic geometry, invariant theory, representation theory, and many others, leading to an extremely rich mathematical theory which we will explore in this class.

Also, by the end of the class, we will see three different ways to (potentially) prove that $\mathrm{P} \neq \mathrm{NP}$ using middle-school algebra.

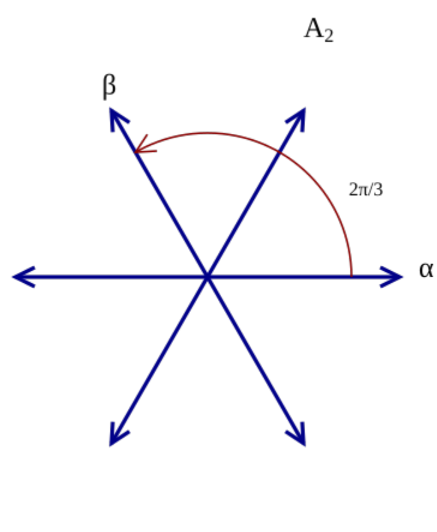
Homework: Recommended.

Class format: Interactive lecture.

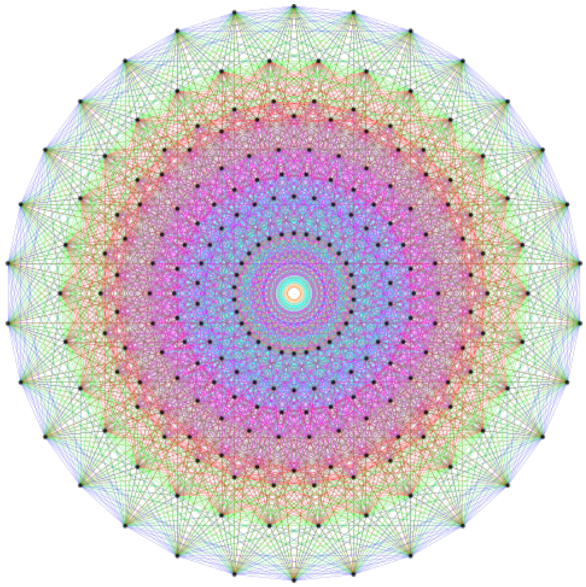
Prerequisites: It would be helpful, but not strictly necessary, to know what determinants are and to have a rough sense of what the P vs. NP question asks. It would also be helpful to have seen big-O notation.

Root systems. 🌶️🌶️ (Kevin) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

Root systems are arrangements of vectors in Euclidean space with lots of symmetries and lots and lots of applications (group theory, Lie groups and Lie algebras, representation theory, combinatorics, algebraic geometry, mathematical physics, and more!). Here's a simple example, called A_2:



Here's a less simple example, called E_8:



Even though they appear in so many different areas of math, root systems themselves are very concrete objects. We'll study their combinatorial properties, construct a bunch of examples like the ones above, and learn about their beautiful classification by Dynkin diagrams.

Homework: Recommended.

Class format: Lecture.

Prerequisites: None.

11:10 AM CLASSES

A 3D reconstruction problem. 🌶️🌶️ (Tim!) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

In 2020, YouTube star and recreational mathematician Matt Parker wired his Christmas tree with 500 individually addressable LED lights, and turned his tree into an eight-foot tall programmable 3D display.

The big problem he needed to solve: He strung up his LEDs in an arbitrary an haphazard way, so he didn't know where in 3D space they all ended up. How could he figure out the 3D coordinates of all his LEDs?

His solution: He found the coordinates by putting his tree in a pitch dark room, lighting up one LED at a time, and taking a picture of the tree. For each LED, he read off the x- and y-coordinates of the LED by taking the pixel in the image that was the brightest. Then he rotated the tree exactly 90 degrees and repeated the whole process to get the z-coordinate. You can watch the video for yourself, if you'd like: `https://www.youtube.com/watch?v=TvlpIojusBE`

Matt's approach to finding 3D coordinates had some advantages and disadvantages. Disadvantages:

- Even though he wrote code to automate the process, he was only lighting one LED at a time, so it took a long time to get through all 500 LEDs.
- He needed the room to be very dark, because he was assuming that any light that reached the camera was coming from the one currently lit LED.
- He had to be careful where he put the camera; his code assumed that the pictures were being taken from head on and then directly from the side.
- Sometimes a branch would obscure the camera's view of an LED, so the camera would not register the right coordinate. He tried to account for this by taking pictures from the back and

the other side as well, which took more time. The LEDs that still weren't correctly located after this he had to fix one by one.

