Applying to Mathcamp 2010

Ready to Apply to Mathcamp?

We invite applications from every student aged 13 through 18 who is interested in mathematics, regardless of racial, ethnic, religious, or economic background.

Mathcamp accepts applications both on the web and by regular mail. We strongly encourage all students with Internet access to use the online application process.
The $20 application fee is waived for online applications.

Online Application:
Go to http://www.mathcamp.org/application/ and follow the instructions. You'll still have the opportunity to submit your quiz or recommendation letters by postal mail.

Postal Application:
Please mail the following items in a single envelope.

1) Some **introductory information** about yourself:
- First name, Nickname (if any), Last Name
- Email address (This is our primary means of contact.)
- Do you check your email at least twice a week?
- Do you authorize Mathcamp to share your email address with other campers (e.g., future roommates)?
- Telephone and postal address
- Country of citizenship
- Date of birth (Must be between 8/1991 and 7/1997.)
- Current grade in school and name of school
- How did you hear about Mathcamp?
- Have you applied to Mathcamp before? (If so, when?)
- Can you attend for all five weeks? (If not, explain.)

Note: all planned absences from the camp must be discussed with us in advance. Ordinarily, we do not want students to miss more than a few days of camp.

2) Your answers to the **qualifying quiz** (see below). Please include the following statement and your signature: "I declare that my solutions to the qualifying quiz are my own work. I did not receive any form of assistance from other people, and I have referenced every instance where I looked something up in a book or on the web."

3) A brief **personal statement** about your interest in math and why you want to come to Mathcamp. Some things you could talk about: What would you like to gain from a summer at Mathcamp? What do you like about math? Which of the problems on our quiz did you enjoy most? Are there specific areas or kinds of math that you're especially interested in? If you've done any other math programs, projects, or independent reading, tell us about them!

4) A **list of math courses** that you've taken at the high-school level or above, with brief descriptions of what was covered. Also, if you have done any math competitions, please include scores and awards.

5) **Two recommendation letters**. The first letter should be from a teacher who knows you well, preferably (but not necessarily) a math teacher. The letter should comment on your creativity, initiative, and ability to work with others, as well as on your academic achievements. The second letter should be from another adult who knows you personally (e.g. an employer, pastor, soccer coach, etc. – preferably someone outside of school and not a relative). This letter should address your maturity, independence, social and personal qualities. We are looking for students who are not only good at math, but who will thrive in the atmosphere of freedom and responsibility that characterizes Mathcamp, and who will make a positive contribution to the camp community. To ensure confidentiality, ask each recommender to seal their letter in an envelope and to sign across the seal. Include the sealed envelopes with your application.

6) If you would like to be considered for financial assistance, please include the **scholarship application** (see instructions below). Note that admission to Mathcamp is need-blind.

7) A **US $20 application fee** (check or money order made out to Mathematics Foundation of America) or a note signed by your parent or guardian explaining that your family cannot afford it.

All applications received by April 26, 2010 will be given equal consideration. Rolling admissions thereafter.

Contact Us

Email: info10@mathcamp.org

Telephone/Fax: 888-371-4159

Postal address:
Mathcamp 2010
129 Hancock Street
Cambridge, MA 02139

Please do not send certified mail to Mathcamp; we will email you to verify that your application has arrived.

Student Care Policy

Dear parent: Student safety and enjoyment are Mathcamp's first priorities. Students will be housed in secure campus dormitories, with male and female students in designated sections of the same building. In case of a medical problem, the hospital is minutes away. Students will have access to university athletic facilities and computers. Every effort will be made to enable students who so desire to attend weekly religious services of their faith. Mathcamp is committed to an atmosphere of mutual tolerance, responsibility, and respect, and is proud of its past record in helping to create such an atmosphere.

- Mira Bernstein, Executive Director, Mathcamp

Cost and Scholarships

Full Camp Fee: $3500

(This includes tuition, room, board, and extracurriculars.)

Admission to Mathcamp is need-blind. We are deeply committed to enabling every qualified student to attend, regardless of financial circumstances.

Mathcamp awards over $100,000 in need-based scholarships every year. In the past five years, no admitted applicant has been unable to attend the camp for financial reasons. We give several full scholarships each year, and occasionally even help students with travel expenses. Please do not let financial considerations prevent you from applying! If you'd like to be considered for a scholarship, just complete the short application at right.

