Applying to Mathcamp 2011

Ready to Apply to Mathcamp?

We invite applications from every student aged 13 through 18 who is interested in mathematics, regardless of racial, ethnic, religious, or economic background.

Mathcamp accepts applications both on the web and by regular mail. We strongly encourage all students with Internet access to use the online application process. The $20 application fee is waived for online applications.

Online Application:
Go to http://www.mathcamp.org/apply/ and follow the instructions. You'll still have the opportunity to submit your quiz or recommendation letters by postal mail.

Postal Application:
Go to http://www.mathcamp.org/applybymail/ and print out the application packet.

An application to Mathcamp consists of the following:

1) Some basic information about **yourself and your math background**. We will ask you to describe the math courses that you've taken at the high-school level or above, along with scores and awards from any math competitions you've done.

2) A brief **personal statement** about your interest in math and why you want to come to Mathcamp.

3) Your solutions to the 2011 **Qualifying Quiz** (see below).

4) **Two recommendation letters**, academic and personal.

- The first letter should be from a teacher who knows you well, preferably a math teacher. The letter should comment on your creativity, initiative, and ability to work with others, as well as on your academic achievements.
- The second letter should be from another adult who knows you personally (e.g. an employer, pastor, soccer coach, etc. – preferably someone outside of school and not a relative). This letter should address your maturity, independence, social and personal qualities. We are looking for students who are not only good at math, but who will thrive in the atmosphere of freedom and responsibility that characterizes Mathcamp, and who will make a positive contribution to the camp community.

5) If you would like to be considered for financial assistance, please include the **scholarship application** (see instructions below). Note that admission to Mathcamp is need-blind.

6) For postal applications only: A **US $20 application fee** (check or money order made out to Mathematics Foundation of America) or a note signed by your parent or guardian explaining that your family cannot afford it.

All applications received by April 27, 2011 will be given equal consideration.

Contact Us

Email: info11@mathcamp.org

Telephone/Fax: 888-371-4159

Postal address:
Mathcamp 2011
129 Hancock Street
Cambridge, MA 02139

***Please do not send certified mail to Mathcamp;** we will email you to verify that your application has arrived.*

Student Care Policy

Dear parent: Student safety and enjoyment are Mathcamp's first priorities. Students will be housed in secure campus dormitories, with male and female students in designated sections of the same building. In case of a medical problem, we have a camp nurse on call, and the hospital is minutes away. Students will have access to university athletic facilities and computers. Every effort will be made to enable students who so desire to attend weekly religious services of their faith. Mathcamp is committed to an atmosphere of mutual tolerance, responsibility, and respect, and is proud of its past record in helping to create such an atmosphere.

- Mira Bernstein, Executive Director, Mathcamp

Cost and Scholarships

Full Camp Fee: $4000

(This includes tuition, room, board, and extracurriculars.)

Admission to Mathcamp is need-blind. We are deeply committed to enabling every qualified student to attend, regardless of financial circumstances.

Mathcamp awards over $100,000 in need-based scholarships every year. In the past six years, no admitted applicant has been unable to attend the camp for financial reasons. We give several full scholarships each year, and occasionally even help students with travel expenses. Please do not let financial considerations prevent you from applying! If you'd like to be considered for a scholarship, just complete the short application at right.

Scholarship Application

Please have a parent or guardian provide the following information, along with her or his email address:

- 2010 family income (all sources).
- Expected family income for 2011. (If significantly different from 2010, please explain.)
- A list of all members of your household (supported by the above income) and their relationships to the applicant. For siblings, please provide ages.
- The cost of schooling, if any, for household members (private school, college, etc).
- The estimated cost of round-trip travel to Mathcamp for the applicant.
- The portion of the cost of Mathcamp (including both tuition and travel) that your family can afford to pay.
- Any special circumstances you want us to consider.

Mathcamp 2011 Qualifying Quiz

Instructions

We call it a quiz, but it's really a challenge: a chance for you to show us how you approach new problems and new concepts in mathematics. What matters to us are not only your final results, but also your reasoning. Correct answers on their own will count for very little: you have to justify all of your assertions and ***prove*** to us that your solution is correct. (For some tips on writing proofs, see www.mathcamp.org/proofs.) Sometimes it may take a while to find the right way of approaching a problem. Be patient: there is no time limit on this quiz.