- He assumed that the camera was infinitely far away, which it was not, so all the measurements were a bit distorted.

On the other hand, there was really only one main advantage of his approach: he was able to finish this project by working for just an hour a night, achieving what he set out to do in an efficient way; thus, he avoided letting the project descend into an obsession that takes over his life.

I realized that I could use a bit more math to make my own 3D display that avoided all of his project's disadvantages, as well as its advantage.

This class goes into the math we need to make a better 3D Christmas tree light display. And there's so much juicy math! *Error-correcting codes* help us speed up the scanning process. *Principal component analysis* helps us remove the background of a picture taken in a brightly-lit room. We'll run optimization algorithms to stitch 2D pictures taken from arbitrary, unknown angles into a 3D model. The specific algorithm we need is pretty clever: we can't just use *least squares* because the problem is not linear, but there is a workaround called *alternating least squares* that is pretty beautiful.

The trickiest part of the whole puzzle is projecting 3D coordinates down to 2D coordinates (for the purposes of plugging into the optimization algorithm). This projection is within the realm of linear algebra if we assume that the camera is infinitely far away, but for a camera in the real world, calculating this projection involves dividing by a variable, something that is distinctly not allowed in linear algebra. But, as if by magic, if we view the problem using *projective geometry*, we can bring the problem back into the world of linear algebra.

And all this is not to mention all the cool fun and/or mathy animations you can make once you have a 3D display: You can make objects that move around in 3D. You can make images that only appear to someone standing in a specified location of your choosing. You can illustrate algorithms on graphs. I'll leave you with an animation I made on my tree last winter (everything programmed from scratch): `https://www.youtube.com/shorts/RvQ3GBOcBYk`

Homework: Recommended.

Class format: Interactive Lecture.

Prerequisites: Linear Algebra.

Infinite games. 🌶🌶 (Krishan) $\boxed{\mathrm{TW}\Theta}$FS 50 minutes

Do you like to play boardgames? Do you wish they lasted longer? Well this is the class for you! In this class we'll be learning about infinite versions of existing games (for example Wordle and Absurdle) as well as proving theorems about large classes of infinite games. We'll start by studying 2-player, perfect information games. These are game where one each player knows everything about the game state and there is no randomness (think chess rather than poker). We'll see that every such game can be associated with a finite tree, and use this idea to prove that every finite game is determined (one of the players has a winning strategy). Then we'll prove that certain infinite games are determined by studying topological properties of their game trees. We'll use the axiom of choice to cook up a weird infinite game where no player has a winning strategy, and we'll see that things can get even weirder without choice.

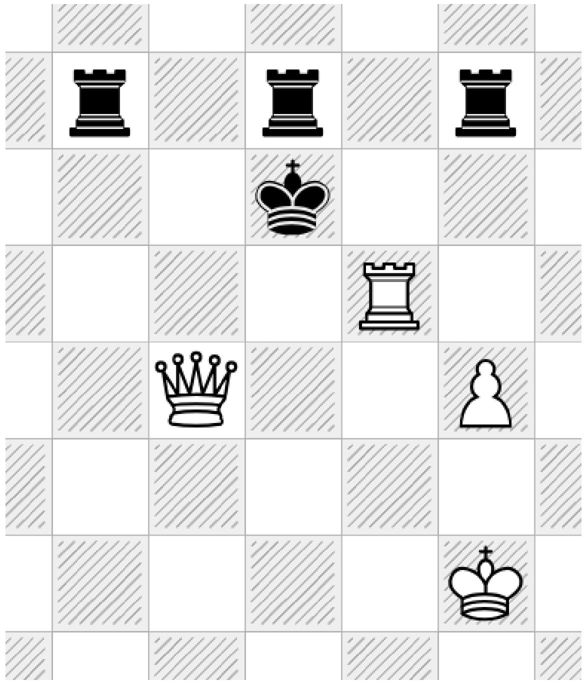
Homework: Optional.

Class format: Lecture plus some group work in class.

Prerequisites: None.

Infinite chess. 🌶🌶 (Della) TWΘ FS 50 minutes

Black to move, white to mate, on an infinite board.[3] How many turns does it take White to win?



Answering this question will require a careful definition of "how many turns", which will lead us to the ordinal numbers and ever more terrifying chess positions.