Scholarship Application

Please have a parent or guardian provide the following information, along with her or his email address:

- 2009 family income (all sources).
- Expected family income for 2010. (If significantly different from 2009, please explain.)
- A list of all members of your household (supported by the above income) and their relationships to the applicant. For siblings, please provide ages.
- The cost of schooling, if any, for household members (private school, college, etc).
- The estimated cost of round-trip travel to Mathcamp for the applicant.
- The portion of the cost of Mathcamp (including both tuition and travel) that your family can afford to pay.
- Any special circumstances you want us to consider.

Mathcamp 2010 Qualifying Quiz

Instructions

We call it a quiz, but it's really a challenge: a chance for you to show us how you approach new problems and new concepts in mathematics. What matters to us are not only your final results, but also your reasoning. Correct answers on their own will count for very little: you have to justify all of your assertions and ***prove*** to us that your solution is correct. (For some tips on writing proofs, see www.mathcamp.org/proofs.) Sometimes it may take a while to find the right way of approaching a problem. Be patient: there is no time limit on this quiz.

The problems start out easier and get harder. (At least we think so – but you may disagree.) None of the problems require a computer; you are welcome to use one if you'd like, but first see www.mathcamp.org/computers.

We don't expect every applicant to solve every problem: in the past, we have sometimes admitted people who could do only half of them, occasionally even fewer. However, don't just try four or five problems and declare yourself done! The more problems you attempt, the better your chances.We strongly recommend that you try all the problems and send us the results of your efforts: partial solutions, conjectures, methods – everything counts.

If you need clarification on a problem, please email quiz10@mathcamp.org. *You may not consult or get help from anyone else.* You can use books or the Web to look up definitions, formulas, or standard techniques, but any information obtained in this way must be clearly referenced in your solution. Please do not try to look for the problems themselves: we want to see how well you can do math, not how well you can use Google! Any deviation from these rules is considered plagiarism and may disqualify you.

Problems

(1) This problem is a variation of the famous Tower of Hanoi problem. If you don't know the original problem and its solution, check out http://mathforum.org/library/drmath/view/55956.html.

Suppose now that the three pegs are arranged in order – A, B, and C – and on a single move you can only move rings between A and B or between B and C. A ring can move from A to C or vice versa only in two moves (and then only provided that there isn't a smaller ring on B).

(a) What is the minimum number of moves required to move a stack of n rings from A to C? Prove your answer.

(b) What is the minimum number of moves required to move a stack of n rings from A to B? Prove your answer.

(2) Consider the following two-player game. Starting with the number 0, players take turns adding to the current sum; on your turn, you can add either 4 or 7. If on your turn you can make the new sum end in two zeros (i.e., if your turn leaves a multiple of 100), you win.

Assuming best play, is there a winning strategy for either player, or will the game go on indefinitely? If there is a winning strategy, should you move first or second, and how do you play from there?

(3) Suppose r and s are positive integers. Let F be a function from the set of all positive integers $\{1, 2, 3, \ldots\}$ to itself with the following properties:

- F is one-to-one and onto. (If you don't know these terms, look them up online.)
- For every positive integer n, either $F(n) = n+r$ or $F(n) = n-s$.

(a) If $r = 5$ and $s = 8$, what is $F(2010)$?

(b) Find, with proof, the smallest positive integer k such that the k-fold composition of F with itself is the identity function; that is, $F(F(\ldots F(n))) = n$ for all n (where there are k copies of F on the left-hand side). The answer will depend on r and s.

(4) For some positive integer n, the numbers 2^n and 5^n begin with the same 10 digits when written in base 10. What are these digits? You do not need to show that such an n exists, but you do need to prove that, assuming such an n exists, your answer is the only possible one.

Extra: Do you have any ideas about how to prove that such an n exists?

(5) Let a_n be the number of strings of length n that can be formed from the symbols X and O, with the restriction that a string may not consist entirely of multiple copies of one string of shorter length. For example, for $n = 4$, the string XXOX is allowed, while the strings XXXX and XOXO are not allowed. It is not hard to check that $a_4 = 12$.

