CLASS DESCRIPTIONS—WEEK 5, MATHCAMP 2025

Contents

9:30 Classes

Continued Fractions: The Irrationalest Number (🌶🌶, Susan, TW ΘF)
Continued fractions are a tool we can use to approximate irrational numbers. But some irrational numbers are irrationaler than others. And the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$ is as irrational as it is possible to be. Come to this class to find out why!

Class format: Interactive Lecture

Prerequisites: None

Homework: Recommended

Elliptic Functions (🌶🌶🌶🌶, Mark, TWΘF)
Functions of a complex variable, meet elliptic curves! Actually, you don't need to know anything about elliptic curves to take this class, but they will show up along the way. Meanwhile, if you like periodic functions, such as cos and sin, then you should like elliptic functions even better: They have two independent (complex) periods, as well as a variety of nice properties that are relatively easy to prove using some complex analysis. Despite the name, which is a kind of historical accident (it all started with arc length along an ellipse, which comes up in the study of planetary motion; this led to so-called elliptic integrals, and elliptic functions were first encountered as inverse functions of those integrals), elliptic functions don't have much to do with ellipses. Instead, they are closely related to cubic curves, and also to modular forms. If time permits, we'll use some of this material to prove the remarkable fact that

$$\sigma_7(n) = \sigma_3(n) + 120\sum_{k=1}^{n-1} \sigma_3(k)\sigma_3(n-k),$$

where $\sigma_i(k)$ is the sum of the ith powers of the divisors of k. (For example, for $n = 5$ this comes down to

$$1 + 5^7 = 1 + 5^3 + 120[1(1^3 + 2^3 + 4^3) + (1^3 + 2^3)(1^3 + 3^3) + (1^3 + 3^3)(1^3 + 2^3) + (1^3 + 2^3 + 4^3)1],$$

which you are welcome to check if you run out of things to do.)

Class format: Interactive lecture

Prerequisites: Functions of a complex variable; in particular, Liouville's theorem

Homework: Optional

Entropy (, *Milan Haiman*, TWΘF)
Suppose Alice rolls a standard dice N times and wishes to send Bob a binary string encoding the result. With an optimal strategy, what is the minimum expected number of bits needed for the message? In this class we will introduce the concept of entropy to answer this question, and then we will use entropy to prove various results that are seemingly unrelated. For example, consider the following setup: Let $G \subseteq V \times V$ be a directed graph on V and let

$$A = \{(x, y, z) \in V^3 \colon (x, y), (y, z), (z, x) \in G\},$$
$$B = \{(x, y, z) \in V^3 \colon (x, y), (x, z) \in G\}.$$

It turns out that we always have $|A| \leq |B|$, but the only proof I know uses entropy!

Class format: Lecture

Prerequisites: Nothing required. Knowing a bit about random variables and graph theory may be helpful.

Homework: Recommended

Factoring Huge Integers (, *David Roe*, TWΘF)
Do you dream of factoring 50 digit integers? Do you want to know how to send encrypted messages to people you've never met? Then this class is for you! On the first day we'll learn about public key cryptography, starting with the RSA cryptosystem and touching on more modern methods that are designed to be secure against quantum computers. On the second day we'll go into detail on the quadratic sieve, the second-fastest known general purpose factoring algorithm.

Class format: Interactive Lecture

Prerequisites: We'll be using modular arithmetic extensively. The quadratic sieve uses linear algebra, but you can block-box it as long as you believe it's possible to solve linear equations in many variables.

Homework: Recommended

Into the Quantum World of Computers (, Sam, TWΘF)
It seems these days, that the words 'quantum computing' are buzz words that get thrown around everywhere, mostly by influencers on LinkedIn that don't have a clue what they're talking about... However, in this class, we will take a quick dip into the quantum world to explore the cool sort of things that a quantum computer can do, from Shor's algorithm for polynomial time decryption to faster than light propogation of information. Not only this, but we will get to use actual quantum computer :o (yes, they already exist!)

Class format: Interactive class with computer sessions

Prerequisites: Knowledge of very basic linear algebra for $\mathbb{R}^n$, like matrices.

Homework: Recommended

Mathematics of LessWrong (, Nikita, TWΘF)
LessWrong is a nerdy forum with people interested in rationality and AI safety/alignment. It was a source of some developments in mathematics as well. While thinking of the hard problem of controlling AI that is much smarter than you, lesswrongers made progress in mathematics of decision theory and these advances are interesting in its own right.