Homework: Optional.

Class format: Lecture.

Prerequisites: The basic rules of chess.

Topological Tverberg's theorem. 🌶🌶🌶🌶 (*Viv Kuperberg*) TWΘFS 50 minutes

The convex hull of a set X of points is the smallest set C containing X and all lines between points in C. Given four points in the plane, I can *always* partition them into two sets whose convex hulls intersect. And if I live in any Euclidean space and I'm given enough points, I can do the same thing. And if, before starting my partitioning process, I draw my convex hulls however I like, rather than through such silly procedures as "following the definition", I can still do the same thing. And if I'd like to split my set into three, four, five, or seven subsets instead of two, I can still do the same thing... but I can't do six subsets.

If this seems really weird, that's because it's *really weird.* We'll prove these statements and discuss the recent astonishing counterexample to the composite case.

Homework: Recommended.

Class format: Lecture

Prerequisites: Linear algebra. Enough group theory to know what a group action on a set is.

Intro to elliptic curves. 🌶🌶🌶 (Chloe) TWΘFS 50 minutes

Why is a torus like a cubic curve? Find out in intro to elliptic curves! Elliptic curves are fascinating mathematical objects with wide ranging applications and very active research. Not only was the

[3]Black will open by moving the middle rook up.

theory of elliptic curves essential in proving Fermat's Last Theorem, but they also have applications to cryptography! It's interesting that objects which are so simple to define, i.e. a non-singular curve of the form $y^2 = x^3 + ax^2 + bx + c$, have such surprisingly deep theory.

In this class we'll study the points on elliptic curves by defining a group law, discussing points of finite order, and the discriminant of the curve. We'll use what we learn about these topics to prove the Nagell-Lutz Theorem which says exactly when we find rational points of finite order. For the last day we'll jump ahead to get the flavor of some extended topics like Complex multiplication, the broad strokes of the proof of Fermat's Last Theorem, and a Galois theory connection!

Homework: Recommended.

Class format: Interactive lectures

Prerequisites: Soft prereq: definition of group.

Linear models. 🌶🌶 (*Mira Bernstein*) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

At some point in your high school career, you've probably encountered the concept of "line of best fit" for a set of data points. You may have been told that the line of best fit is the one that minimizes the sum of vertical square distances between the line and the data points. (Though actually — spoiler! — that's not always the case.)

This raises all sorts of questions. What does "best fit" mean: best from what point of view? (You're not allowed to say, "best from the point of view of minimizing the sum of squares": that's circular.) Why do we want to minimize the sum of the *squares* of the distances, as opposed to, say, the sum of their absolute values? Why do we use vertical distance rather than the usual definition of distance from a point to a line? Finally, once we find the line of best fit, what sorts of real-world conclusions can we actually draw from this?

All these questions have good answers if (and only if) you adopt a specific mathematical framework for your data analysis: the framework of linear probabilistic models. This will lead us to some very cool math (and math history) that most introductory statistics courses gloss over. We'll also talk about the difference between the traditional (frequentist) approach to statistics and the modern (Bayesian) approach.

A big part of the course will be working with real data in R. (R is an open-source programming language geared toward statistics and data analysis. I'll teach you all the R you need as part of the course.) We'll see that it's not enough to understand the math behind your models: you also have to pick the *right* model, which can be quite tricky — especially if you're asking questions about causality. (Correlation, famously, does not imply causation, but it's causation that we actually care about most of the time!)

Most statistics classes are either very theoretical (just the math) or very applied (geared toward non-math people). But to me, it's actually the interplay between math and the real world that makes statistics so interesting! So why settle for just one when you can have both?

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: There are no strict prerequisites, but:

- It would be helpful to have at least a bit of programming experience. We'll be using very elementary R, but if you've never written any code at all, it might be challenging.
- It would be *very* helpful to be familiar with the basics of random variables (mean, variance / standard deviation, independence, normal distribution). I'll do a quick review on Day 1, but if all of this is completely new to you, it'll go by really fast. (You'll also have a chance to review/practice some of these basics on Problem Set 1; so if you're completely new to random variables, you should consider Day 1 of this class "homework required".)

