CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2014

Contents

These class descriptions are only for week 1. Selected blurbs for week 2–4 classes coming soon to www.mathcamp.org.

9:10 am Classes

Cubic curves. (🌶🌶🌶, Mark, Tue–Sat)
A curve in the x, y-plane is called a cubic curve if it is given by a polynomial equation $f(x, y) = 0$ of degree 3. Compared to conic sections (which have degree 2), at first sight cubic curves are unpleasantly diverse and complicated; Newton distinguished more than 70 different types of them, and later Pluecker made a more refined classification into over 200 types. However, as we'll see, by using complex numbers and points at infinity we can bring a fair amount of order into the chaos, and cubic curves have many elegant and excellent properties. One of those properties in particular, which is about intersections, will allow us to prove a beautiful theorem of Pascal about hexagons and conic sections, and it will also let us define a group structure on any cubic curve - well, almost. We may have to leave out a singular ("bad") point first, but a cubic curve has at most one such point (although it may be well hidden; for example, $y = x^3$ has one!), and most of them don't have any. Cubic curves without singular points are known as elliptic curves, and they are important in number theory, for example in the proof of Wiles' Theorem, (AKA "Fermat's Last Theorem"). However, in this week's class we probably won't look at that connection at all, and no knowledge of number theory (or even groups) is required. With any luck, along the way you'll pick up some ideas that extend beyond cubic curves, such as how to deal with points at infinity (using "homogeneous coordinates"), what to expect from intersections, and where to look for singular points and for inflection points.

Homework: Recommended

Prerequisites: Mild use of differential calculus, probably including partial derivatives; complex numbers; some use of determinants. Group theory *not* required.

Related to (but not required for): Counting Conics (W2)

Linear Algebra. (🌶🌶, Yvonne, Tue–Sat)
Linear algebra, the study of vectors and matrices, is one of the most fundamental and useful tools in mathematics. It appears in nearly every area of pure and applied math, and is also frequently used in the physical and social sciences. Much of what mathematicians (and physicists, and engineers, and

economists) do is try to reduce hopelessly complicated non-linear problems to linear ones that can actually be solved.

One cannot hope to cover all of linear algebra in one week, but this class will give you a basic background, as well as a preview of some of the most important results. We're going to start out in the cartesian plane, where linear algebra springs out of geometry. We'll define linear maps and give an intuitive preview of one of the central themes of linear algebra: eigenvectors and their eigenvalues. Then we'll leave our two-dimensional pictures behind and introduce the more general concepts of vector spaces, linear independence, dimension, inner products, orthonormal bases, and diagonalization. (If you don't know what any of these words mean, that's great: come to the class! If you know all of them, then you probably don't need this class.)

Later in the class, we'll introduce a big theorem about eigenvectors of symmetric matrices, the Spectral Theorem. This result is fundamental to a variety of applications, ranging from population genetics and image processing to personality psychology. We won't have a chance to go into these applications, but campers have done projects in the past on these applications!

Homework: Required

Prerequisites: None

Required for: Galois Theory; Geometry of Spacetime; Category Theory; Compressed Sensing; Graph Coloring; Counting Conics; Quantum Mechanics; Latin Squares; PS: Polynomials

Mathcamp Crash Course. (🌶, Paddy, Tue–Sat)
This is a "crash course" in the essential bits of mathematics your other classes will typically assume familiarity with. In this course, we'll discuss basic logic and mathematical notation, proof techniques (induction, contradiction, pigeonhole, and combinatorial), equivalence relations, modular arithmetic, and the concept of cardinality. If you are new to advanced mathematics or just want to shore up your foundations, this course is highly recommended!

If you want to test your knowledge of these concepts, try a few of the following problems:

(1) Can you find three statements A_1, A_2, B such that neither A_1 nor A_2 are strong enough to imply B, but A_1 and A_2 together imply B? How about statements $A_1, \dots A_n, B$ such that no proper subset of $A_1, \dots A_n$ implies B, but all of $A_1, \dots A_n$ together collectively imply B?

(2) Look at $\mathbb{Z}/57\mathbb{Z}$, i.e. the integers modulo 57. Does every nonzero number here have a multiplicative inverse? How about for 59?

(3) What is wrong with the following argument (aside from the fact that the claim is false)?