Here is a table showing a_n and approximate values of the ratio a_{n+1}/a_n for different values of n.

n	1	2	3	4	5	6	7	8
a_n	2	2	6	12	30	54	126	240
a_{n+1}/a_n	1	3	2	2.5	1.8	2.333	1.905	2.1

The table suggests two conjectures:

(a) For any $n > 2$, the number a_n is divisible by 6,

(b) $\lim_{n\to\infty} a_{n+1}/a_n = 2$.

Prove or disprove each of these conjectures.

(6) For $0 \le x \le 1$, let $T(x) = x$ if $x \le 1/2$, and $T(x) = 1-x$ if $x \ge 1/2$. In other words, $T(x)$ is the distance from x to the nearest integer. Let

$$f(x) = \sum_{n=1}^{\infty} T(x^n).$$

Does there exist a value of x (still between 0 and 1) such that $f(x) = 2010$? Find all such values or show that none exist.

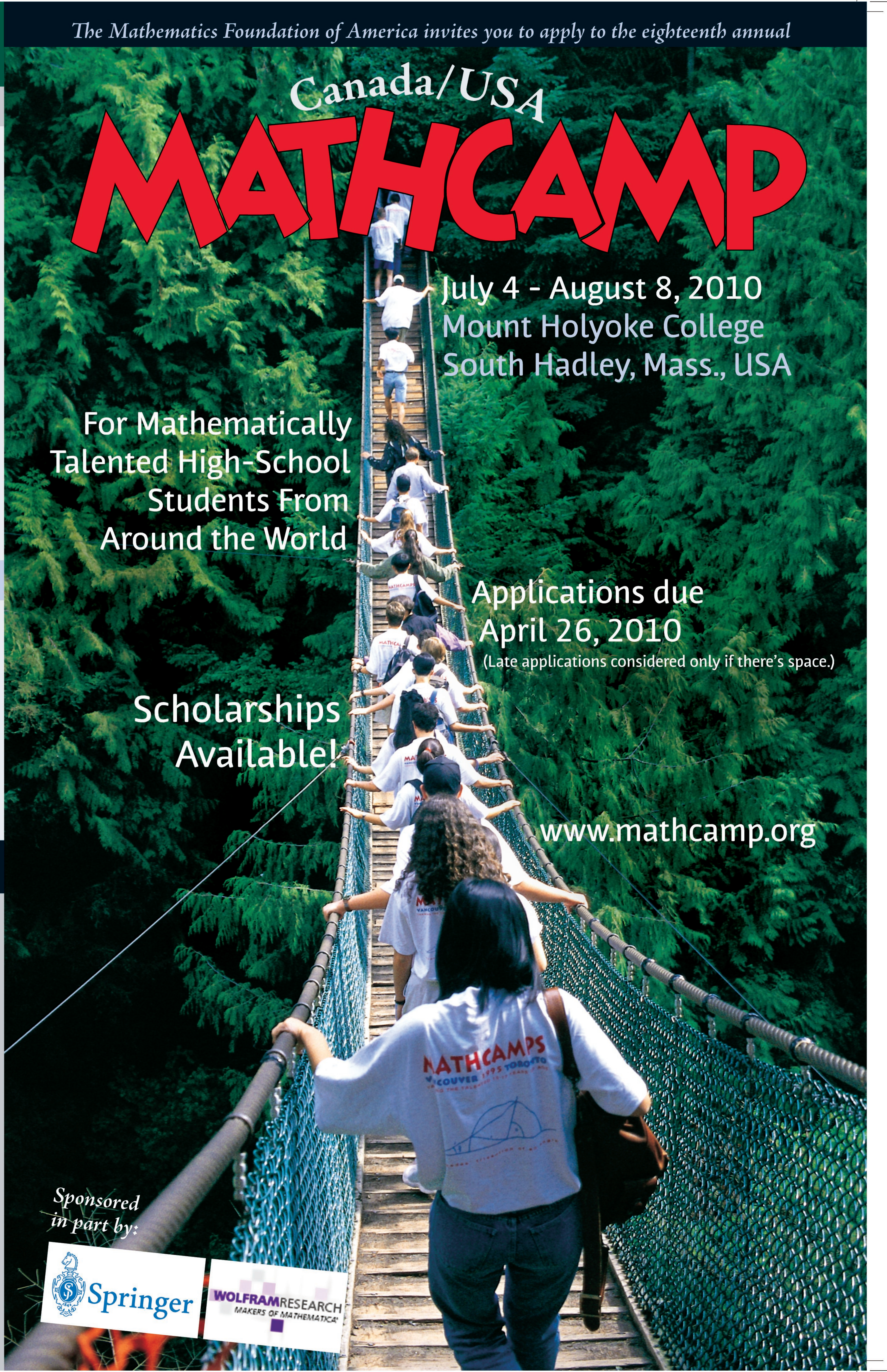
(7) For what values of M and N can an $M \times N$ chessboard be covered by an equal number of horizontal and vertical dominoes? (A domino always covers two adjacent squares on the board.)