Class format: None.

Prerequisites: None.

Homework: Optional

Mathematics of Modern Music (, Michael R, TW[Θ]F)
In the twentieth century, composers began to write weird music. Really weird music. So weird that music theorists couldn't analyze it anymore. Some theorists panicked and decided to ignore this new music; others decided to deal with the problem. In their quest, they created what is known as transformational theory – a look at music through mathematical groups and without any central element, such as a key. In this class, we will briefly introduce transformational theory and attempt to apply it to one of those strange 20th century pieces.

Class format: Interactive lecture

Prerequisites: Intro group theory, basic music theory (12 different notes, intervals, major vs. minor), +1 chili if you don't know concepts such as parallel/relative keys or the circle of fifths

Homework: Optional

Introduction to ~~Algebraic Topology~~ Hex (, Michael I, TWΘ[F])
The Brouwer Fixed Point Theorem is one of the most famous results in mathematics: any continuous function from a ball to itself must have a fixed point. Most proofs use some heavy-duty algebraic topology — homology groups, chain complexes, and more.

But what if we could do away the machinery and play a board game instead?

In this class, we'll dive into Hex, a deceptively simple connection game, and discover the topology behind winning the game. We'll then use Hex to prove the Brouwer fixed point theorem — a fun proof with no topological prerequisites. Bring your combinatorial wits and get ready to play your way to a topological victory.

Class format: Lecture

Prerequisites: None

Homework: None

10:30 Classes

Hyperbolic Hyperbolic Geometry (, Arya, T[WΘF])
A conversation between Assaf and Arya, 2022 — "Hey Arya, what's a hyperbolic hyperbolic geometer?" "I don't know, Assaf." "It's someone who thinks all graphs are trees."

The goal of this class is to explain this joke. Last week, Dan introduced a bunch of ideas in defining different models for hyperbolic geometry, such as the Poincaré disk, or the upper half plane. We shall abstract those ideas to talk about the intrinsic geometry of various metric spaces which "behave like the hyperbolic space". In particular, we can say a bunch of things about the isometries of such spaces, talk about how parallel lines behave, define "a boundary at infinity" and so on. Come find out!

(I briefly spoke about this content on Day 9 of my Geometric Group Theory class, but specifically in the setting of Cayley graphs of groups.)

Class format: Interactive Lecture

Prerequisites: Dan's Week 4 Hyperbolic Geometry class, or more generally, some familiarity with hyperbolic geometry. +1 chili if this is the first time you're dealing with metric spaces.

Homework: None

Random Graphs and Various Properties Thereof, Having Been Compiled by the Obedient Servant of this Most Fair and Noble Numerical Pastime (, Lucas, T[WΘF])
We will prove non-probabilistic things about colourings of finite graphs using probabilistic things. I think this is the most interesting idea in mathematics.

Day 1: Intro to the probabilistic method. Hypergraphs, Ramsey Theory.

Day 2: The Local Lemma + applications. This allows you to prove things about graphs knowing only what they look like locally.

Day 3: Entropy compression. A very complicated way of proving skibidi simplest solution you can think of will work.

The number of chilis is the day number plus one.

Class format: Lecture

Prerequisites: Some graph theory (specifically the definition of the edge colouring number), some probability (specifically Markov's inequality), comfortable with big/little O notation

Homework: Optional

Tensor Products and the Dehn Invariant (🌶🌶, *Preston Bushnell*, TWΘF)
Do you like cutting shapes and putting them back together? Did you take Zach's class[1] on it in week 3? Did you think it was suspicious that Zach seemed to avoid the topic of 3d shapes? Did you wonder... why did he cut that out of his course? Eh?

In 1900, David Hilbert posed a list of 23 of the most important unsolved problems in mathematics. Many on the list are still unsolved and remain influential to this day. Not #3, though... somewhat embarrassingly for Hilbert, his student solved it right after the list was published.

Hilbert's 3rd problem asked whether a polyhedron can always be dissected to form any other one of the same volume. The question was answered in the negative, using the discovery that polyhedra wield two quantities that remain unchanged when you cut them apart: volume, and an abstract algebraic quantity called the Dehn invariant.

On the first day of this class, we'll have an algebra marathon, becoming masters of the tensor product. Then we'll switch gears, applying what we know to this geometric problem and solving Hilbert's problem along the way.