> **Claim:** On a certain island, there are $n \geq 2$ cities, some of which are connected by roads. If each city is connected by a road to at least one other city, then you can travel from any city to any other city along the roads.
>
> **Proof:** We proceed by induction on n. The claim is clearly true for $n = 1$. Now suppose the claim is true for an island with $n = k$ cities. To prove that it's also true for $n = k + 1$, we add another city to this island. This new city is connected by a road to at least one of the old cities, from which you can get to any other old city by the inductive hypothesis. Thus you can travel from the new city to any other city, as well as between any two of the old cities. This proves that the claim holds for $n = k + 1$, so by induction it holds for all n. QED.

(4) Prove that there are infinitely many prime numbers.

(5) Explain what it means to say that the real numbers are uncountable. Then prove it.

(6) Explain why $(0, 1)$ and $[0, 1]$ have the same cardinalities. Prove your claim.

If these problems look particularly difficult, you should take this class! (If they don't, but you're still wondering if you should take this class, come and talk to either me or your academic advisor; we can make recommendations.)

Also: it's going to be fun! (Yay, fun!)

Homework: Required

Prerequisites: None

Required for: Everything!

Through the Eyes of a Prime: An Introduction to p-adic Numbers. (, Holly Swisher, Tue–Sat)
Integers arise in day–to–day life, and from integers we get fractions. But where do we go from there? In this class we investigate number systems that are different from the real and complex numbers. These number systems, called p-adics, are each determined by only one prime number! They have weird and interesting properties that we will explore.

Ever wondered what the world would look like through the eyes of a prime? Take this class and find out!

Homework: Recommended

Prerequisites: None required, but a basic idea of limits and modular arithmetic is very helpful.

Related to (but not required for): Real Analysis (W1)

Universal Algebra. (, Steve, Tue–Sat)
Throughout mathematics, we frequently find ourselves studying classes of algebras (that is, sets with interesting functions) defined by systems of equations:

- Groups: sets with a constant (zero-ary function) e, binary function $*$, and unary function $^{-1}$, satisfying the equations $(a * b) * c = a * (b * c)$ (associativity), $x * e = e * x = x$ (identity), and $x * x^{-1} = x^{-1} * x = e$ (inverses). For example, positive real numbers with 1, multiplication, and reciprocals.
- Rings: two constants, 0 and 1, two binary functions, $+$ and $\times$, and a unary function $-$, satisfying the commutative and associative laws for $+$ and $\times$, the distributive law $(x+y) \times z = x \times z + y \times z$, and the equations saying that 0 and 1 are units for $+$ and $\times$, and that $-$ is the additive inverse. For examples, the integers with the usual meanings of 0, 1, $+$, $\times$, and $-$.
- Lattices: sets with two binary functions, $\vee$ and $\wedge$, satisfying the commutativity equations $x \vee y = y \vee x$ and $x \wedge y = y \wedge x$, the associativity equations $(x \vee y) \vee z = x \vee (y \vee z)$ and $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, and the equations $a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$ (these last two are called *absorption laws*). For example, sets of real numbers with $\cup$ and $\cap$.

A collection of algebras defined by a system of equations — such as groups, rings, and lattices — is called a *variety*. Universal algebra is the general study of varieties, and in this class we will survey as much of the state of the art of universal algbera as we can, giving lots of examples along the way.

We will begin by characterizing when a collection of algebras is a variety; this amounts to studying certain functions on sets of algebras, which themselves have interesting algebraic structure that we will investigate! Then we will turn to the structure of individual algebras, and develop the notion of a *congruence lattice*. Finally, we will return to looking at varieties, and investigate one of the most important problems in universal algebra about the structure of varieties — and we will show that this is intimately connected with the study of congruence lattices of individual algebras. So all the levels of the hierarchy — algebras, classes of algebras, functions of sets of algebras — are tied together.

Homework: Required

Prerequisites: None

Required for: None

Related to (but not required for): Introduction to Ring Theory (W1); Group Theory (W1); Category Theory (W2); On Beyond i (W3)

10:10 am Classes

Bayesian Statistics. (🌶🌶, Ruthi, Tue–Sat)
Statistics is the science of analyzing data in the presence of uncertainty or with incomplete information. Since there is little in the world that is certain, and information is always scarce, we humans can't go a day without doing some kind of statistics – in our routine cognitive functions, in science, in the political arena, etc.

