CLASS DESCRIPTIONS—WEEK 4, MATHCAMP 2013

Contents

9:10 AM Classes

Category Theory. (🌶🌶🌶🌶, Waffle, Tue–Sat)
In the Category of Sets, you got an introduction to category theory, but we focused on only one category: <u>Set</u>. But there are lots of other categories! For every kind of object studied in abstract algebra (and also many other areas of math), there's a category. Every poset is a category. Every group is itself a category. There's even a category of categories!

In this class, we'll explore this wider world of categories. Accompanying us on this journey will be our familiar operations of addition and multiplication from sets, and we'll see what guises they take in different categories. We'll also investigate their cousins, a very general class of operations called colimits and limits which include (among other things) kernels, quotient objects, and inverse limits.

Homework: Recommended

Prerequisites: Category of Sets and a strong familiarity with abstract algebra; it's recommended to have taken at least two abstract algebra classes before. The more of groups, rings, and vector spaces you feel comfortable thinking about, the better. If you want to refresh your memory of Category of Sets, the notes are available at: `http://math.harvard.edu/~waffle/catset13.pdf`.

Curvature of Polyhedra. (🌶, Nic, Tue–Sat)
How could you prove the Earth is curved without ever leaving the surface? Here's one completely ridiculous and impractical way. Start walking south from the north pole holding a stick directly in front of you the whole time. When you get to the equator, turn left, but keep the stick pointing in the same direction, so that now it points to your right. Walk a quarter of the way around the planet, then turn left again, walking due north until you hit the north pole. You'll find that, even though you kept the stick pointing the same way the whole time, once you get back where you started the stick has made a ninety-degree turn! (Try this with your finger on a small ball if you're not convinced.)

This could never have happened if the Earth were flat. In fact, there's another strange thing about this example: the path you traced is a triangle with three right angles. These two facts—that the sum of the angles is too big and that your stick isn't pointing the same way it was when you started—turn out to be fundamentally related to each other; they're two different manifestations of the fact that the surface of the Earth is *positively curved.*

In this class we'll explore the phenomenon of curvature from the perspective of polyhedra. We'll look at and draw on lots of paper models of polyhedra to learn the different ways to determine their curvature, and we'll explore the strange things that can happen when a surface is very positively or negatively curved.

Homework: Recommended

Prerequisites: None.

Elections: Influence and Stability. (🌶🌶, Tim!, Tue–Sat)
People often complain that their vote doesn't matter. And indeed, in a majority election, your vote can only change the outcome if all the other votes are split evenly. But we don't decide all of our elections by majority; for example, another voting system we use is the electoral college. Does the electoral college or a straight majority give each voter more influence over the outcome of the election? Is there any voting system that can do better than both of them?

In this class, we'll assign cold, hard scores to different voting systems (for elections with two candidates)—"influence" will be one kind of score, and another will be "noise stability"—and we'll see which voting systems do best. It turns out that there are simple, beautiful formulas for these quantities.

And, oh yeah, those formulas are in terms of Fourier coefficients. This class is secretly about Fourier analysis on the boolean cube (shhhh). But don't worry if you haven't seen the Fourier transform before; you've seen *this* Fourier transform even if you've never heard the word "Fourier", and no calculus is involved.

This class is disjoint from Alfonso's Voting Theory class, with the exception that we'll also prove Arrow's impossibility theorem at the end of the week, but it'll be a completely different proof, using the Fourier coefficients. *Warning:* Your faith in democracy may be restored after this class.

Homework: Required

Prerequisites: None

Related to: Harmonic Analysis, Voting Theory, Extreme Probability

Representations of the Symmetric Group. (🌶🌶🌶, Alex & Kevin, Tue–Sat)
The representations of a group carry a lot of structure that tells you a lot about the group itself. Unfortunately, representation theory is really hard. Fortunately, the representation theory of symmetric groups, groups of permutations on finite sets, is quite elegant and carries a number of combinatorial interpretations. In this class, we'll use combinatorial ideas, specifically those from the study of tableaux, to do the hard work of representation theory for us. By the end of the class, we will succeed in constructing all of the irreducible representations of symmetric groups via tableaux, and conversely, we will obtain combinatorial results about tableaux through representation theory.