The problems start out easier and get harder. (At least we think so – but you may disagree.) None of the problems require a computer; you are welcome to use one if you'd like, but first see www.mathcamp.org/computers.

We don't expect every applicant to solve every problem: in the past, we have sometimes admitted people who could do only half of them, occasionally even fewer. However, don't just try four or five problems and declare yourself done! The more problems you attempt, the better your chances. We strongly recommend that you try all the problems and send us the results of your efforts: partial solutions, conjectures, methods – everything counts.

If you need clarification on a problem, please email quiz11@mathcamp.org. *You may not consult or get help from anyone else.* You can use books or the Web to look up definitions, formulas, or standard techniques, but any information obtained in this way must be clearly referenced in your solution. Please do not try to look for the problems themselves: we want to see how well you can do math, not how well you can use Google! Any deviation from these rules is considered plagiarism and may disqualify you.

Have fun and good luck!

Problem 6 copyright Mark Krusemeyer.

Problems

(1) A betting company offers the possibility to bet on any of the n contestants in a race, with odds $a_1, \ldots, a_n$, respectively. This means that if you bet X dollars on contestant j (for any positive real number X), then:

- If contestant j loses, you lose your X dollars.
- If contestant j wins, you get your X dollars back, together with a profit of $a_j X$ dollars.

If the company does not choose the odds wisely, it may be possible to bet on all of the contestants in a manner that guarantees a profit no matter what. For instance, if there are 3 contestants with odds 2, 2, and 10, you could bet $1 on each of the first two and $0.50 on the third one, guaranteeing a profit no matter who wins the race.

(a) What relation do the real numbers $a_1, \ldots, a_n$ need to satisfy so that it is impossible to guarantee a profit in this way?

(b) What is largest profit that you can guarantee on a total bet of $1 if the relation is ***not*** satisfied?

(2) The function f is defined on the positive integers by the three formulas $f(1) = 1$, $f(2n) = 2f(n) + 2n$, and $nf(2n+1) = (2n+1)f(n)$ for all $n \geq 1$.

(a) Prove that $f(n)$ is always an integer.

(b) For what values of n does the equality $f(n) = n$ hold? (Be sure to prove that no other value works.)

(3) (Proposed by Michael Wu, a student at Mathcamp 2009 & 2010.) There are n children equally spaced around a merry-go-round with n seats, waiting to get on. The children climb onto the merry-go-round one by one (but not necessarily going in order around the circle), always using the seat in front of them and only taking a seat if it is empty. After one child climbs on and takes a seat, the merry-go-round rotates $360/n$ degrees counterclockwise so that each remaining child is again lined up with a seat. For what values of n is it possible for the children to climb on, in some order, so that everyone gets a seat? (You must prove both that it's possible for the values you claim and that it's impossible for all other values.)

(4) A country has n airports served by k airlines. Not every pair of airports is connected by a direct flight, but if you don't mind stopovers, you can get from any airport to any other. Also, if a direct flight between a pair of cities exists, you can travel on it in either direction.

(a) To connect n airports, you need to have at least $n-1$ direct flights. (You may assume this without proof.) Unfortunately, airlines in this country often go bankrupt. The airlines are required to coordinate their routes so that if any one of the airlines goes bankrupt, a traveler can still get from any airport to any other. What is the smallest number of direct flights that could be offered by all the airlines together? (Note: whenever you are asked to find the smallest number with a certain property, you need to prove both that your number has the required property and that no smaller number works.)

(b) Suppose $n = 7$, $k = 5$, and you want to be able to get from any airport to any other even if any two airlines go bankrupt. What is the smallest number of direct flights needed in this scenario?

(5) Bianca and Yuval are playing a game. Yuval thinks of an integer greater than or equal to 1000, but does not reveal it to Bianca. Then Bianca names an integer greater than 1. If Yuval's number is divisible by Bianca's, Bianca wins. Otherwise, Yuval subtracts Bianca's number from his own; the resulting difference is his new number. The game continues, with Bianca naming a different integer each time (no repeats allowed!) and Yuval subtracting each integer she names, until either Yuval's number is divisible by Bianca's (in which case Bianca wins) or Yuval's number becomes negative (in which case Yuval wins).

(a) Obviously, Yuval does not have a winning strategy in this game: whatever number he picks, Bianca might get lucky. Does Bianca have one?