On the one hand, good statistics is a way of making our belief about whether something is true rigorous. On the other hand, statistics can also, through negligence or malice, be manipulated to show all kinds of not-particularly-true things, which is how we get the famous quote by Mark Twain: "There are three kinds of lies: lies, damned lies, and statistics."

In this class, we'll talk about some of the pitfalls statistics often leads us to fall into, how to think critically about the data we're given, and how Bayesian statistics can be a clean way of analyzing our intuitions. If you're interested at all in how math can be used in real life, this is the class for you!

Homework: Recommended

Prerequisites: None. Knowing basic probability and/or calculus will be helpful, but is not necessary.

Related to (but not required for): Reasoning about Knowledge and Uncertainty (W2); Knowledge and the Mind (W2)

Complex Analysis. (🌶🌶🌶, Kevin, Tue–Sat)
Complex analysis studies functions whose input and output are both complex numbers of the form $z = a + bi$ rather than real numbers. Many of the same concepts that come up in calculus extend to the complex setting, like derivatives and integrals.

But miraculous things happen with complex functions that don't happen in the real setting! For example, if a complex function is differentiable in a region, then it's automatically infinitely differentiable there and hence has a power series expansion. Further, for nice enough functions, integrals around a closed curve in the complex plane are entirely determined by the function's behavior near the points inside the curve where the function is undefined. And these integrals have applications in all sorts of areas of math—they even help us evaluate difficult real integrals!

We'll see all of this in the first week, setting the foundation for the next week of the class and other analytic classes at camp this summer.

Homework: Required

Prerequisites: Single-variable calculus (differential and integral)

Required for: Analytic Number Theory; Polynomial Fermat's Last Theorem

Related to (but not required for): Real Analysis (W1); Scandalous Curves (W4)

Group Theory. (🌶🌶🌶, Don, Tue–Sat)
What do the hydrogen atom, crystal structures, and Rubik's cube have in common? How many different ways can you paint the sides of a cube with three colors? Why isn't there a formula for solving fifth-degree polynomial equations like $x^5 - x - 1 = 0$, even though we have the quadratic formula for second degree polynomials and there's even a formula for third and fourth degree polynomials? The answers to all of these questions can be found in group theory (although some of them are beyond the scope of this class).

Group theory is useful in, and often critical to understanding, many different areas of mathematics. This class will be an introduction to the theory of groups. We'll define groups and related concepts, like subgroups, cosets, homomorphisms, and learn about the symmetric group, Lagrange's theorem, and the first isomorphism theorem.

Homework: Recommended

Prerequisites: None

Required for: Galois Theory; Category Theory; Evasiveness; How to Cut a Sandwich; Sylow Theorems; Geometry of Groups; Tilings, Groups, and Orbifolds

How Fast Can We Multiply? (🌶🌶→🌶🌶🌶, Matt, Fri–Sat)
Way back in elementary school, you (probably) learned how to multiply two numbers. A little thought should convince you that the amount of time it takes to multiply two n-digit (or n-bit) numbers that way is roughly proportional to n^2, and for a long time people thought that that was about the best you could do.

Surprisingly, though, we can do better! We'll see how a pretty simple trick lets us break the n^2 barrier, and how taking the trick further leads us to multiplication based on the Fast Fourier Transform—an algorithm often used for things like sound processing, but which gives a way to multiply that is far faster for large numbers.

Homework: None

Prerequisites: None

Infinite Trees. (🌶🌶🌶, Susan, Tue–Thu)
König's infinity lemma states that any tree of countably infinite height with finite levels has a countably infinite branch. Its obvious generalization one cardinality up turns out to be false: there exist trees of uncountable height with countable levels with no uncountable branches! And this isn't the weirdest thing we'll see in this class. We'll see trees that may not even exist—we have to go outside of normal Zermelo-Fraenkel set theory to find them! In this class, you'll find out what it means for a tree to be uncountably tall, delve into the mysteries of the Diamond Axiom, and learn how to pronounce the word Aronszajn.

Note: This class has some really gorgeous tie-ins with the Continuum Hypothesis class that is being offered in weeks 3 and 4. Both classes are self-contained, but they work extremely well as a pair. In particular, if you are interested in the Continuum Hypothesis class but a little intimidated by the four-chili label, this class will give you some practice following the kinds of arguments that come up a lot in the later class.