(8) If we tile the plane with black and white squares in a regular checkerboard pattern, then every square has an equal number (four) of black and white neighbors. (Two squares are considered neighbors if they are not the same but have at least one common point; squares that touch just at a vertex count as neighbors.) But if we try the analogous pattern of cubes in 3-dimensional space, it no longer works this way.

(a) How many neighbors of each color does a white cube have?

(b) Find a coloring pattern for a grid of cubes in 3-dimensional space so that every cube, whether black or white, has an equal number of black and white neighbors.

(c) What happens in n-dimensional space for $n > 3$? Is it still possible to find a color pattern for a grid of hypercubes, so that every hypercube, whether black or white, has an equal number of black and white neighbors?



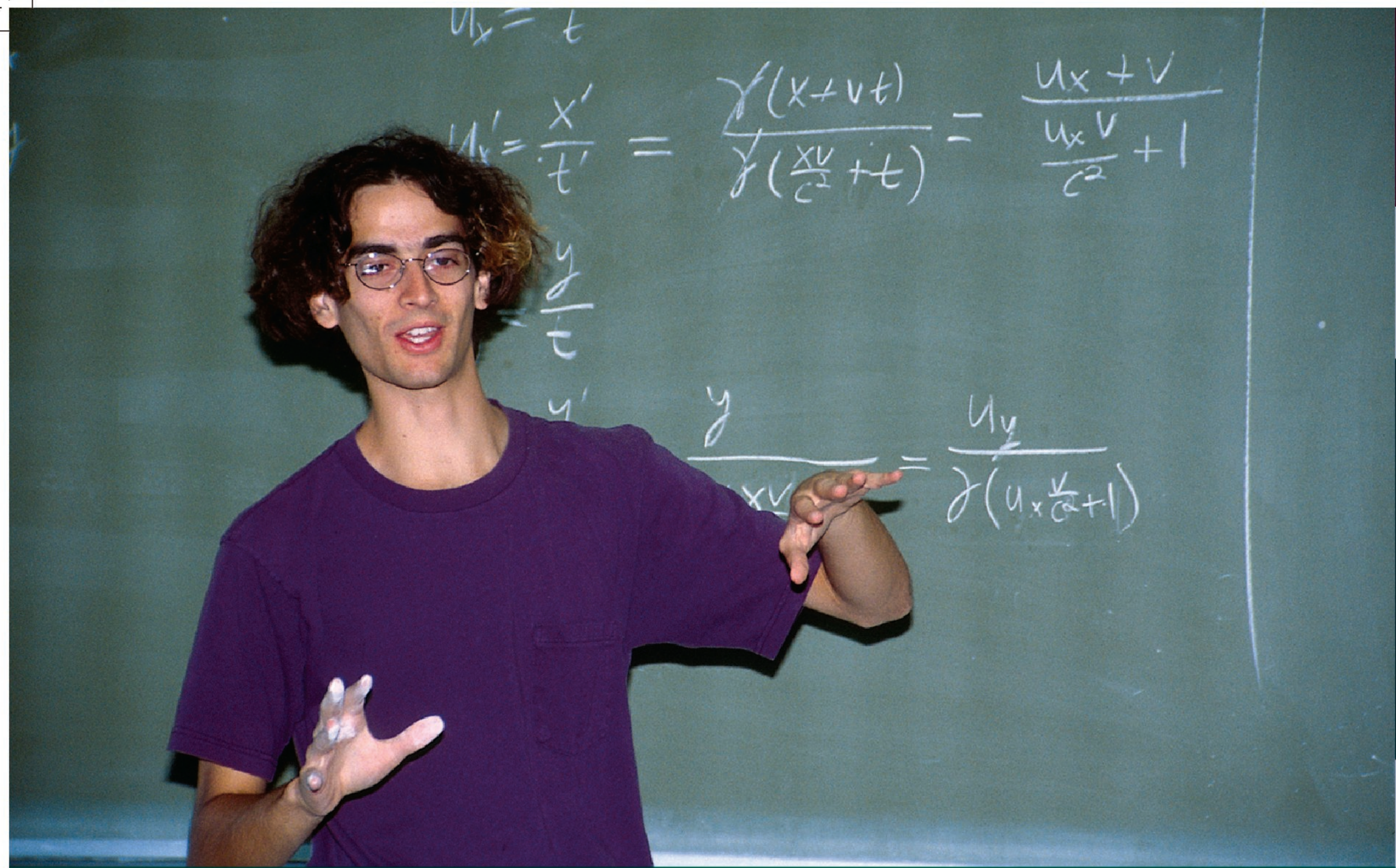
Discover Mathcamp!