Class format: Lecture

Prerequisites: Linear algebra would be helpful, but isn't required

Homework: Recommended

The Riemann Zeta Function (🌶🌶🌶, Mark, TWΘF)
Many highly qualified people believe that the most important open question in pure mathematics is the Riemann hypothesis, a conjecture about the zeros of the Riemann zeta function. Having been stated in 1859, the conjecture has outlived not only Riemann and his contemporaries, but a few generations of mathematicians beyond, and not for lack of effort! So what's the zeta function, and what's the conjecture? By the end of this class you should have a pretty good idea. You'll also have seen a variety of related cool things, such as the probability that a random positive integer is not divisible by a perfect square (beyond 1) and the reason that $-691/2730$ is a useful and interesting number.

Class format: Interactive lecture

Prerequisites: Some single-variable calculus (including integration by parts) and some familiarity with complex numbers and infinite series; in particular, geometric series.

Homework: Optional

What Math Teachers are Taught (🌶, Riley, TWΘF)
This is a class where students read and discuss materials that middle and high school math teachers (at least, at Portland State University) get taught. Every day except the first day students will read

[1] Not a prerequisite

some article before class and discuss it in class. Suggested topics include the following: what it means to do mathematics, teaching math for social justice, the place of definitions in the math classroom. Topics may be changed depending on student interest.

Class format: Discussion

Prerequisites: None

Homework: Required

11:30 Classes

Computability Theory (🌶🌶🌶, Amelia & Spencer, RWΘF)
We often study *how* computers can compute things (algorithms, etc.), but rarely the limits of *what* computers can compute. In this class, we explore how a set can be beyond the reach of programs, as well as a way to compare the non-computability of different sets. While this is a one day course, there are two optional follow-up lectures which will be offered uncolloquium style as scheduleboard events.

Class format: Lecture

Prerequisites: None

Homework: None

Einstein's Theory of Gravity 3: Black Holes (🌶🌶🌶, Laithy, TWΘF)
Having understood Einstein's geometric perspective of spacetime, we will study the most famous solution to Einstein's field equations, the Schwarzschild solution, which predicted what we now call black holes.

The Schwarzschild solution describes spacetime around a spherical, non-rotating mass and predicts something extraordinary: regions where spacetime curvature becomes so extreme that nothing, not even light, can escape.

Starting with the history: In 1916, just months after Einstein published his theory, Karl Schwarzschild found this solution while serving on the Russian front in WWI. For decades, physicists debated whether black holes were mathematical curiosities or physical reality. The answer came in the 1960s-70s with the discovery of quasars and X-ray binaries — indeed, black holes are real and abundant in our universe.

We'll derive the Schwarzschild metric ourselves. By imposing spherical symmetry, we'll reduce Einstein's equations to just a few ODEs that we can solve explicitly. The solution reveals strange properties: as you approach the event horizon, your clock runs slower relative to distant observers — in fact, they see your time freeze completely at the horizon. Yet if you fall in, you experience nothing special at the horizon and reach the singularity in finite proper time.

We'll discuss Birkhoff's theorem: any spherically symmetric vacuum spacetime must be Schwarzschild. This means even a collapsing star's exterior spacetime is identical to a static black hole's—a remarkable rigidity result.

Time permitting, we'll discuss Hawking's discovery that black holes emit thermal radiation due to quantum effects near the horizon, slowly evaporating over time. This connects gravity, quantum mechanics, and thermodynamics in unexpected ways.

Class format: Interactive lectures

Prerequisites: General Relativity

Homework: Recommended

Hard Hat Puzzles (🌶🌶🌶, Della, TWΘF)
Nikita's hat puzzles are all finite, and therefore boring. What can we do with infinitely many people,

or with infinitely many different hats? Make sure you wear PPE, since we'll be in constant danger of set theory.

Class format: Lecture (or ignoring me to solve puzzles)

Prerequisites: Ordinal numbers, axiom of choice

Homework: Optional

How to Prove Things NP-complete (🌶, Della, TWΘF)

If you've talked to me, or Glenn, or Nikita, or Misha, or Zach, you might have heard a comment like "this is NP-complete" as though saying that solves a problem. I'll explain what it means to be NP-complete, why it means you should stop trying, and how to prove it. You can also use this to torture Glenn's SAT solvers!