(b) Suppose we replace the number 1000 in the problem by a different positive integer N. For which values of N, if any, does Bianca have a winning strategy?

(6) Three numbers are arranged around a circle; at time $t = 0$, the numbers are 0, 1, 2, reading counterclockwise. Thereafter, the numbers are adjusted each unit of time, as follows: If the two neighbors of a number m are equal, then that number m will stay the same for the next unit of time. If the counterclockwise neighbor of a number m is less than its clockwise neighbor, then the number m will be raised by 1. If the counterclockwise neighbor of a number m is greater than its clockwise neighbor, then the number m will be reduced by 1. The first few steps of this process yield the following:

Time	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
Numbers	0, 1, 2	1, 0, 3	2, −1, 2	3, −1, 1	4, 0, 0	4, 1, −1

Including $t = 0$, what is the 2011th time that at least one of the three numbers is zero?

(7) (Proposed by Josh Frisch, a student at Mathcamp 2010.) Consider the interval $(0, 1]$, which is the set of real numbers x with $0 < x \leq 1$. A subset S of the interval $(0, 1]$ is called *evil* if, for each positive integer n:

- Its reciprocal $1/n$ is in S.
- If x is in S then so is the average of x and $1/n$.

Must an evil set contain every rational number in the interval $(0, 1]$?

(8) There are N cyclists riding counterclockwise around a circular track, each one at a different constant speed. The track is very narrow: there is only one point on it where passing is possible (but at this point, the track is wide enough that any number of bicycles can pass there simultaneously). Somehow the cyclists have arranged it so that they can keep going as long as they want, always passing each other at exactly this point.

(a) For $N = 2$, what condition must the two cyclists' speeds satisfy?

(b) For what values of N is this situation possible?

The Mathematics Foundation of America invites you to apply to the nineteenth annual

Discover Mathcamp!

"Out of nothing I have created a strange new universe."
– Janos Bolyai, co-discoverer of hyperbolic geometry

Mathcamp is a chance to...

- Live and breathe mathematics: fascinating, deep, difficult, fun, mysterious, abstract, interconnected (and sometimes useful).
- Gain mathematical knowledge, skills and confidence – whether for a possible career in math or science, for math competitions, or just for yourself.
- Set and pursue your own goals: choose your classes, do a project, learn what you want to learn.
- Study with mathematicians who are passionate about their subject, from internationally known researchers to graduate students at the start of their careers, all eager to share their knowledge and enthusiasm.
- Make friends with students from around the world and discover how much fun it is to be around people who think math is cool.

"Mathcamp was the first place where I really understood the beauty and intricacies of abstract mathematics."
– Paul Hlebowitsch (Iowa City, IA, USA)

Academics

A Variety of Choices

The Mathcamp schedule is full of activities at every level, from introductory to the most advanced:

- Courses lasting anywhere from a few days to five weeks
- Lectures and seminars by distinguished visitors
- Math contests and problem-solving sessions
- Hands-on workshops and individual projects

You can learn more at:
http://www.mathcamp.org/academics

The Freedom to Choose

Mathcamp does not have a set curriculum or a list of requirements. We encourage the faculty to teach what they are most passionate about, and we let the students choose what they are interested in learning. With the help of an academic advisor, you will design a program of study that reflects your own interests and goals. You can take any classes you want, and even the number of classes that you attend each day is up to you: you can use your time to review what you've learned, talk to one of your professors, work on problems, do a project, or just take a break. For many students, the freedom to take charge of their own education is one of the aspects of Mathcamp that they value most.

Classes

Course offerings vary from year to year, depending on the interests of the students and faculty. Some of the topics taught in previous years have included:

Discrete Mathematics: Combinatorics • Generating functions • Partitions • Graph theory • Ramsey theory • Finite geometries • Polytopes and Polyhedra • Combinatorial Game Theory • Probability

Algebra and Number Theory: Primes and factorization algorithms • Congruences and quadratic reciprocity • Linear algebra • Groups, rings, and fields • Galois theory • Representation theory • p-adic numbers

Geometry and Topology: Euclidean and non-Euclidean (hyperbolic, spherical, projective) geometries • Geometric transformations • Combinatorial topology • Algebraic geometry • Knot theory • Brouwer Fixed-Point Theorem