"Out of nothing I have created a strange new universe."
– Janos Bolyai, co-discoverer of hyperbolic geometry

Mathcamp is a chance to...

- Live and breathe mathematics: fascinating, deep, difficult, fun, mysterious, abstract, interconnected (and sometimes useful).
- Gain mathematical knowledge, skills and confidence – whether for a possible career in math or science, for math competitions, or just for yourself.
- Set and pursue your own goals: choose your classes, do a project, learn what you want to learn.
- Study with mathematicians who are passionate about their subject, from internationally known researchers to graduate students at the start of their careers, all eager to share their knowledge and enthusiasm.
- Make friends with students from around the world, and discover how much fun it is to be around people who think math is cool.

"Mathcamp was the first place where I really understood the beauty and intricacies of abstract mathematics."
– Paul Hlebowitsch (Iowa City, IA, USA)



Academics

A Variety of Choices...

The Mathcamp schedule is full of activities at every level, from elementary to the most advanced:

- Courses lasting anywhere from a few days to five weeks
- Lectures and seminars by distinguished visitors
- Math contests and problem-solving sessions
- Hands on workshops and individual projects

You can learn more at:
http://www.mathcamp.org/academics

The Freedom to Choose

Mathcamp does not have a set curriculum or a list of requirements. We encourage the faculty to teach what they are most passionate about, and we let the students choose what they are interested in learning. With the help of an academic advisor, you will design a program of study that reflects your own interests and goals. You can take any classes you want, and even the number of classes that you attend each day is up to you: you can use your time to review what you've learned, talk to one of your professors, work on problems, do a project, or just take a break. For many students, the freedom to take charge of their own education is one of the aspects of Mathcamp that they value most.

Classes

Course offerings vary from year to year, depending on the interests of the students and faculty. Some of the topics taught in previous years have included:

Discrete Mathematics: Combinatorics • Generating functions • Partitions • Graph theory • Ramsey theory • Finite geometries • Polytopes and Polyhedra • Combinatorial Game Theory • Probability

Algebra and Number Theory: Primes and factorization algorithms • Congruences and quadratic reciprocity • Linear algebra • Groups, rings, and fields • Galois theory • Representation theory • p-adic numbers

Geometry and Topology: Euclidean and non-Euclidean (hyperbolic, spherical, projective) geometries • Geometric transformations • Combinatorial topology • Algebraic geometry • Knot theory • Brouwer Fixed-Point Theorem

Calculus and Analysis: Fourier analysis • Complex analysis • Real analysis • Measure theory • Dynamical systems • Non-standard analysis

Computer Science: Cryptography • Algorithms • Complexity • Information Theory • P vs. NP

Logic and Foundations: Cardinals and ordinals • Gödel's Incompleteness Theorem • The Banach-Tarski Paradox • Model theory • Category theory

Connections to Science: Relativity and quantum mechanics • Dimensional physics • Voting Theory • Bayesian Statistics • Neural networks • Mathematical biology • Cognitive Science

