Applying to Mathcamp 2013

Ready to Apply to Mathcamp?

We invite applications from every student aged 13 through 18 who is interested in mathematics, regardless of racial, ethnic, religious, or economic background.

Mathcamp accepts applications both on the web and by regular mail. We strongly encourage all students with Internet access to use the online application process. The $20 application fee is waived for online applications.

Online Application (free):
Go to http://www.mathcamp.org/apply/ and follow the instructions. You'll still have the opportunity to submit your Quiz or recommendation letters by postal mail or fax.

Postal Application ($20 application fee):
Go to http://www.mathcamp.org/applybymail/ and print out the application packet.

An application to Mathcamp consists of the following:

1) Some basic information about **yourself and your math background**. We will ask you to describe the math courses that you've taken at the high-school level or above, along with scores and awards from any math competitions you've done.

2) A brief **personal statement** about your interest in math and why you want to come to Mathcamp.

3) Your solutions to the 2013 **Qualifying Quiz** (see below).

4) **Two recommendation letters**, academic and personal.

- The first letter should be from a teacher who knows you well, preferably a math teacher. The letter should comment on your creativity, initiative, and ability to work with others, as well as on your academic achievements.
- The second letter should be from another adult who knows you personally (e.g. an employer, pastor, soccer coach, etc. – preferably someone outside of school and not a relative). This letter should address your maturity, independence, social and personal qualities. We are looking for students who are not only good at math, but who will thrive in the atmosphere of freedom and responsibility that characterizes Mathcamp, and who will make a positive contribution to the camp community.

5) If you would like to be considered for financial assistance, please include the **scholarship application** (see instructions below). Note that admission to Mathcamp is need-blind.

Mathcamp will offer two application deadlines in 2013. The options are simply for your convenience - we have no preference!

Early Action: Submit your application by March 15th; admissions decisions will be announced April 1st.

Regular Action: Submit your application by April 15th; admissions decisions will be announced May 1st.

Contact Us

Email: info13@mathcamp.org

Telephone/Fax: 888-371-4159

Postal address: Mathcamp 2013
129 Hancock Street
Cambridge, MA 02139

Please do not send certified mail to Mathcamp; we will email you to verify that your application has arrived.

Student Care Policy

Dear parent: Student safety and enjoyment are Mathcamp's first priorities. Students will be housed in secure campus dormitories, with male and female students in designated sections of the same building. In case of a medical problem, we have a camp nurse on call, and the hospital is minutes away. Students will have access to college athletic facilities and computers. Every effort will be made to enable students who so desire to attend weekly religious services of their faith. Mathcamp is committed to an atmosphere of mutual tolerance, responsibility, and respect, and is proud of its past record in helping to create such an atmosphere.

- Mira Bernstein, Executive Director, Mathcamp

Cost and Scholarships

Full Camp Fee: $4000

(This includes tuition, room, board, and extracurriculars.)

We are deeply committed to enabling every qualified student to attend, regardless of financial circumstances.

Mathcamp awards over $100,000 in need-based scholarships every year. In the past eight years, no admitted applicant has been unable to attend the camp for financial reasons. We give several full scholarships each year, and occasionally even help students with travel expenses. Please do not let financial considerations prevent you from applying! If you'd like to be considered for a scholarship, just complete the short application at right.

Scholarship Application

Please have a parent or guardian provide the following information, along with her or his email address:

- 2012 family income (all sources).
- Expected family income for 2013. (If significantly different from 2012, please explain.)
- A list of all members of your household (supported by the above income) and their relationships to the applicant. For siblings, please provide ages.
- The cost of schooling, if any, for household members.
- The estimated cost of round-trip travel to Mathcamp for the applicant.
- The portion of the cost of Mathcamp (including both tuition and travel) that your family can afford to pay.
- Any special circumstances you want us to consider.

Mathcamp 2013 Qualifying Quiz

Instructions

We call it a quiz, but it's really a challenge: a chance for you to show us how you approach new problems and new concepts in mathematics. What matters to us are not only your final results, but also your reasoning. Correct answers on their own will count for very little: you have to justify all of your assertions and ***prove*** to us that your solution is correct. (For some tips on writing proofs, see www.mathcamp.org/proofs.) Sometimes it may take a while to find the right way of approaching a problem. Be patient: there is no time limit on this quiz.