Class format: Lecture

Prerequisites: None

Homework: Recommended

n^{n-2} **More Proofs of Cayley's Tree Theorem** (🌶🌶, Zach, TWΘF)

How many trees exist with vertices $\{1, 2, \ldots, n\}$? The (perhaps surprising) answer is n^{n-2}, and there are soooo many neat ways to show this! Last year we filled 3 days with many different proofs, but I still have more! Let's look at two more proofs: one using a pure bijection that turns trees inside-out at a parking lot, and another that runs backwards from infinity.

Last year's class is *not* a prereq.

Class format: Lecture

Prerequisites: None.

Homework: Optional

Susan Teaches Ben's Class (🌶, Susan, TWΘF)

If this class runs, Ben will email Susan a slide deck five minutes before class starts. Good luck, Susan!

Class format: Revenge slideshow telephone

Prerequisites: None. There is already too much risk of Susan figuring things out.

Homework: Recommended

The Sound of Proof (🌶, Eric, TWΘF)

Can you hear what a proof sounds like? I'll present five proofs from Euclid's Elements, and then play (recordings of) five pieces of music written to capture each proof in sound. You'll get to try and work out which piece of music lines up with which proof, and then we'll dissect how a couple of the compositions "sonify" the proofs. All of the material I'm drawing on is from an art piece entitled *The Sound of Proof* by mathematician Marcus du Sautoy and composer Jamie Perera at the Royal Northern College of Music in Manchester.

Class format: Interactive lecture feat. listening to music and filling out a short survey

Prerequisites: None

Homework: None

This Class (🌶🌶🌶🌶, Steve, TWΘF)

In this class, we will cover the material covered by this class. The teacher teaching this class, along with the staff member responsible for writing this blurb, will instruct those students who choose to study this material.

Translation from Steve-ish: this is a class on self-reference in mathematics, specifically self-referential sentences in the context of arithmetic. If you've heard of Godel's incompleteness theorem, that's the sort of jazz we'll be listening to hear! If there's a subfield of math that feels more like a pun than this, I'm not aware of it.

After blackboxing some important but tedious technicalities (and I'll provide handouts of these that folks interested can work through at TAU), we'll cover basically everything I know about self-reference; besides Godel's incompleteness theorem itself, we'll see Lob's theorem ("This sentence is provable" is definitely provable), Kreisel's counterexample (... unless you write it stupidly), and at least one open question (due to me because I really want someone to solve it someday).

Class format: Lecture

Prerequisites: None

Homework: None

Unicorns and Poland (🌶🌶, The JCs, TWΘF)

"Unicorn paths" were defined by Polish mathematician Piotr Przytycki (now at McGill), because he initially wanted to define "one-cornered paths", and the word for "one-cornered" in Polish is very similar to the word "unicorns", and "unicorns" is way cooler. Polish people are cool :P

But why do we care about these (and what are these)? Associated to every surface, there is a graph called the "curve graph", which very literally is a graph of curves on the surface. For deep reasons, this graph is very cool. Unicorns guide the way to travel along the unyielding terrains of this graph, and tell us a lot about the geometry of this graph.

Come to this class to get a feel about stuff people study in modern geometric topology!

Class format: The JCs will teach Arya's class. There might be performances halfway through each class.

Prerequisites: None

Homework: None

Will It Flatten? (🌶🌶, Della, TWΘF)

This is an origami class! We'll think about which crease patterns can be folded perfectly flat. Unfortunately, this is a hard problem[2], much too hard for a 1-day class. To make our lives easier, we will exclusively fold one-dimensional paper!

Class format: Lecture

Prerequisites: None

Homework: None

1:30 Classes

A Taste of the Snark (🌶🌶🌶, Antonio, TWΘF)

A little puzzle for you:

You are a very ambitious graduate student in mathematics at Frog University. You have written a proof for a very prestigious problem, and if it were correct, you would become a very famous mathematician frog. You would want to ask your very smart mentor, the Evil Frog, to check your proof but you are scared that they would steal it and publish it themselves. Additionally, the Evil

[2] NP-hard in fact!

Frog is very busy and has no time to read all your thousands of pages of symbols: for every solution that you want him to check, they will only read 1 page from you and that's it, no matter how hard the problem is. How do we manage to make our proof small enough that it fits on one page and written in such a way that the Evil Frog can't steal the clever math we've created but can still check it?