Discussions: History and philosophy of mathematics • Math Education • "How to Give a Math Talk" • College, Graduate School and Beyond

Problem Solving: Proof techniques • Elementary and advanced methods • Contest problems of various levels of difficulty • Weekly "Math Relays" and team competitions

Projects

Every student at Mathcamp is encouraged to do a project, supervised by one of the mentors or faculty. Projects range in scope from creative applications of simple techniques to advanced problems connected to faculty research. Project topics in previous years have included:

- Periodicity of Fibonacci numbers mod *n*
- Information theory and psychology
- Knight tours on an *m*-by-*n* chessboard
- Cellular automata
- Cops and robbers on a graph
- Constructing the regular 17-gon
- Admissible covers of algebraic curves
- Mathematical Finance
- Algorithmic composition of music
- Intelligent ways of searching the web
- Probability in sports
- The elasticity equation of string
- Digital signal processing
- Light paths in universes with alternate physics
- Playing 20 Questions with a Liar
- Dirichlet's Theorem on Arithmetic Progressions
- Non-Orientable Knitting

"One cannot compare my idea of what `I'm interested in math' meant before and after Mathcamp."
– Asaf Reich (Vancouver, BC, Canada)

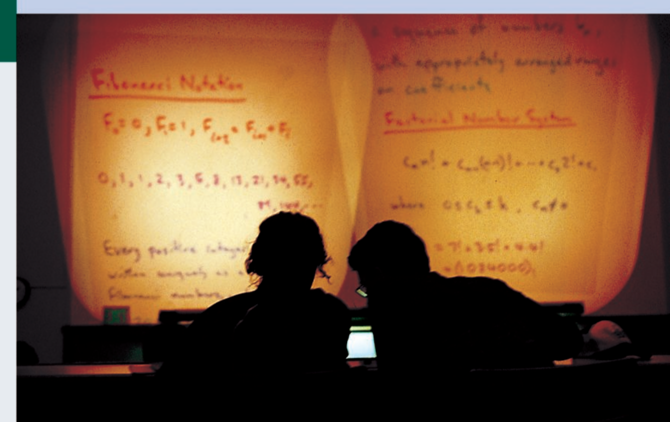
"There was no pressure: the incentive to learn came from within."
– Keigo Kawaji (Toronto, ON, Canada)

"I wish I had found Mathcmap years before... it is truly a wonderful place and a haven for young mathematicians."
– Amit Hazi (Los Angeles, CA, USA)

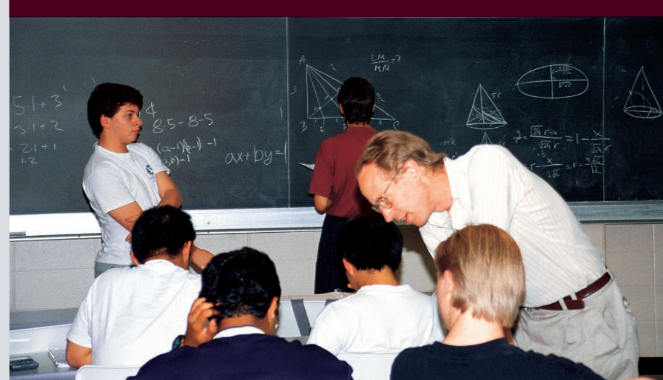
Spotlight on a Class

Set Theory (2009) • What is infinity? You may already know what the "size" of an infinite set is, and how it's possible for one infinity to be bigger than another. In this class, we'll go a lot further: we'll play with infinite numbers, and we'll learn how to do arithmetic with them. Infinite arithmetic turns out to be easier than ordinary arithmetic: if κ and λ are infinite numbers, then $\kappa+\lambda = \kappa\lambda = \max(\kappa, \lambda)$ is just whichever of κ and λ is larger.

Along the way, we'll develop a tool called transfinite induction, which is like a cross between induction and the Energizer Bunny: instead of stopping when you've gone through all the natural numbers, it just keeps going and going. Just as ordinary induction is very useful for proving theorems about the natural numbers, transfinite induction is an essential tool for proving things about infinite sets. It also has lots of applications outside of set theory (e.g., to prove the intermediate value theorem in analysis, or that any vector space has a basis in linear algebra). Finally, we'll look at what the universe of all sets looks like. We'll study the Zermelo-Fraenkel Axioms for set theory, and how each one of them is important for building up all the sets we need to do math (or in some cases, we'll find the axioms are unnecessary and can be proven from the other axioms!). We'll also see how to show that we can't prove an axiom from the other axioms.