- Calculus will come up a couple of times but is not a prerequisite. You can definitely enjoy this course without knowing any calculus, as long as you're willing to take a few things on faith.
- At one point, we'll need some very rudimentary linear algebra: just matrix multiplication and dot product. But if you tune out briefly at this point, you can still enjoy the rest of the class.
- Statistics is definitely **not** a prerequisite. In fact, if you've taken AP Stats, there will be some things you'll need to unlearn. :)

1:10 PM CLASSES

Building mathematical sculptures. 🌶️ (Zach) TWΘFS 80 minutes

Come transform ordinary items into elaborate geometric sculptures! In these small yet intricate construction projects (that you can take home when complete), we will assemble dizzying drinking straw weavings, decorative spheres made from playing cards, precarious binder clip constructions, and more! Browse `http://zacharyabel.com/sculpture/` for examples of the types of projects this course may feature. Assembling these mathematical creations requires scrutiny of their mathematical underpinnings from such areas as geometry, group theory, and knot theory, so come prepared to learn, think, and build!

Homework: None.

Class format: Projects.

Prerequisites: None.

Ghostly graphs: or, Why bother with combinatorics when you could do linear algebra instead? 🌶️🌶️🌶️ (Travis) TWΘFS 80 minutes

This class is all about what matrices and linear algebra can tell you about graphs, and it turns out that it's a *lot*: How many different paths of length k does a graph contain? How well-connected is it? How do you draw a huge graph so that it doesn't look like a pile of spaghetti spilled on the Wheelock floor? Is there another kind of graph limit that Travis has kept hidden from the campers? Is there a connection to discrete geometry?

We'll answer some of these questions (though we probably won't have time for all of them), not by using combinatorics, but linear algebra.

Homework: Recommended.

Class format: Groupwork (IBL).

Prerequisites: You should know: the dot product for real vectors; eigenvectors; and what an orthonormal basis is. (Week 1 of Intro to Linear Algebra is sufficient.) If you've seen some of these things but not others, talk to me—I might be able to catch you up.

Building (weird) topological spaces. 🌶️🌶️🌶️ (*Dan Zaharopol*) TWΘFS 80 minutes

Topology is such an amazing subject. It lets us rigorously study wild and crazy spaces that shed light on deep parts of math, not to mention physics, robotics, and more. But when you first study the field, it's kind of a morass of formalism—cool, but it's easy to miss the forest for the trees.

Instead, join us on a beautiful hike with some wonderful views of the forest canopy: tools like product spaces, quotient spaces, and homotopy equivalences that shed light on projective spaces of any dimension, tori, cell complexes, and even the infinite-dimensional sphere. We'll even gaze out a bit at the valley of category theory.

Point-set topology isn't a requirement for this class. If you've already seen point-set topology, this will show you how to build up to the beautiful stuff algebraic topologists study. And if you haven't, this will still be rigorous—we'll just black box a few ideas, so that when you see the basics in another class you'll have context and they'll be more than just a pile of definitions.

It's topology without the poison ivy.[4] Let's go!

Homework: Recommended.

Class format: Interactive lecture, with occasional breaks for students (in groups) to consider a new space and share their analyses.

Prerequisites: You'll have a deeper understanding of the class if you know point-set topology, but you'll be fine without.

Representation theory. 🌶🌶🌶🌶 (*Aaron Landesman*) TWΘFS 50 minutes

What do the pictures on the floor of the Puget Sound science building mean? It turns out they are character tables. Given a finite group, the associated character table is a square grid of complex numbers satisfying certain rules. These numbers come from representation theory, which describes the symmetries of your favorite shapes. It can be pithily summarized as "group theory meets linear algebra."

Perhaps the best advertisement for this class is that, 6 years ago, one Mathcamp student had never heard of representation theory, but took this class, and he loved it so much that he is heading to grad school next year to study representation theory!

Homework: [**HW**] Required.

Class format: The first two days I will lecture to introduce the basic notions. The next two days we students will work through examples of character tables. If there is a 5th day, I will lecture on some of the proofs, though perhaps I'll make an option to allow students to continue working on problems.

Prerequisites: Linear algebra and group theory. From group theory, familiarity with group actions is essential. One should also be familiar with symmetric groups, conjugacy classes, commutators, and quotient groups, among other things. From linear algebra, one should be familiar with linear transformations, eigenvalues, trace, and direct sum, among other things.