In this class, we will answer the puzzle above. To do so, we will be studying some of the theory behind "zero-knowledge succinct non-interactive arguments of knowledge" (zk-SNARKs), and learning how we can use Quadratic Arithmetic Programs (QAPs) and the KGZ commitment scheme to construct Groth'16, a popular zk-SNARK proof system which is actively used in industry (especially in the blockchain space).

Class format: Interactive Lecture

Prerequisites: Group theory

Homework: None

Everything is the Cantor Set (🌶️🌶️🌶️🌶️, Eli G, TWΘF)

We'll prove what the title says: that (almost) everything is (the continuous image of) the Cantor set. Specifically, we'll give a beautiful and surprisingly visual proof of Hausdorff's Theorem, which says that every closed, bounded subset of $\mathbb{R}^n$ is the image of the Cantor set under a continuous map. The Cantor set is probably the most discontinuous object you can think of (it's basically just a bunch of points), yet Hausdorff's Theorem says that the Cantor set maps surjectively and continuously onto almost anything you can think of; so $[0, 1]$, a square, a 4D-hypercube—they're all just the Cantor set in disguise. Along the way, we'll encounter interesting ideas about what the Cantor set really is, what it really means to be continuous, and the infinite tree hiding inside of every closed, bounded subset of $\mathbb{R}^n$. At the end, I'll also talk about how Hausdorff's Theorem relates to space-filling curves—how the Cantor set helps us construct space-filling curves, and why the Cantor set is really what gives space-filling curves their magic.

Class format: Interactive lecture

Prerequisites: You should be fluent with the abstract definition of a continuous function, and know what closed and open balls in $\mathbb{R}^n$ are. Minus one chili if you are familiar with the notion of compactness.

Homework: None

Extending a Knot Invariant (🌶️🌶️, Audrey, TWΘF)

We will look at my undergraduate research that was recently accepted for publication. I will introduce knots and links before talking about how to calculate the invariant and how we know that this method of calculating is indeed an invariant.

Class format: Lecture

Prerequisites: None

Homework: Optional

How to Teach (🌶️, *Ari Nieh*, TWΘF)

I probably don't need to sell you on the idea that learning from a great teacher is a transformative experience. After all, you're here at Mathcamp. But I might need to sell you on the claim that you- yes, you- can be a great teacher. To my mind, calling somebody a great teacher is like calling them a great pianist. It doesn't mean they made beautiful music the first time they touched the keyboard. It means they've spent countless hours practicing and refining their skills, and gotten feedback and coaching from more experienced pianists, so that what once felt impossible is now second nature.

I have three main goals for this class:

- Provide you with the beginnings of a toolkit for teaching: how to speak, write, interact, and plan for the classroom
- Invite you to think critically about tough questions: Am I being understood? What's the right mental model for this abstraction? How safe do my students feel speaking up?
- Give you a framework for future improvement: What sort of feedback do you seek out, and from whom? How do you practice and refine your techniques? How do you stay motivated in the face of difficulties?

We'll explore these topics through a variety of class formats. Exactly what we cover will depend on your priorities, feedback, and questions. Let's play some Mozart together! (Or Chopin, if you prefer.)

Class format: Variety

Prerequisites: None

Homework: None

How We Solved an Open Problem Posed by Knuth (🌶🌶🌶, Purple, TWΘF)

My research is in pretty abstract stuff, but it can sometimes be reduced to concrete questions of combinatorics. In this class we will discuss saturated transfer systems, a relatively straightforward object whose enumeration has surprising connections to homotopy theory and the classification of 4-manifolds (don't worry, we won't talk about these connections except potentially during TAU). In the short version of this class, we will just talk about these objects and several open questions related to them. In the long version of this class, we will work through the proof of a forthcoming (joint with several collaborators) result on their enumeration, which also happens to answer an open question first posed by Donald Knuth.

Class format: IBL

Prerequisites: Know what a binomial coefficient is

Homework: Recommended

Infinite Ramsey Theory (🌶🌶🌶, Susan, TWΘF)

Suppose you throw a party and invite six people, and some of those people know each other and some don't. A well known result from graph theory tells us that we can guarantee a group of three mutual friends, or three mutual strangers. But suppose you wanted to throw a much, much bigger party, with infinitely many attendees? Can you guarantee an infinite group of friends or strangers? Come to this class to find out!