Calculus and Analysis: Fourier analysis • Complex analysis • Real analysis • Measure theory • Dynamical systems • Non-standard analysis

Computer Science: Cryptography • Algorithms • Complexity • Information Theory • P vs. NP

Logic and Foundations: Cardinals and ordinals • Gödel's Incompleteness Theorem • The Banach-Tarski Paradox • Model theory • Category theory

Connections to Science: Relativity and quantum mechanics • Dimensional physics • Voting Theory • Bayesian Statistics • Neural networks • Mathematical biology • Cognitive Science

Discussions: History and philosophy of mathematics • Math Education • "How to Give a Math Talk" • College, Graduate School and Beyond

Problem Solving: Proof techniques • Elementary and advanced methods • Contest problems of various levels of difficulty • Weekly "Math Relays" and team competitions

Projects

Every student at Mathcamp is encouraged to do a project, supervised by one of the mentors or faculty. Projects range in scope from creative applications of simple techniques to advanced problems connected to faculty research. Project topics in previous years have included:

- Periodicity of Fibonacci numbers mod n
- Information theory and psychology
- Knight tours on an m-by-n chessboard
- Cellular automata
- Cops and robbers on a graph
- Constructing the regular 17-gon
- Admissible covers of algebraic curves
- Mathematical Finance
- Algorithmic composition of music
- Intelligent ways of searching the web
- Probability in sports
- The elasticity equation of string
- Digital signal processing
- Light paths in universes with alternate physics
- Playing 20 Questions with a Liar
- Dirichlet's Theorem on Arithmetic Progressions
- Non-Orientable Knitting

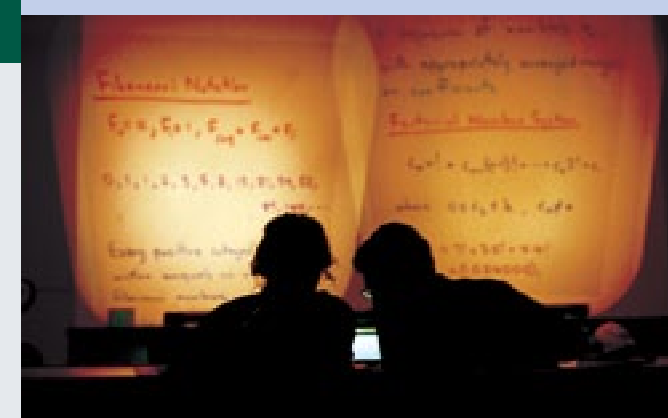
"One cannot compare my ideas of what `I'm interested in math' meant before and after Mathcamp."
– Asaf Reich (Vancouver, BC, Canada)

"There was no pressure: the incentive to learn came from within."
– Keigo Kawaji (Toronto, ON, Canada)

"Mathcamp took every limitation I thought I had—social, academic, and personal—and shattered it."
– Andrew Kim (Dover, MA, USA)

Spotlight on a Class

Information Theory (2010) • In 1948, Claude Shannon published a paper called "A Mathematical Theory of Communication." By the time the paper came out as a book in 1949, its name had changed to "The Mathematical Theory of Communication": it took only a year for people to realize that what Shannon had invented was *the* theory. His book, dubbed "the Magna Carta of the Information Age", was the mathematical foundation of the digital revolution. Shannon's insight was to look beyond the variety of communication channels available in his day—telegraph, radio, TV, plain old writing and speech—to formulate a single mathematical framework for the transmission of information (a concept that he was the first to define mathematically). Shannon showed that any channel, even a very noisy one, with lots of errors and distortion, has a certain rate at which it can transmit information virtually error-free. Anything up to that rate is possible, at least in theory; anything beyond it is hopeless. In our course, we started by rederiving Shannon's definition of information (and learned a lot of probability theory along the way). We talked about the "redundancy of English" (which Shannon estimated in a really clever way) and what this has to do with file compression. Finally, we tackled the long and difficult proof of Shannon's amazing result, the "noisy channel coding theorem", and discussed its connection to error-correcting codes.