Homework: Recommended

Prerequisites: Week 1 of Representation Theory or equivalent

Related to: Symmetric Polynomials, Representation Theory

10:10 AM Classes

Auction Theory. (🌶🌶🌶, *Glenn Ellison*, Fri–Sat)
Auctions are used to buy and sell all kinds of things: people sell millions of items a year through eBay; Sotheby's auctions million-dollar paintings; governments use auctions to award paving contracts and licenses to provide cell phone service; etc. The study of bidding in auctions is a subfield of game

theory—what's optimal to bid depends on what your rivals are bidding, which reflects what you're doing, and so on. In this class I'll discuss both equilibrium bidding theory for several classic auctions—first-price, second-price, and all-pay auctions—and discuss the field of auction design which studies what types of auctions raise more revenue in different circumstances.
Homework: Optional
Prerequisites: Calculus (differentiation and integration)

Fractal Geometry. (🌶, Julian, Tue–Fri)

> Tyger Tyger. burning bright,
> In the forests of the night;
> What immortal hand or eye.
> Could frame thy fearful symmetry? *William Blake* (1794)

For thousands of years, humans studied the perfect forms: straight lines, circles, conics and the like. But the world is filled with far more exotic shapes than these, and only in the 20th century did we begin to develop the tools to study them. Fractals are infinitely detailed, highly symmetrical shapes, the most famous being the Mandelbrot set (look up "Mandelbrot zoom" on YouTube for a crazy video!).

We will explore some fractals and learn some of the tools used to understand and describe them. There are lines which fill space, snowflakes, dust and more besides, making the concept of "dimension" far less clear than we usually think it is. On Friday, I aim to explain what the Mandelbrot set is.
Homework: Recommended
Prerequisites: Logarithms are essential; some familiarity with epsilons and deltas (for example from "Epsilon the enemy") would be helpful.

Hacking Heads off Hydras. (🌶🌶, Susan, Sat)
A hydra is a mythical being with many heads. Whenever a head gets cut off two grow back in its place! A mathematical hydra is a game that we play on trees. On the nth turn, we cut off a "head," causing n new "heads" to grow back in this place. In this class we will learn about the hydra game. How is it played? For which graphs is there a winning strategy? And what do the ordinal numbers have to do with it?

Representation Theory of Finite Groups. (🌶🌶🌶🌶, Mark, Tue–Sat)
This is a continuation of last week's class. For more information, see last week's blurb and/or talk to Mark.
Homework: Recommended
Prerequisites: Week 1 of this class.

Root-Finding Algorithms. (🌶🌶🌶, *Paddy Bartlett*, Tue–Thu)

- Thing you know: a general expression for the roots of $ax^2 + bx + c = 0$ in terms of radicals. (Yay, quadratic formula!)
- Thing you probably don't know: a general expression for the roots of $ax^4+bx^3+cx^2+dx+e = 0$ in terms of radicals.
- Thing you definitely[1] don't know: a general expression for the roots of $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$ in terms of radicals.

[1] Ask a mentor why I'm so sure about this!

- Thing it turns out is almost as good: **approximations** of roots!

In this class, we're going to talk about root-finding algorithms used by numerical analysts to approximate the roots of various functions. In particular, we're going to introduce a handful of the most frequently-used algorithms, come up with good notions of robustness and speed for each algorithm, and (where relevant) look at pathological cases where these algorithms "break down."

Homework: Recommended

Prerequisites: Simple derivatives and limits.

A Taste of Chaos. (🌶🌶, *Craig Sutton*, Tue–Sat)
Dynamical systems are all around us: weather patterns, the interactions of species in an ecosystem, financial markets and the evolution of a particle under the laws of classical mechanics are just a few examples. And, chaotic dynamical systems are those dynamical systems which (among other properties) exhibit sensitive dependence on initial conditions (aka. the "butterfly effect"). In this course we will familiarize ourselves with the study of dynamical systems and the concept of chaos by focusing primarily on one-dimensional systems.

After a brief review of (or introduction to) useful concepts from calculus we will explore topics such as periodic points, weak-stable sets, hyperbolic points, symbolic dynamics, chaos and more (as time permits).

Homework: Optional

Prerequisites: Single-variable calculus (and a predilection for ϵ's and δ's).

11:10 AM Classes

Flag Varieties. (🌶🌶🌶🌶, Asi, Wed–Sat)
Some of you may know that mathematicians like *projective space*: the space of all linear subspaces of a given vector space V. This is a kind of parameter space. This class is all about more general kinds of parameter spaces that are defined along the same lines.

Instead of just looking at linear subspaces, we will consider chains of subspaces of V, beginning at $\{0\}$ and ending at V. Such a configuration is called a *flag*. We will try to construct the parameter space of flags, describe it as a geometric object, and explore connections with group theory, algebra, and representation theory.