We don't expect every applicant to solve every problem: in the past, we have sometimes admitted people who could do only half of them, occasionally even fewer. However, don't just do four or five problems and declare yourself done! The more problems you attempt, the better your chances. The problems are roughly in increasing order of difficulty, but the later problems often have some easier parts. We strongly recommend that you try *all* the problems and send us the results of your efforts: partial solutions, conjectures, methods – everything counts. None of the problems require a computer; you are welcome to use one if you'd like, but first see www.mathcamp.org/computers.

If you need clarification on a problem, please email your question to quiz13@mathcamp.org. *You may not consult or get help from anyone else.* You can use books or the Web to look up definitions, formulas, or standard techniques, but any information obtained in this way must be clearly referenced in your solution. Please do not try to look for the problems themselves: we want to see how well you can do math, not how well you can use Google! Any deviation from these rules is considered plagiarism and may disqualify you.

Have fun and good luck!

Problems

(1) A teacher asks 100 students to help her with a math contest that she's organizing: each student is supposed to come up with one or two problems to include in the contest. Surprisingly, all 100 students do as requested; the teacher actually needed only n problems, with $n<100$, so now she has to decide which problems to use. She announces that she will call on the students one by one, at random, and collect all the problems from each student she calls on. Once she has n problems, she will stop. (If the last student wrote two problems and the teacher only needs one of them, she'll pick one at random.)

(a) Angela thought of one problem, Bill thought of two. Bill tells Angela, "Because I wrote more problems than you, I have a better chance of getting a problem on the contest." Angela says, "But the teacher is picking people completely at random! My chances of getting picked are as good as yours." Who is right? Does it depend on n? Does it depend on how many problems the other students came up with? (Be sure to explain any special cases.)

(b) Catherine only thought of one problem, but she overhears Bill talking to Angela and gets worried. She quickly composes another problem, just in case. Does this change her chances of having a problem on the contest? Does it change Bill's or Angela's chances, and if so, in what direction? (Note: you do not need to compute the exact probabilities.)

(2) The subscript ! sometimes indicates that a string of numbers is to be interpreted in factorial base: the i-th number from the right ranges from 0 to i and tells you what multiple of $i!$ to add. For example,

$$20301_! = 2\cdot5! + 0\cdot4! + 3\cdot3! + 0\cdot2! + 1\cdot1! = 240 + 18 + 1 = 259.$$

(a) Warm-up: show that a number is even if and only if its factorial base representation ends in 0. State and prove a similar condition for divisibility by 3.

(b) The number 27 has the property that its binary representation 11011_2 ends with its factorial base representation $1011_!$. Show that there are infinitely many numbers with this property.

(3) Charles and Lilly are playing a game. They start with an empty pot, to which a piece of candy is automatically added at the beginning of every turn. The player whose turn it is then has a choice: he/she can either take all the candy in the pot or pass. (For instance, if Lilly goes first and takes the pot on her first turn, she gets one piece of candy. If she passes and Charles takes the pot on the next turn, he gets two pieces, etc.) If a player decides to take the pot, he/she must pass on his/her next k turns. The winner is the first player to collect n pieces of candy.

Charles unwisely offers Lilly the choice of going first or second. Which should she choose in order to be sure of winning, and how should she play? (Her choice may depend on n and/or k. Describe Lilly's strategy completely and prove that Charles cannot win against it, no matter what he does. We suggest you start with the case $k=1$.)

This is a variant of the online game Reaper. Playing the original version of Reaper will not help you solve this problem, but you can check it out just for fun: http://www.artofproblemsolving.com/Edutainment/r2.

(4) A *binary block* is a rectangle whose side-lengths are powers of two.

(a) Show that any $m\times n$ rectangle can be chopped up into binary blocks no two of which are the same. For example, a 5×5 rectangle can be cut into binary blocks of dimensions 4×4, 4×1, 1×4, and 1×1. (Note that a 4×1 block is considered different from a 1×4 block: rotation is not allowed.)

(b) If $m=1$ or $m=2$, show that you get the same set of binary blocks from your $m\times n$ rectangle no matter how you arrange the cuts. (In other words, the set of blocks is uniquely determined by m and n.)

(c) On the other hand, if $m=n=5$, there is more than one set of binary blocks that works. Can you demonstrate this?

(d) Ideally, we'd like to find a set of rectangular blocks with integer sides (not necessarily powers of two) such that (i) every $m\times n$ rectangle can be cut into blocks from this set that are all different, and (ii) the blocks obtained in this way are uniquely determined by m and n. As we just saw, the set of binary blocks satisfies condition (i) but not (ii). Can you find a different set of blocks that satisfies both conditions?