"Mathcamp was definitely the most fun I've ever had."
– Avichal Garg (Cincinatti, OH, USA)

Reed College

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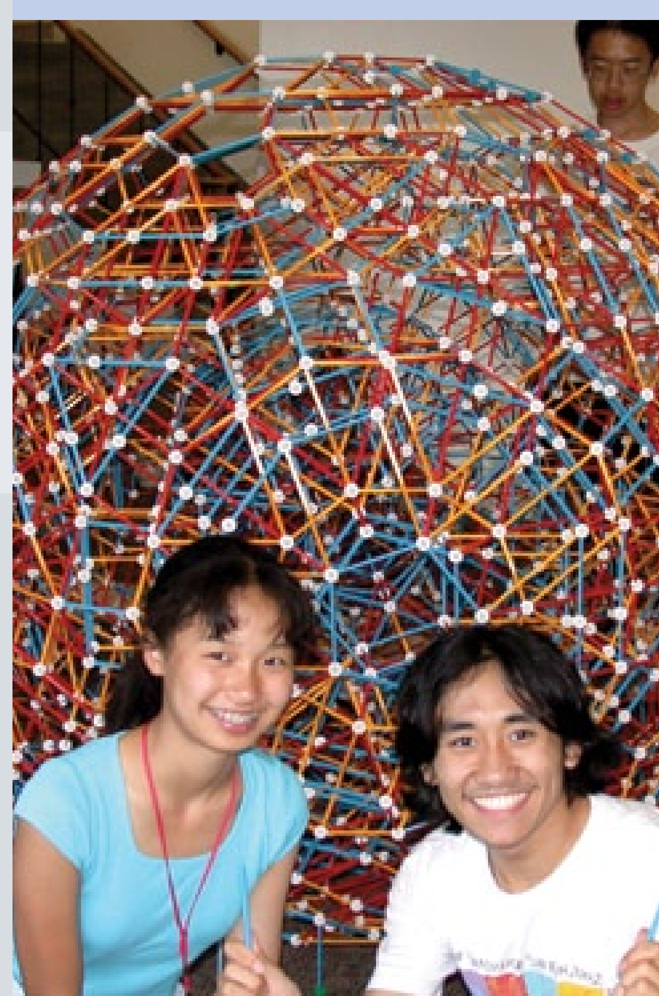
Site of Mathcamp 2011

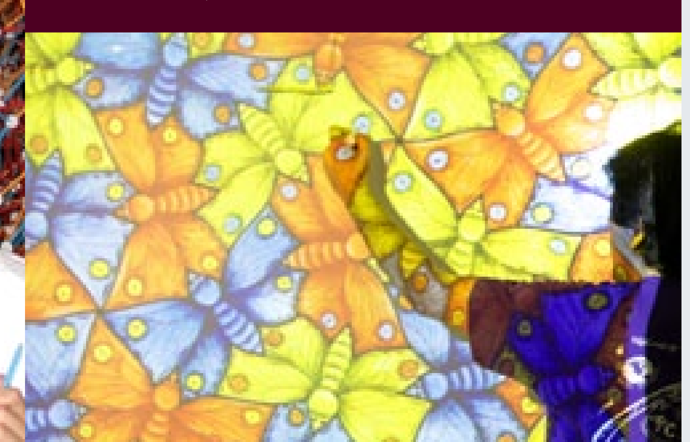
"Mathcamp isn't really a camp. It's more of a five-week long festival - a congregation of people who celebrate math, enjoy math, learn math and essentially live math. Through it all I've discovered cool theorems that I wouldn't have understood before and cool people I didn't know existed. I've learnt that I actually know close to nothing about the weird and wonderful subject that is mathematics, and that I will probably pursue it for the rest of my life. Math on, Mathcamp!"
– Yongquan Lu (Singapore)

"It's not often that you find a place that is exciting to the mind and liberating to the spirit. Mathcamp is both."
– Greg Burnham (Memphis, TN, USA)

"I've changed so much in my two years here. I think about math in a new, deeper way. I approach problems differently. I've gained perseverence and learned to ask for help without shame and give it with joy."
– Hallie Glickman-Hoch (Brooklyn, NY, USA)



"Go, just go! Trust me!"
– Jian Xu (Toronto, ON, Canada)



People and More

Regular Faculty

Mira Bernstein (Executive Director, Mathcamp)
Interests: Algebraic Geometry, Mathematical Biology, Information Theory

Mark Krusemeyer (Carleton College)
Interests: Abstract Algebra, Combinatorics, Number Theory, Problem Solving

David Savitt (University of Arizona)
Interests: Number Theory, Arithmetic Geometry

Yvonne Lai (University of Michigan)
Interests: Geometric Group Theory, Hyperbolic Geometry, Mathematical Knowledge for Teaching

Visiting Faculty

John H. Conway (Princeton) • One of the most creative thinkers of our time, John Conway is known for his ground-breaking contributions to such diverse fields as knot theory, geometry of high dimensions, group theory, transfinite arithmetic, and the theory of mathematical games. Outside the mathematical community, he is perhaps best known as the inventor of the "Game of Life."