Homework: Recommended

Prerequisites: none

Required for: none

Related to (but not required for): The Continuum Hypothesis (W3–4)

Introduction to Graph Theory. (🌶, Marisa, Tue–Sat)
There is a theorem that says that for any map of, say, countries on your favorite continent, you can color the countries so that any two countries that share a border (not just meet at a point, but actually share some boundary) get different colors, and that the number of colors you will need is no more than 4. (Try inventing a complicated political landscape and coloring: no matter how crazy the scene, you'll always be able to color the map with four colors.)

Mathematicians have been pretty convinced about the truth of this Four Color Theorem since the late 1800s, but despite many false starts, no one gave a proof until 1976, when two mathematicians wrote a very good computer program to check 1,936 cases. (To this day, we have no human-checkable proof.)

In this class, we will definitely not prove the Four Color Theorem. We will, however, prove the Five Color Theorem, which is a whole lot shorter (and which was successfully proven by hand in 1890). Along the way, we'll meet many other cool concepts in Graph Theory. You can expect to write a lot of proofs by induction on the problem sets.

Homework: Recommended
Prerequisites: None
Required for: Graph Coloring

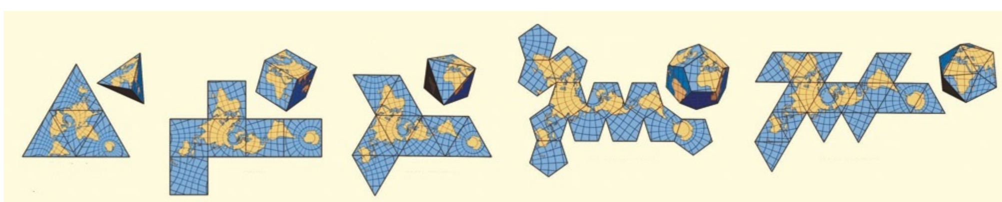
11:10 am Classes

A Quick Introduction to Number Theory. (🌶🌶, Mark, Tue–Sat)

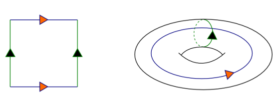
How do you find the GCD of two large numbers without having to factor them? What postages can you get (and not get) if you have only 8 cent and 17 cent stamps available? What is the mathematics used when you send confidential information, such as your credit card number, over the Internet? Besides the answers to such questions, number theory offers insight into many beautiful and subtle properties of our old friends, the integers. For thousands of years, professional and amateur mathematicians have been fascinated by the subject (by the way, some of the amateurs, such as the 17th century lawyer Fermat and the modern-day theoretical physicist Dyson, are not to be underestimated!) and chances are that you, too, will enjoy it quite a bit.
Homework: Recommended
Prerequisites: None (modular arithmetic, but I can catch people up on that individually if needed; may use pigeonhole principle, same comment)
Required for: Problem Solving: Number Theory; Congruent Numbers & Elliptic Curves; Quadratic Forms in Number Theory; Geometry of Numbers; Error-Correcting Codes; When Factoring Goes Wrong; Bernoulli Numbers
Related to (but not required for): Quadratic Forms in Number Theory (W2)

Combinatorial Topology. (🌶🌶, Jeff, Tue–Sat)

Imagine that you wanted to describe a sphere to somebody who lived on a sheet of paper. You might start by giving them folding instructions:



However, there are several different instructions that you could use to describe the same topological shape! How could a flatlander know that all of these different folds describe the sphere? As opposed to this fold, which describes a torus instead...



In combinatorial topology, we describe topological spaces with discrete data called simplicial complexes. With these descriptions, we will start exploring the classical results of topology, including the Euler characteristic. Then we will turn this model in reverse, using topology to study combinatorics, and showing how problems involving partially ordered sets and incidence algebras can be described with shapes. Finally, we will bring together these two languages when exploring discrete Morse theory, which will give us a glimpse of the methods and intuitions that topologists use today.

Homework: Required

Prerequisites: None

Related to (but not required for): All of the Topology classes

Models of Computation Simpler than Programming. (🌶🌶, Pesto, Tue–Sat)
Almost all programming languages are equally powerful—anything one of them can do, they all can. We'll talk about two *less* powerful models of computation—ones that can't even, say, tell whether two numbers are equal. They'll nevertheless save the day if you have to search through 200MB of emails looking for something formatted like an address.[1]

This is a math class, not a programming one—we'll talk about clever proofs for what those models of computation can and can't do.