(e) (Hard!) Our results in (b) and (c) raise the question: for what values of m and n is the set of binary blocks unique? Try to find a necessary and sufficient condition for uniqueness. (You'll need to prove that if m and n satisfy the condition, uniqueness is guaranteed, whereas if they don't satisfy it, there are at least two different sets of blocks that work. If the general version seems too hard, try for some partial results: are there classes of numbers m and n for which you can prove that there is only one set of binary blocks? Are there classes of numbers for which you can prove that there is more than one?)

(5) A farmer living on the xy-plane wants to fence off a rectangular field with sides parallel to the axes and area at least 1. A malicious king tries to stop the farmer by preemptively placing stakes at infinitely many points in the plane (thereby staking his own claim to those points). It's a constitutional monarchy, so the king can't just claim every point for himself: the law dictates that each point where he places a stake must have a circle of non-zero radius around it that contains no other stakes. (These unclaimed circles around each stake can be of different sizes and can be as small as the king likes.) Can the king place his stakes in such a way that the farmer can't find a rectangle of area 1 with no stakes anywhere in its interior or boundary?

(6) Isosceles triangle ABC has $AB = AC$ and $\angle BAC = \alpha$. The triangle is originally situated in the plane with A at (0,0) and midpoint M of side BC at (1,0). You may move the triangle by reflecting it across any of its edges. Your goal is to come up with a sequence of moves that switches A and M – that is, brings A to (1,0) and M to (0,0).

(a) Show that such a switch is impossible if $\alpha=60°$ or $90°$.

(b) If $\alpha = 45°$, show that such a switch is still impossible, but that the triangle can come close to its destination: we can simultaneously get A within distance 0.1 of (1,0) and M within distance 0.1 of (0,0). Can we get even closer? How close?

(c) Is there an angle α for which switching A and M is possible?

(7) Let f be a function on the nonnegative integers such that $f(2n) = f(f(n))$ and $f(2n+1) = f(2n)+1$.

(a) If $f(0) = 0$, find a simple formula for $f(n)$.

(b) Show that $f(0)$ cannot equal 1. For what nonnegative integers k (if any) can $f(0)$ equal 2^k?

(c) For what nonnegative integers k (if any) can $f(0)$ equal 2^k+2?

(d) (Open question!) What else can you say about the possible values of $f(0)$? Are there values of $f(0)$ for which more than one such function exists? If so, how many different functions are there? Partial results are welcome.

The Mathematics Foundation of America invites you to apply to the twenty-first annual

Discover Mathcamp!

"Out of nothing I have created a strange new universe."
– Janos Bolyai, co-discoverer of hyperbolic geometry

Mathcamp is a chance to...

- Live and breathe mathematics: fascinating, deep, difficult, fun, mysterious, abstract, interconnected (and sometimes useful).
- Gain mathematical knowledge, skills and confidence – whether for a possible career in math or science, for math competitions, or just for yourself.
- Set and pursue your own goals: choose your classes, do a project, learn what you want to learn.
- Study with mathematicians who are passionate about their subject, from internationally known researchers to graduate students at the start of their careers, all eager to share their knowledge and enthusiasm.
- Make friends with students from around the world and discover how much fun it is to be around people who think math is cool.

"Mathcamp was the first place where I really understood the beauty and intricacies of abstract mathematics."
– Paul Hlebowitsch (Iowa City, IA, USA)

Academics

A Variety of Choices

The Mathcamp schedule is full of activities at every level, from introductory to the most advanced:

- Courses lasting anywhere from a few days to five weeks
- Lectures and seminars by distinguished visitors
- Math contests and problem-solving sessions
- Hands-on workshops and individual projects

You can learn more at:
http://www.mathcamp.org/academics

The Freedom to Choose

Mathcamp does not have a set curriculum or a list of requirements. We encourage the faculty to teach what they are most passionate about, and we let the students choose what they are interested in learning. With the help of an academic advisor, you will design a program of study that reflects your own interests and goals. You can take any classes you want, and even the number of classes that you attend each day is up to you: you can use your time to review what you've learned, talk to one of your professors, work on problems, do a project, or just take a break. For many students, the freedom to take charge of their own education is one of the aspects of Mathcamp that they value most.