Class format: Interactive Lecture

Prerequisites: None

Homework: Recommended

The Universe is Kind of Lazy (🌶🌶🌶, Kaia, TWΘF)

To understand classical mechanics, we'd like to move beyond Newton's three laws. This will lead us to something called the Principle of Least Action– in short, when anything moves, it likes to be lazy and solve a minimization problem (which perhaps isn't such a lazy thing to do at all).

So let's learn how to solve this minimization problem ourselves, shall we? This will bring us to the Calculus of Variations– we'll ask what happens when we perturb a function a little bit.

Ever wanted to find geodesics, the shortest paths between points? Ever wondered how to maximize the area of a shape with a fixed perimeter? Well, the universe certainly likes solving these sorts of problems.

Class format: Interactive lecture

Prerequisites: Derivatives and integrals

Homework: Recommended

2:30 Classes

Communication Complexity (🌶🌶, Nikita, T[W]ΘF)
We didn't have enough time in my hat puzzles class to talk about communication complexity – a field in computer science that studies how hard it is to compute a function whose inputs are distributed among several parties, when communication between them is limited. In this class, we'll solve the following problem: The tsar writes a 1000-digit number on each prisoner's hat. The prisoners take turns: on each turn, a prisoner announces a digit. After 100 turns, they must collectively decide which of the two numbers is bigger.

Devise a strategy that ensures they guess correctly with at least 99% probability.

Class format: Interactive lecture

Prerequisites: Preferably my hat puzzles class

Homework: Recommended

Cyclotomic Polynomials and Migotti's Theorem (🌶🌶🌶, Mark, [T]WΘF)
The cyclotomic polynomials form an interesting family of polynomials with integer coefficients, whose roots are complex roots of unity. Looking at the first few of these polynomials leads to a natural conjecture about their coefficients. However, after the first hundred or so cases keep confirming the conjectured pattern, eventually it breaks down. In this class we'll prove a theorem due to Migotti, which sheds some light on what is going on, and in particular on why the conjecture finally fails just when it does.

Class format: Interactive lecture

Prerequisites: Some experience with complex numbers, preferably including complex roots of unity; some experience with polynomials.

Homework: None

Infinite Field Trip (🌶🌶🌶, Eric, TW[Θ]F)
In this class we'll prove a cool theorem about polynomials over $\mathbb{C}$ by reducing to the case of finite fields, where we can just count things! (The Ax-Grothendieck theorem for $\mathbb{C}^n$.) If there's time after proving the main theorem we'll gesture towards the more general philosophy in model theory (the Lefschetz principle) that is the idea behind the proof.

Class format: Interactive lecture

Prerequisites: Ring theory: the statement that an ideal I in a commutative ring R is maximal iff R/I is a field should be comfortable. You should have seen the words "algebraically closed field" before.

Homework: Optional

Markov Processes (🌶🌶🌶🌶, Nikita, [T]WΘF)
Continuing our discussion on percolation, this class will introduce the Markov property — the principle that the future evolution of a system depends only on its present state, not its past.

We'll solve the following problem:

You shuffle a deck of cards. You draw cards until you get your first spade, then draw one more card. What is the probability that this extra card is also a spade?

Time permitting, we'll also explore applications of Markov processes to percolation and discuss why site (vertex) percolation >> bond (edge) percolation.

Class format: Lecture

Prerequisites: Percolation

Homework: Recommended

Monads, Categories, and the Structure of Programming (1 chili, Glenn, TWΘF)

"A monad is a monoid in the category of endofunctors." If you haven't heard of this saying before, I think that most people understand it as a joke that pokes fun at mathematicians being crazy and lost in abstract nonsense. But the joke actually goes one step further—it was actually first popularized by a programmer, and it is funny because a monad is actually an extremely common, basic, and easy-to-understand concept in programming! In other words, the category theorists weren't just saying nonsense to define hard, abstract concepts: they were saying nonsense to explain easy things.[3]

We will learn what programmers know monads and monoids and endofunctors to be, which are all very straightforward patterns that appear in most programming languages. At the end, I'll say a few words on how mathematicians define these things and how they are equivalent.

Why 1 chili? Unlike previous category theory classes at Mathcamp this summer, this class will deal with very concrete objects without much abstraction. In particular, we will just look at some functions and types in a programming language called Haskell. So I think it will actually be very accessible to everyone (with a bit of programming background)! At the same time, I think the connection between programming and category theory is something that should be interesting for people who typically like more chilis, too.