Homework: Recommended

Prerequisites: Linear algebra, Group theory, experience with topology or metric spaces is recommended.

Related to: Representation theory, Symmetric polynomials and Schubert calculus.

Flows. (🌶🌶 if you've taken Matchings; 🌶🌶🌶 if not., Pesto, Wed–Sat)

(1) We can color the vertices, edges, or faces of a planar graph, but nonplanar graphs don't have any faces to color, which makes them feel left out. We'll talk about how to color them anyway.
(2) A bear appears at the main entrance to the dorms. Many Campers Scatter Panickedly, trying to escape through the other doors. How should we evacuate? Can anything go wrong other than people not being able to fit through the doors fast enough?
(3) Hall's Marriage Lemma: if each of a bunch of boys has a certain set of girls they want to marry and vice versa, can everyone be happily married off?

These questions have in common that they can be approached as questions about *flows*, a way of assigning numbers to the edges of a digraph so that every vertex has the same amount coming in and going out. We'll see how.

Homework: Recommended

Prerequisites: Intro Graph Theory
Related to: Matchings.

The Littlewood-Offord Problem. (🌶🌶, Susan, Wed–Sat)
Suppose I give you seven differently-colored fuzzy-wuzzies, each worth a positive real number of points. How many different teams of fuzzy-wuzzies can you construct that are worth exactly 100 points? What if you're the one assigning the points? How many different 100-point teams of fuzzy-wuzzies you can create?

We will begin by finding a solution to this problem. But wait! There's more! What if we aim for fuzzy-wuzzy teams worth r points? What if we have n fuzzy-wuzzies? What if we allow fuzzy-wuzzies to be worth any nonzero real number of points? What if instead of a real number of points, we assign each fuzzy-wuzzy a vector in d-space, and want teams of fuzzy-wuzzies that add up to the $(100, 100, \ldots, 100)$ vector?

Finding upper bounds for these problems isn't hard, but proving that these bounds are optimal is tricky. These questions about sums of real numbers quickly transform into questions about posets. We'll talk about chains, antichains, Dilworth's theorem, and more. If you love fuzzy-wuzzies, posets, and stunningly beautifuly proofs, then this is the class for you!
Homework: Optional
Prerequisites: None!

Mathematical Origami. (🌶🌶🌶, Zach, Wed–Sat)
By "Mathematical Origami," I do not mean that we will use origami as a medium for discussing math. Quite the contrary: we will use math to study origami! For example, what is the largest perimeter shape you can make by folding a 1 ft by 1 ft square sheet of paper (without cutting!)? Is it possible to cut out any shape you want by folding a sheet of paper and making one straight cut through all the layers? The very activity of folding paper is mathematically quite rich, and this course will delve deeply into the theoretical and practical limitations of the art. For homework, you will fold, cut, or otherwise construct an object that demonstrates the next day's topic of discussion.
Homework: Required
Prerequisites: None.

Trail Mix. (🌶–🌶🌶🌶, Mark, Wed–Sat)
Is your mathematical hike getting a little too hardcore? Would you like to relax a bit with a class that offers an unrelated topic every day, so you can pick and choose which days to attend, and that does not expect you to do homework? If so, how about some Trail Mix? Individual descriptions of the four topics follow. There is basically no homework, although there will certainly be some things you could look into if you like.
Homework: None
Prerequisites: See individual mini-blurbs below.

Trail Mix Day 1: Perfect Numbers. (🌶, Mark, Wed)
Do you love 6 and 28? The ancient Greeks did, because each of these numbers is the sum of its own divisors, not counting itself. Such integers are called *perfect*, and while a lot is known about them, other things are not: Are there in finitely many? Are there any odd ones? Come hear about what is known, and what perfect numbers have to do with the ongoing search for primes of a particular form, the so-called Mersenne primes—a search that has largely been carried out, with considerable success, by a far-flung network of individual "volunteer" computers.
Prerequisites: None.

Trail Mix Day 2: Integration by Parts and the Wallis Product. (🌶🌶, Mark, Thu)
Integration by parts is one of only two truly general techniques known for finding antiderivatives (the other is integration by substitution). In this class you'll see (or review) this method, and two of its applications: How to extend the factorial function, so that there is actually something like (1/2)! (although the commonly used notation and terminology is a bit different), and how to derive the famous product formula

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots$$

which was first stated by John Wallis in 1655.