Avi Wigderson (Institute for Advanced Study) • Avi Wigderson studies Computational Complexity Theory, the mathematical foundations of computer science. This field tries to understand questions like "is multiplication harder than addition?" and "is discovery harder than verification?" (the latter is the famous P vs. NP problem). Favorite result: a zero-knowledge proof of every theorem.

Allan Adams (MIT) • Allan Adams works on quantum versions of algebraic and differential geometry, and uses black holes in 5 spacetime dimensions to study high-temperature superconductors in the usual 4.

Jim Belk (Bard College) • Jim Belk works in the areas of topology, geometry, group theory, and dynamical systems. He is fascinated by the connections between group theory and fractal geometry, and he looks forward to exploring these connections with the participants at Mathcamp.

Moon Duchin (University of Michigan) • Moon Duchin works in geometric topology and geometric group theory. She particularly looks at the large-scale geometric structure of groups and unusual metric spaces. She also thinks about philosophy, cultural studies, gender theory, what they have to say about math, and what math has to say back!

Sarah Koch (Harvard) • Sarah Koch loves to look at pictures of the beautiful fractals that emerge from her study of complex dynamical systems. She also studies connections of group theory to Teichmüller theory and topology; in particular, she is quite fond of classifying twisted rabbits.

Josh Tenenbaum (MIT) • Josh Tenenbaum is a professor of Cognitive Science and a member of the MIT Computer Science and Artificial Intelligence Lab. In his research, he builds mathematical models of human and machine learning, reasoning, and perception. He is also interested in neural networks, information theory, and statistical inference.

Students

We never cease to be amazed at what a varied and interesting bunch of young men and women our students are! While everyone at camp shares a love of mathematics, their other interests run the gamut. Each year's camp is a collection of 110 students who are musicians and writers, jugglers, dancers, athletes and actors, artists, board game players, hikers, computer programmers, students of science and philosophy - all sharing their interests and experiences with each other.

Most of the students at camp come from North America, but many come from overseas. Students have come to camp from Bulgaria, India, Japan, Lithuania, Macedonia, Mexico, Poland, Romania, Russia, Serbia, South Korea, Tanzania, Turkey, and many other places around the globe.

It is a testament to our students' maturity and independence that they can be serious about doing math, while still finding so many different ways to have fun. Many camp activities are organized entirely by campers, and students routinely cite each others' company as one of the best aspects of camp.

Mentors and Junior Counselors

The residential staff at camp is made up of Mentors and Junior Counselors ("JCs"). Mentors are graduate students in mathematics and computer science; they teach most of the classes at camp, picking the course topics freely from among their favorite kinds of math. JCs, all of them camp alumni, are undergraduates who run the non-academic side of camp (from field trips to first aid to frisbee games). Each student is assigned a Mentor or JC as their residential advisor; RAs live on the same hall as their advisees and look out for them on a day-to-day basis. Like campers, the Mentors and JCs often return year after year to Mathcamp.

Beyond Math

Mathematical activities are scheduled for five days a week; whatever math happens on the other two days is purely informal. The weekend is reserved for relaxation and the incredible number of activities that quickly fill the schedule. All of these activities are optional, and students can choose simply to relax with friends.

Field trips in the past have included hiking, sea kayaking, whitewater rafting, amusement parks, and museums. Lots of activities happen on-campus, too: there are rehearsals for the choir and the contemporary a cappella group, juggling and salsa dancing lessons, improv, and even making berry pies. There is an annual team "puzzle hunt" competition, a talent show, and ice cream made with liquid nitrogen. Campers also organize many events themselves—from sports and music to chess and bridge tournaments—and each year, a group of students creates the camp yearbook.

"Coming to Mathcamp has given me a community with which to interact, not just five weeks a year, but all year round."
– Eric Wofsey (St Louis, MO, USA)