Class format: Lecture

Prerequisites: Some programming in any language, ideally a language that requires types like C++ or Java. If you only know Python/JavaScript, be comfortable that some languages require you to declare the types of your variables and functions.

Homework: Recommended

Outro to Group Theory (3 chilis, Narmada, TWΘF)

This is the last group theory class you will ever want to take! We'll learn about the Burnside problems, one of which is a long-standing open conjecture sometimes called the Fermat's Last theorem of group theory. (Don't look up the problems if you want to take this class!) Be prepared to leave the world of finite groups behind and see just how badly behaved infinite groups can be.

Class format: Interactive lecture

Prerequisites: Intro to Group Theory

Homework: Optional

Space-Filling Curves: Why You Should ~~Hate~~ Love Analysis Even More (3 chilis, Charloutte, TWΘF)

I am retroactively branding my Week 3 class on nowhere ~~continuous~~ differentiable but ~~differentiable~~ continuous functions as Everyone Should ~~Hate~~ Love Analysis. Do you know what's even worse than a nowhere differentiable but continuous function? A continuous function that maps $[0,1]$ onto $[0,1]^2$, that's what.

Class format: Worksheets and lecture

[3] See also: `https://ncatlab.org/nlab/show/carrying`.

Prerequisites: You should be comfortable with the definitions of a metric space, Cauchy sequences, and what it means for a space to be complete.

Homework: Recommended

Statistical Oddities (, *Jane Wang*, TW$\boxed{\Theta}$F)

A 1973 study into admissions at Berkeley showed that men had a much larger chance of being admitted into a graduate program than women. However, on the department by department level, there was a small but significant bias in favor of admitting women. How was this possible? Statistics is full of examples such as this one that can challenge our thinking and run contrary to our intuition. In this course, we'll analyze some counterintuitive statistical phenomena as well as some ways that people try to deceive with statistics.

Class format: Interactive Lecture

Prerequisites: Basic probability (computing discrete probabilities and expectations)

Homework: None

Summation by Parts on the Battlefield (, Zach, TW$\boxed{\Theta}$F)

Integration by parts provides a handy method to integrate a product $f(x) \cdot g(x)$ by instead considering the integral of $f'(x) \cdot G(x)$, where f' is the derivative of f and G is an antiderivative of g. Its discrete analog, known as "Summation by Parts" or "Abel Summation"[4], deals instead with differences and partial sums, and it is a simple but unusually powerful weapon to have in your arsenal—especially, perhaps, when doing battle with some olympiad problems.

In this class you will receive basic weapons training, learning how to use this innocuous-seeming hand grenade to blow some enemies (enemy sums?) into, well, parts. You will also learn to recognize when such a maneuver could be add-vantageous. Finally, we will apply this training in simulated combat against incomplete units of Egyption fractions.

Class format: Lecture

Prerequisites: N/A

Homework: Optional

This Makes No Sense (, Laithsey (Laithy, Linsey), $\boxed{\text{TW}}$ΘF)

You have probably felt this way in Laithy's general relativity classes. In this special one Laithy-chili class, prepare to be confused. We cover two problems with very counterintuitive results, one in geometry and one in probability:

- What happens to the volume of a sphere as we increase the number of dimensions? You might guess that a unit sphere would have more volume in higher dimensions, but the actual answer is surprising. We'll find out how our intuition fails in higher dimensions.
- What if the best way to win is to lose, repeatedly? We'll explore a phenomenon in which you can combine two losing games and end up winning.

Class format: Interactive lecture

Prerequisites: Calculus and integration techniques

Homework: Required

[4]Yes, it is really called "Abel Summation". I did not just name it after myself. Some Norwegian mathematician named it after me instead...

Colloquia

My Favourite Bug (Eric, Tuesday)
Alas, this is not a colloquium about entomology, but rather bugs in software. I'll tell you a story about some research I did as a graduate student, some cool open problems in number theory, some fun linear algebra, and how I lost several days of computations to a major linear algebra software package's failure to obey rule 4.

Everything is Illuminated (*Jane Wang*, Wednesday)
The illumination problem asks the following: given a room with mirrors for walls, can we always place a single point light source somewhere in the room so that the whole room is illuminated? In this colloquium, we will survey various solutions to this problem given different constraints on the rooms (e.g. polygonal or not). In doing so, we will draw lots of pictures, consider many strange-looking rooms, and see how modern techniques at the intersection of geometry and dynamical systems can help answer this classical question.