Classes

Course offerings vary from year to year, depending on the interests of the students and faculty. Some of the topics taught in previous years have included:

Discrete Mathematics: Combinatorics • Generating functions • Partitions • Graph theory • Ramsey theory • Finite geometries • Polytopes and Polyhedra • Combinatorial game theory • Probability

Algebra and Number Theory: Primes and factorization algorithms • Congruences and quadratic reciprocity • Linear algebra • Groups, rings, and fields • Galois theory • Representation theory • p-adic numbers

Geometry and Topology: Euclidean and non-Euclidean (hyperbolic, spherical, projective) geometries • Geometric transformations • Combinatorial topology • Algebraic geometry • Knot theory • Brouwer Fixed-Point Theorem

Calculus and Analysis: Fourier analysis • Complex analysis • Real analysis • Measure theory • Dynamical systems • Non-standard analysis

Computer Science: Cryptography • Algorithms • Complexity • Information theory • P vs. NP

Logic and Foundations: Cardinals and ordinals • Gödel's Incompleteness Theorem • The Banach-Tarski Paradox • Model theory • Category theory

Connections to Science: Relativity and quantum mechanics • Dimensional physics • Voting theory • Bayesian statistics • Neural networks • Mathematical biology • Cognitive science

Discussions: History and philosophy of mathematics • Math education • "How to Give a Math Talk" • College, Graduate School and Beyond

Problem Solving: Proof techniques • Elementary and advanced methods • Contest problems of various levels of difficulty • Weekly "Math Relays" and team competitions

Projects

Every student at Mathcamp is encouraged to do a project, supervised by one of the mentors or faculty. Projects range in scope from creative applications of simple techniques to advanced problems connected to faculty research. Project topics in previous years have included:

- Periodicity of Fibonacci numbers mod *n*
- Information theory and psychology
- Knight tours on an *m*-by-*n* chessboard
- Cellular automata
- Cops and robbers on a graph
- Constructing the regular 17-gon
- Admissible covers of algebraic curves
- Mathematical Finance
- Algorithmic composition of music
- Intelligent ways of searching the web
- Probability in sports
- The elasticity equation of string
- Digital signal processing
- Light paths in universes with alternate physics
- Playing 20 Questions with a Liar
- Dirichlet's Theorem on Arithmetic Progressions
- Non-Orientable Knitting

Spotlight on a Class

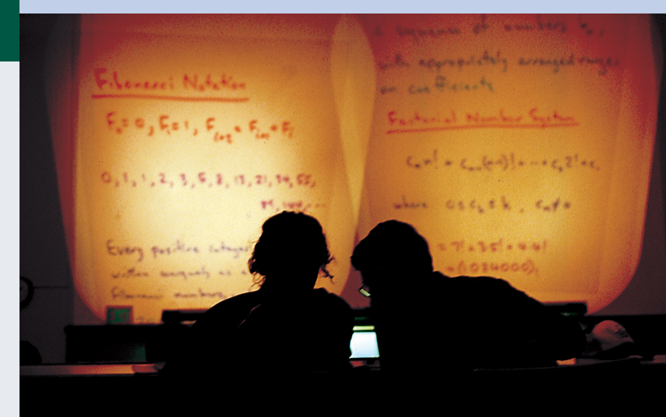
The Shape of Infinity (2012) • What does space look like at infinity? There is no unique answer: it turns out there are lots of different ways to add "points at infinity" to a space. In projective geometry, we add a circle at infinity, with one destination point for each "direction" in which we could go off to infinity. But with "stereographic projection" we add only a single point at infinity, so that no matter which direction we go off to infinity in, we end up at the same place. A better question is: in how many different ways can we add points at infinity? Can we classify these ways, or describe them explicitly in terms of the finite part of space? Is there a best one, or a worst? The answers come from a general theory of "adding points at infinity", called *compactification*. We'll use *gauge spaces*, which are like metric spaces but more general. Finally, we'll construct the *Stone-Cech compactification*, which is the best (or worst) compactification of a space: it has the most possible points at infinity.