Prerequisites: Basic single-variable calculus

Trail Mix Day 3: Intersection Madness. (🌶🌶, Mark, Fri)
When you intersect two ellipses, you can get four points, right? So why can't you get four points when you intersect two circles? Well, you can, and two of the four points *are always in the same place*! If this seems paradoxical, wait until we start intersecting two cubic curves (given by degree 3 polynomial equations). The configuration of their intersection points has a "magic" property (known as the Cayley-Bacharach theorem) that leads to proofs of various other cool results, such as Pascal's hexagon theorem and the existence of a group law on a cubic curve. I can't promise that we'll have time for all these things, but we'll do some of them.

Prerequisites: None, although a little bit of linear algebra might come in.

Trail Mix Day 4: Hensel's Brave New World. (🌶🌶🌶, Mark, Sat)
In one of Euler's less celebrated papers, he started with the formula for the sum of a geometric series:

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

and substituted 2 for x to arrive at the apparently nonsensical formula

$$1 + 2 + 4 + \cdots = -1$$

More than a hundred years later, Hensel described a number system in which this formula is perfectly correct. That system and its relatives (for each of which 2 is replaced by a different prime number p), the p-adic numbers, are important in modern mathematics; we'll have a quick look around this strange "world".

1:10 PM Classes

The John Conway Hour. (🌶-🌶🌶🌶🌶, *John Conway*, Tue–Sat)
If you want to know what this class is about, come listen to John Conway talk about a topic of his choice!

Homework: None

Prerequisites: None

Hilbert's Geometries. (🌶🌶, *Moon Duchin*, Tue–Fri)
The mathematician David Hilbert came up with a "machine" for turning any convex shape (like a disk, an ellipse, a triangle, a regular n-gon for any n, or any of a large variety of blobs) into a metric space—a world where distances can be measured. Among planar figures, the triangle and the disk are special in various ways—one is a so-called Banach space, while the other is equivalent to the very

famous hyperbolic plane—and all the rest of the spaces should be thought of as mildly exotic geometric worlds that transition between those extremes. I'll explain all this from scratch in very hands-on ways, with lots of examples.
Homework: Optional
Prerequisites: None

Hyperreal Analysis. (🌶🌶🌶, *Don Laackman*, Tue–Sat)
The calculus done by Newton and Leibniz in the 17th century utilized infinitesimals, positive amounts smaller than any real number, in order to define the concepts of derivatives and integrals. So, when differentiating $y = x^2$, they said $\frac{dy}{dx} = \frac{(x+dx)^2 - x^2}{dx} = \frac{2xdx}{dx}$, and then cancelled the dx on the right hand side to get $\frac{dy}{dx} = 2x$. While these methods were incredibly useful, they rested on unstable foundations, and ultimately fell out of fashion. Cauchy was the last mathematician to do important calculus work using infinitesimals before they were cast aside in favor of $\epsilon - \delta$ limits.

However, in the mid-20th century, mathematicians re-discovered infinitesimals, this time with a new, rigorous foundation, based on the then-new logical tool known as an ultrafilter. In this class we'll use ultrafilters to define the hyperreals, an extension of the reals that contain infinitesimal (and unbounded) quantities, and then see how these hyperreals can be used to elucidate important theorems and concepts in calculus.
Homework: Recommended
Prerequisites: Calculus; knowing the $\epsilon - \delta$ definition of a limit would be useful.

A Mathematical Card Trick. (🌶, Abi & Mira, Sat)

> Welcome to Mira's Card Selection Performance! Here, take this ordinary deck of cards, and draw a hand of five cards from it. Choose them deliberately or randomly, whichever you prefer—but do not show them to me! Show them instead to my lovely assistant Abi, who will now give me four of them, one at a time. Let's see—the 7♠, then the $Q\heartsuit$, the 8♣, and finally the $3\diamondsuit$. Hmmm... There is one card left in your hand, known only to you and Abi. And that card, my friend, is the $K\diamondsuit$.

Think that's too easy? What if in addition to choosing the 5 cards you flip a coin, and after seeing four out of your five cards, I also tell you if the coin came up heads or tails? Could I do that with a deck larger than 52 cards? How large?
Prerequisites: None.

9:10 AM Superclasses

Expander Graphs. (🌶🌶🌶, *Avi Widgerson*, Sat)
Expander graphs are among the most ubiquitous and useful objects in mathematics and computer science. In math, they arise and are used in group theory, number theory, metric embeddings, combinatorics, topology and more. In CS, they arise and are used in network theory, error correcting codes, algorithms, computational complexity and more. The study of expanders is a most active current research area, mixing ideas from all these disciplines.