"Mathcamp was definitely the most fun I've ever had."
– Avichal Garg (Cincinatti, OH, USA)



Mt. Holyoke College

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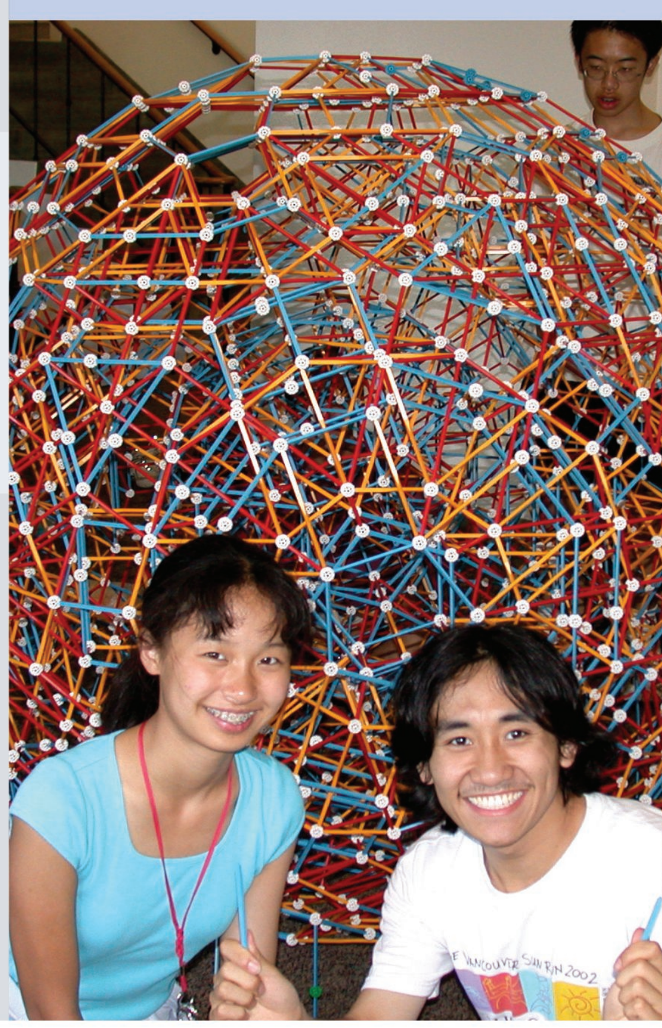
Site of Mathcamp 2010

"Mathcamp isn't really a camp. It's more of a five-week long festival - a congregation of people who celebrate math, enjoy math, learn math and essentially live math. Through it all I've discovered cool theorems that I wouldn't have understood before and cool people I didn't know existed. I've learnt that I actually know close to nothing about the weird and wonderful subject that is mathematics, and that I will probably pursue it for the rest of my life. Math on, Mathcamp!"
– YQ Lu (Singapore)

"It's not often that you find a place that is exciting to the mind and liberating to the spirit. Mathcamp is both."
– Greg Burnham (Memphis, TN, USA)

"I've changed so much in my two years here. I think about math in a new, deeper way. I approach problems differently. I've gained perserverance and learned to ask for help without shame and give it with joy."
– Hallie Glickman-Hoch (Brooklyn, NY, USA)



"Go, just go! Trust me!"
– Jian Xu (Toronto, ON, Canada)

People and More

Regular Faculty

Mira Bernstein (Executive Director, Mathcamp)
Interests: Algebraic Geometry, Mathematical Biology, Information Theory

Mark Krusemeyer (Carleton College)
Interests: Abstract Algebra, Combinatorics, Number Theory, Problem Solving

David Savitt (University of Arizona)
Interests: Number Theory, Arithmetic Geometry

Kenny Easwaran (University of Southern California)
Interests: Formal Epistemology, Philosophy of Math, Philosophy of Probability, Decision Theory

Visiting Faculty

John H. Conway (Princeton) • One of the most creative thinkers of our time, John Conway is known for his groundbreaking contributions to such diverse fields as knot theory, geometry of high dimensions, group theory, transfinite arithmetic, and the theory of mathematical games. Outside the mathematical community, he is perhaps best known as the inventor of the "Game of Life."