Teichmueller theory of the torus. 🌶🌶 (Arya) TWΘFS 80 minutes

Take a paper square, and glue opposite sides. If done correctly (i.e., in $\mathbb{R}^4$), you will get a torus which is flat—just like the paper you used to create it. In this class, we will study the geometry of this type of construction. We will look at the "space of all flat tori" (Teichmüller space) and study it using Lattices (in $\mathbb{R}^2$), Loops (on the torus), and Linear algebra. Along the way, we'll meet some beautiful critters like the curve graph, the Farey tessalation of the circle, and Möbius transformations in the upper-half plane. Be warned—this class *will* involve some divison by zero, under staff supervision.

Homework: Optional.

Class format: Interactive lecture.

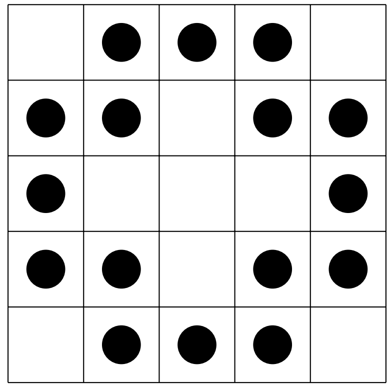
Prerequisites: None.

Colloquia

Peg solitaire to infinity. (Misha) TWΘFS 50 minutes

Here is a quick puzzle. You have a 5×5 checkerboard with checkers placed on it as follows:

[4] Continued forest puns not guaranteed.



The only moves you are allowed to make are moves in which one of the pieces jumps horizontally or vertically over another to an empty space, removing the piece that was jumped over. Your goal is to make 15 moves, ending with only one piece remaining.

In this colloquium, we will solve a puzzle (made up by Conway) which is infinitely harder than this puzzle. In fact, we will prove that Conway's puzzle is impossible to solve. But who are we to be stopped by mere impossibility!? We will end by taking peg solitaire to infinity and solve Conway's impossible puzzle.

Math education, access, and opportunity. (*Dan Zaharopol*) T$\boxed{\text{W}}$ΘFS 50 minutes

After spending over ten years at Mathcamp, I left my academic career to pursue work in math education, creating the Bridge to Enter Advanced Mathematics (BEAM) program. In the course of that work, I've learned a lot about the education space, seen the successes and failures of schools, programs, and research.

What are we to make of differing levels of achievement in education? How do we parse everything we see, and our instinct to "just fix it?" How do we reconcile the persistent ties between education, inequality, and power in the US?

I'll try to bring some order to the chaos, and talk about not just where things are now but how you might be able to get involved and make a difference. Don't expect easy answers, but do expect some thought provoking questions and a better sense of just what the challenges are.

The law of anomalous numbers. (*Yuval Wigderson*) TW$\boxed{\Theta}$FS 50 minutes

30.1% of all powers of two begin with a one. Also, 30.1% of all powers of three begin with a one. And don't even get me started on powers of five.

On the other hand, 30.8% of the world's countries have a population beginning with one. Whereas 28.1% of countries have an area beginning when one when measured in square kilometers, but 34.2% of countries have an area beginning with one when measured in square miles.

There are 57 fundamental physical constants listed on Wikipedia. Of them, 19 (33.3%) begin with a one.

What's up with this? And what does it have to do with the fact that $\log_{10}(2) \approx 0.301$?

A mathematical headache (and perhaps a cure). (*Mira Bernstein*) TWΘ$\boxed{\text{F}}$S 50 minutes

The legendary mathematician and long-time Mathcamp visitor John H. Conway passed away in 2020 from COVID-19. Just before his death, the American Mathematical Monthly published a short paper that Conway had written back in the 1970's, called "A Headache-Causing Problem". The authors listed on the paper are "Conway (J.H.), Paterson (M.S.), and Moscow (U.S.S.R)" — yes, you read that correctly. The abstract of the paper is: "After disproving the celebrated Conway-Paterson-Moscow theorem [1], we prove that theorem and make an application to a well-known number-theoretic problem." The reference [1] is to the paper itself; the number-theoretic problem is Fermat's Last Theorem.

As you can tell, this paper is joke piled upon joke — but the math is real and quite elegant. It concerns a logic game that involves thinking about what other people are thinking. The "headache" comes from the fact that both the proof and the disproof of the main theorem seem correct, and it is hard to see the flaw in either argument. The paper doesn't give away the answer — you have to figure it out for yourself.

In this talk, I will tell you the story of this funny little paper (the first mathematical publication by a major metropolitan area!) and introduce you to the Headache-Causing Problem. Hopefully, by the end of the hour, we can vanquish the headache together.