In these two hours I plan to give (at least) two definitions, two applications and two constructions of expanders.

Some references, for those who want to find out (much) more, before or after the class:

- "Expander graphs and their applications", a survey by Hoory, Linial and Wigderson.
`http://www.cs.huji.ac.il/~nati/PAPERS/expander_survey.pdf`

- "What is... an expander", a short note by Peter Sarnak.
  `http://www.ams.org/notices/200407/what-is.pdf`
- "Basic theory of expander graphs", course notes of Terry Tao.
  `http://terrytao.wordpress.com/2011/12/02/245b-notes-1-basic-theory-of-expander-graphs/`

Homework: None.
Prerequisites: Basic notions of linear algebra and graph theory will be helpful.

Metric Spaces. (🌶🌶, Alfonso & Nina, Tue–Fri)
A metric space is a "space with a distance". We can formalize this by selecting out the essential properties that any kind of "distance" should have. When we consider general spaces with these properties, we can create all kinds of new and unfamiliar spaces which do strange and wacky things.

When you study metric spaces carefully, you suddenly have access to many different areas of math (for example, metric spaces are your gateway drug to topology, and they are an essential tool for proving existence and uniqueness of solutions to differential equations).

The specific set of topics we will learn in this course is not fully fixed yet, and may depend on students' interests. They may include a rigorous construction of the reals, the p-adic numbers, compactness, and the Cantor set. For coolness points, we may use the Cantor set to construct a curve that fills up a plane. And for meta points, we may figure out how to make the set of metrics on a space into a metric space itself.

A big chunk of class time is devoted to working on problems. If you do not complete them in class, we expect you to finish them as homework.
Homework: See above.
Prerequisites: None

PRIMES is in P. (🌶🌶🌶, Matt & Mira, Tue–Sat)
How can we tell if a number is prime? The naïve way is to try dividing it by all smaller numbers, but that takes exponential time—to test an n-bit number, we need about 2^n divisions. Over the years, people have come up with a lot of clever ways to test primality, but until recently they either didn't give a huge improvement in speed or weren't guaranteed to succeed every time. But in 2002, three people (one of them an undergraduate!) came up with a polynomial-time algorithm for determining primality, which is guaranteed to succeed. We'll see how it works!
Homework: None
Prerequisites: Number theory, knowledge of abstract algebra (such as Mark's group theory or ring theory classes).
Related to: All the number theory classes

1:10 PM Superclasses

Algebraic Number Theory. (🌶🌶🌶🌶, Ruthi, Tue–Sat)
Think you know number theory do you, young padawan? You know but the beginning... here we travel to the world of number fields, where we will attempt (and fail at times!) to take rings and do number theory with them!
Homework: Recommended
Prerequisites: Intro to Ring Theory, some linear algebra
Related to: Elementary number theory, Group theory, Multilinear algebra, Commutative algebra, Primes is in P, Continued fractions.

Computing with Everything. (🌶, Aaron, Tue–Sat)
Lots of people seem to think that computation is something you do with numbers, but they're missing all the fun! In this course, we'll learn to compute with words, graphs, surfaces, knots, 11th-century Chinese architecture, and anything else we can get our hands on. We'll design, test, and play with *rewrite systems*—computers of a very general kind—that do things like. . .

- Undoing knots.
- Topologically classifying surfaces.
- Producing and recognizing grammatical sentences.
- Finding minimal spanning trees for graphs.
- Drawing blueprints for buildings.

We can also explore more technical questions, like whether rewrite systems can be emulated by more familiar types of computers.

Homework: None

Prerequisites: None, but there will be extra fun for people who have taken (Th)ink Machine, and people who know some graph theory.

Colloquia

The Cross-ratio. (*Moon Duchin*, Tue)
The cross-ratio is a way of inputting four numbers and outputting a single number that expresses something fundamental about the way they are spaced out—something that is invisible to the naked eye! Projections from one number line onto another preserve this mysterious cross-ratio, and I'll explain that it is in fact the ONLY quantity that stays invariant under projection. This has surprising applications in art (perspective drawing), in non-Euclidean geometry, and elsewhere . . .

NTBA. (*John Conway*, Wed)
Not to be announced. . .