Allan Adams (MIT) • Allan Adams works on quantum versions of algebraic and differential geometry, and uses black holes in 5 spacetime dimensions to study high-temperature superconductors in the usual 4.

Moon Duchin (University of Michigan) • Moon Duchin works in geometric topology and geometric group theory. She particularly looks at the large-scale geometric structure of groups and unusual metric spaces. She also thinks about philosophy, cultural studies, gender theory, what they have to say about math, and what math has to say back!

Tina Eliassi-Rad (Center for Applied Scientific Computing, Lawrence Livermore National Lab) • Tina Eliassi-Rad is a computer scientist, and her research interests include data mining, artificial intelligence, and machine learning. Her work has been applied to the World-Wide Web, text corpora, complex networks, and large-scale scientific simulation data.

Branden Fitelson (U.C. Berkeley) • Branden Fitelson is a professor of Philosophy and a core faculty member in Cognitive Science and Logic and in Methodology of Science. His research is at the intersection of philosophy of science, epistemology, logic, and cognitive science. He is writing a book on the historical, philosophical, and cognitive-scientific significance and applications of Confirmation Theory.

George Hart (SUNY Stony Brook) • George Hart is both a computer science professor and a mathematical sculptor. He leads hands-on workshops in which participants explore the geometry of three- (and four-) dimensional space using the mathematical construction set Zometool.

Josh Tenenbaum (MIT) • Josh Tenenbaum is a professor of Cognitive Science and a member of the MIT Computer Science and Artificial Intelligence Lab. In his research, he builds mathematical models of human and machine learning, reasoning, and perception. He is also interested in neural networks, information theory, and statistical inference.

Students

We never cease to be amazed at what a varied and interesting bunch of young men and women our students are! While everyone at camp shares a love of mathematics, their other interests run the gamut. Each year's camp is a collection of 110 students who are musicians and writers, jugglers, dancers, athletes and actors, artists, board game players, hikers, computer programmers, students of science and philosophy - all sharing their interests and experiences with each other.

Most of the students at camp come from North America, but many come from overseas. Students have come to camp from Bulgaria, India, Japan, Lithuania, Macedonia, Mexico, Poland, Romania, Russia, Serbia, South Korea, Tanzania, Turkey, and many other places around the globe.

It is a testament to our students' maturity and independence that they can be serious about doing math, while still finding so many different ways to have fun. Many camp activities are organized entirely by campers, and students routinely cite each others' company as one of the best aspects of camp.

Mentors and Junior Counselors

The residential staff at camp is made up of Mentors and Junior Counselors ("JCs"). Mentors are graduate students in mathematics and computer science; they teach most of the classes at camp, picking the course topics freely from among their favorite kinds of math. JCs, all of them camp alumni, are undergraduates who run the non-academic side of camp (from field trips to first aid to frisbee games). Each student is assigned a Mentor or JC as their residential advisor; RAs live on the same hall as their advisees and look out for them on a day-to-day basis. Like campers, the Mentors and JCs often return year after year to Mathcamp.

Beyond Math

Mathematical activities are scheduled for five days a week; whatever math happens on the other two days is purely informal. The weekend is reserved for relaxation and the incredible number of activities that quickly fill the schedule. All of these activities are optional, and students can choose simply to relax with friends.

Field trips in the past have included hiking, sea kayaking, whitewater rafting, amusement parks, and museums. Lots of activities happen on-campus, too: there are rehearsals for the choir and the contemporary a cappella group, juggling and contra dancing lessons, improv, and even cupcake decorating. There is an annual team "puzzle hunt" competition, a talent show, and a lot of ice cream made with liquid nitrogen. Campers also organize many events themselves – from sports and music to chess and bridge tournaments – and each year, a group of students creates the camp yearbook.

"Coming to Mathcamp has given me a community with which to interact, not just five weeks a year, but all year round."
– Eric Wofsey (St Louis, MO, USA)