"One cannot compare my ideas of what 'I'm interested in math' meant before and after Mathcamp."
– Asaf Reich (Vancouver, BC, Canada)

"There was no pressure: the incentive to learn came from within."
– Keigo Kawaji (Toronto, ON, Canada)

"New understanding of number theory, topology, and real analysis were not the only things I took from Mathcamp. I learned how much I have left to learn, and thanks to my new friends and mentors, I couldn't be more excited about the world of math."
– Rachel Hong (San Jose, CA, USA)

"Mathcamp took every limitation I thought I had—social, academic, and personal—and shattered it."
– Andrew Kim (Dover, MA, USA)



"Mathcamp was definitely the most fun I've ever had."
– Avichal Garg (Cincinatti, OH, USA)

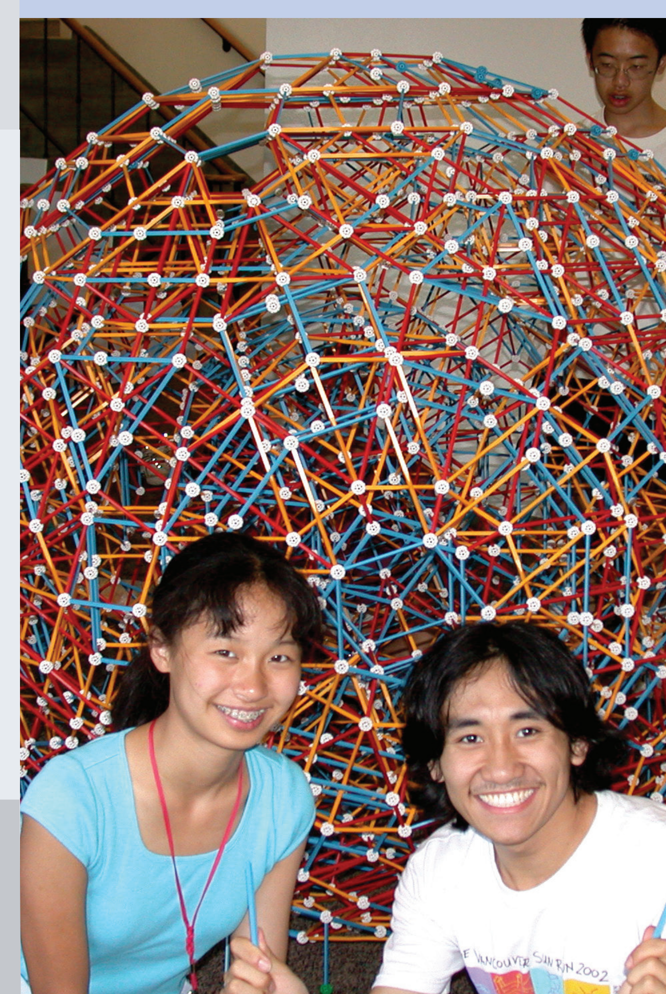
Colby College

Site of Mathcamp 2013

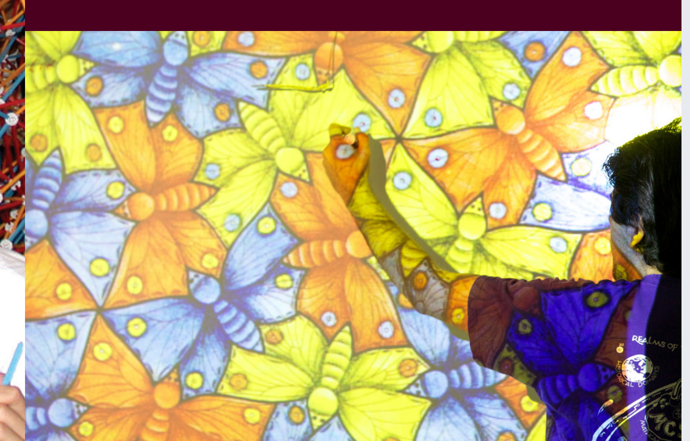
"Mathcamp isn't really a camp. It's more of a five-week long festival - a congregation of people who celebrate math, enjoy math, learn math and essentially live math. Through it all I've discovered cool theorems that I wouldn't have understood before and cool people I didn't know existed. I've learnt that I actually know close to nothing about the weird and wonderful subject that is mathematics, and that I will probably pursue it for the rest of my life. Math on, Mathcamp!"
– Yongquan Lu (Singapore)

"It's not often that you find a place that is exciting to the mind and liberating to the spirit. Mathcamp is both."
– Greg Burnham (Memphis, TN, USA)

"I've changed so much in my two years here. I think about math in a new, deeper way. I approach problems differently. I've gained perseverance and learned to ask for help without shame and give it with joy."
– Hallie Glickman-Hoch (Brooklyn, NY, USA)



"Go, just go! Trust me!"
– Jian Xu (Toronto, ON, Canada)



People and More

Faculty

Mira Bernstein (Executive Director, Mathcamp)
Interests: Algebraic Geometry, Mathematical Biology, Information Theory

Mark Krusemeyer (Carleton College)
Interests: Abstract Algebra, Combinatorics, Number Theory, Problem Solving

David Savitt (University of Arizona)
Interests: Number Theory, Arithmetic Geometry

Nina White (University of Michigan)
Interests: Math Education, Geometric Topology, Geometric Group Theory

Visiting Speakers

John H. Conway (Princeton) • One of the most creative thinkers of our time, John Conway is known for his ground-breaking contributions to such diverse fields as knot theory, geometry of high dimensions, group theory, transfinite arithmetic, and the theory of mathematical games. He is perhaps best known as the inventor of the "Game of Life."