Spectral Geometry & the Link Between Classical and Quantum Mechanics. (*Craig Sutton*, Thur)
As I mentioned last summer, spectral geometry studies the relationship between the eigenvalues of the Laplace operator and the geometry of a manifold. But, what I didn't mention is that some spectral geometers also try to understand the relationship between classical and quantum mechanics. In this talk I will endeavor to give an introduction to spectral geometry which makes connections between the geometric and physical aspects of spectral geometry and then briefly discuss a recent result which has an interpretation in terms of physical chemistry.

Google Advertising. (*Glenn Ellison*, Fri)
Google's "page rank" algorithm for ranking websites is famous. But the way that Google sells "sponsored links" is in a sense much more important—that's how Google makes $10 billion per year. The math behind the advertising auctions is also more interesting than page rank and more enlightening about Google's success. I'll present the Vickrey-Clarke-Groves theorem and its connection to a game-theoretic model of the advertising auction.

Finding Hay in a Haystack. (*Avi Widgerson*, Sat)
Despite the proverbial difficulty of finding needles in a haystack, there are many fundamental problems

in mathematics and computer science (including the $1M worth problems of Riemann's hypothesis and P vs. NP) which rather call for finding a strand of hay instead of a needle. And despite the abundance of hay, it is difficult to avoid the very few needles.

The mathematical notion capturing "hay" is "pseudo-random"—a property which holds for the vast majority of the elements of a given set (of numbers, or functions, or graphs, etc.), just like hay in a haystack. Objects with pseudorandom properties are often desirable, and almost as often hard to construct or ascertain. I will give a gentle introduction to the theory of pseudorandomness, with many examples and open problems.

No specific background is assumed. A guaranteed outcome of the talk (or your money back) will be your ability to explain the Riemann hypothesis to your friends and family.

A popular article on this topic and more can be found in this issue of the IAS Letter:

`http://www.ias.edu/files/pdfs/publications/letter-2009-summer.pdf`

Visitor Bios

Avi Widgerson (Princeton) studies Computational Complexity Theory, the mathematical foundations of computer science. This field tries to understand questions like "is multiplication harder than addition?" and "is discovery harder than verification?" (the latter is the famous "P vs. NP problem"). Favorite result: a "zero-knowledge" proof of every theorem.

Craig Sutton (Dartmouth) works on problems in inverse spectral geometry, where one explores the extent to which the geometry of a manifold is encoded in the spectrum of its associated Laplace operator. Metaphorically speaking, this means he considers the extent to which we can hear the shape of a drum. Of particular interest to him is understanding whether we can "hear" the geometry of spheres and other symmetric spaces. When not working on mathematics, Craig can often be found playing hockey or cycling. He looks forward to exploring spectral geometry and symmetry with Mathcamp participants this summer.

Don Laackman (UCLA) spent four years as a camper and two years as a JC before evolving into his current form, visitor. Outside of Mathcamp, he just finished his first year of grad school at UCLA, where he is interested in algebraic k-theory and category theory.

Glenn Ellison (MIT) is a professor of economics. He is a game theorist who has worked both on developing new models of how people learn to play games and applying game theoretic models to help understand particular industries like Internet search, online auctions, mutual funds, and pharmaceuticals. Hobbies include playing various sports and writing math books for kids.

John Conway (Princeton), one of the most creative thinkers of our time, is known for his groundbreaking contributions to such diverse fields as knot theory, geometry of high dimensions, group theory, transfinite arithmetic, and the theory of mathematical games. Outside the mathematical community, he is perhaps best known as the inventor of the "Game of Life."

Julian Gilbey has been a Mathcamp visitor for many years, and was a mentor in the distant past. He is a high-school teacher full-time, and is working on a problem in mathematical biology in his spare time. And in his spare spare time, he has been known to help with the yearbook. . . .

Moon Duchin (Tufts) works in geometric topology and geometric group theory. She particularly looks at the large-scale geometric structure of groups and unusual metric spaces. One recurring theme is taming the geometric infinite by either attaching a "boundary at infinity" to a space you want to study, or else approaching it dynamically by understanding what happens after you flow or jump around in your space for a really long time. She's also actively interested in history, philosophy, and cultural studies of science.

Paddy Bartlett (UCSB) is a lecturer at the University of California, Santa Barbara. He was a mentor from 2010-2012, and just finished up his Ph.D at Caltech (with the help of many many campers!) His research interests are currently in Latin squares and quasirandom graphs. When he's not teaching, you can usually find him either playing frisbee or DOTA.