Jim Gates (University of Maryland) • Dr. Gates uses mathematical models involving supersymmetry, supergravity, and superstring theory to explore nature. One of his current focus areas includes Adinkras, a new mathematical concept, linking computer codes like those in browsers to the equations of fundamental physics as if our physical reality resides in the science fiction movie "The Matrix."

Allan Adams (MIT) • Allan Adams works on quantum versions of algebraic and differential geometry, and uses black holes in 5 spacetime dimensions to study high-temperature superconductors in the usual 4.

George Hart (Mathematician, Artist) • George Hart is a mathematician and a sculptor whose artwork and videos can be seen at georgehart.com. At Mathcamp, he leads hands-on workshops in which participants explore the geometry of three- (and four-) dimensional space using Zometool.

Moon Duchin (Tufts) • Moon Duchin works in geometric topology and geometric group theory. She particularly looks at the large-scale geometric structure of groups and unusual metric spaces. She also thinks about philosophy, cultural studies, gender theory, what they have to say about math, and what math has to say back!

Glenn Ellison (MIT) • Glenn Ellison is a professor of economics at MIT. He works in game theory, where he focuses on the dynamical systems that arise when many people are simultaneously trying to learn how to play a game, and on using game-theoretic models to understand how Google works and why airlines charge so much for checked bags.

Josh Tenenbaum (MIT) • Josh Tenenbaum is a professor of Cognitive Science and a member of the MIT Computer Science and Artificial Intelligence Lab. In his research, he builds mathematical models of human and machine learning, reasoning, and perception. He is also interested in neural networks, information theory, and statistical inference.

Students

We never cease to be amazed at what a varied and interesting bunch of young men and women our students are! While everyone at camp shares a love of mathematics, their other interests run the gamut. Each year's camp is a collection of 120 students who are musicians and writers, jugglers, dancers, athletes and actors, artists, board game players, hikers, computer programmers, students of science and philosophy - all sharing their interests and experiences with each other.

Most of the students at camp come from North America, but many come from overseas. Students have come to camp from Bulgaria, India, Japan, Lithuania, Macedonia, Mexico, Poland, Romania, Russia, Serbia, Singapore, South Korea, Tanzania, Turkey, and many other places around the globe.

It is a testament to our students' maturity and independence that they can be serious about doing math, while still finding so many different ways to have fun. Many camp activities are organized entirely by campers, and students routinely cite each others' company as one of the best aspects of camp.

Mentors and Junior Counselors

The residential staff at camp is made up of Mentors and Junior Counselors ("JCs"). Mentors are graduate students in mathematics and computer science; they teach most of the classes at camp, picking the course topics freely from among their favorite kinds of math. JCs, all of them camp alumni, are undergraduates who run the non-academic side of camp (from field trips to first aid to frisbee games). Each student is assigned a Mentor or JC as their residential advisor; RAs live on the same hall as their advisees and look out for them on a day-to-day basis. Like campers, the Mentors and JCs often return year after year to Mathcamp.

Beyond Math

Mathematical activities are scheduled for five days a week; whatever math happens on the other two days is purely informal. The weekend is reserved for relaxation and the incredible number of activities that quickly fill the schedule. All of these activities are optional, and students can choose simply to relax with friends.

Field trips in the past have included hiking, sea kayaking, whitewater rafting, amusement parks, and museums. Lots of activities happen on-campus, too: there are rehearsals for the choir and the contemporary a cappella group, salsa dancing workshops, improv, and bread baking (and subsequent eating). There is an annual team "puzzle hunt" competition, a talent show, and ice cream made with liquid nitrogen. Campers also organize many events themselves—from sports and music to chess and bridge tournaments—and each year, a group of students creates the camp yearbook.

"Coming to Mathcamp has given me a community with which to interact, not just five weeks a year, but all year round."
– Eric Wofsey (St Louis, MO, USA)