Homework: Recommended

Prerequisites: None

Related to (but not required for): Models of Computation as Strong as Programming (W2)

The Banach-Tarski Paradox. (🌶🌶🌶, Mark Sapir, Tue–Sat)
About a century ago, Hausdorff discovered that one can cut a 3-dimensional ball into a finite number of pieces, then rotate the pieces to obtain two balls of the same radius. Shortly after, Banach and Tarski proved that one can double almost any 3-dimensional body. Moreover, one can transform one body into another: eg, one can cut an ordinary cat (Garfield) into a finite number of pieces, rearrange the pieces and obtain 101 dalmatians. Von Neumann connected this paradox with group theory, thus discovering the subject of amenable groups which is now an important part of both algebra and analysis. In my course, I will explain Hausdorff's proof. We shall also discuss the question "If one wants to double something, how many cuts and rotations does one need?" I will show that this question is related to (the infinite version of) Hall's marriage theorem which answers the following important question: How to pair up infinitely many boys with infinitely many girls so that each girl likes her partner?

Homework: Recommended

Prerequisites: None

Related to (but not required for): Group Theory (W1)

The Hales–Jewett Theorem. (🌶🌶🌶🌶, Misha, Tue–Sat)
Imprecisely speaking, the Hales–Jewett theorem is about multidimensional tic-tac-toe. If you tell me "I want to play with 19 other players on a board with 100 cells to a side", I can find you some sufficiently large number of dimensions which will guarantee that the game cannot end in a draw, ie. no matter how you play, one of you will definitely win.

This theorem can be used to prove results about arithmetic progressions, set families, vector spaces, et cetera, and is therefore one of the key results in Ramsey theory (the branch of combinatorics that studies such statements). We'll see some of these applications in this class.

We'll also focus on the question: how large is "sufficiently large"? (Spoiler: Large enough for us to need new notation for really large numbers!) I'll cover two proofs of the Hales–Jewett theorem, one of which is a revolutionary improvement on the other even though both involve numbers exceeding the number of particles in the universe. We'll discuss lower bounds for this and other Ramsey results as well.

Homework: Recommended

Prerequisites: None

[1] xkcd.com/208

1:10 pm Classes

Introduction to Ring Theory. (🌶🌶, Daoji Huang, Tue–Sat)
You know integers. You know rational numbers, real numbers, and complex numbers. You also know polynomials, matrices, and maybe power series. In fact we can call any of these things a ring, which is an abstract algebraic structure formed by a set of elements with two "well-behaved" operations: addition and multiplication. Simple as the definition seems, it took mathematicians decades of effort working with concrete theories in algebraic number theory, algebraic geometry, non-commutative generalizations of complex numbers, etc. before coming to a clean axiomatic definition of a ring.

Our goal is to develop a basic understanding of this abstract being in algebra–land. You will get a sense of what questions people typically ask when dealing with an algebraic structure, which will come in handy when you study other types of structures later. You will also get to see a nonconstructive proof of the famous Hilbert's Basis Theorem: we will show that something exists without explicitly demonstrating how to find it. Its nonconstructiveness was controversial at its time.

Homework: Required

Prerequisites: None

Required for: Category Theory; Galois Theory; When Factoring Goes Wrong

Irrationalia. (🌶, Aaron Fenyes, Tue–Sat)
At first glance, a single irrational number is a pretty boring thing: just a point on a line. If you look deeper, though, you'll find a whole world of information and structure, which can be brought to the surface in many different ways. One, which you're probably familiar with, is writing the number as a decimal. Another, which we'll focus on in this class, is using the number as the slope of a line on an infinite chessboard, and watching how the line cuts through the sides of the squares. Exploring these "cutting sequences" will teach us a lot about irrational numbers, and take us to some very unexpected places.

This class will emphasize hands-on experimentation, and you'll discover a lot of the main results yourselves. You're very welcome to go "off-road" and explore topics and examples I haven't planned on visiting, although the stuff we talk about during class will mostly be scheduled ahead of time.

Homework: Recommended

Prerequisites: None

Required for: Nothing

Point-set topology. (🌶🌶🌶🌶, Alfonso, Tue–Sat)
You probably know that the sequence

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$$

has limit 0.

Here is another sequence

Rock, Paper, Rock, Paper, Rock, Paper, Rock, Paper, ...

What is the limit of this sequence? Does this question even make sense? The answer is, surprisingly, "yes!" as long as we have a topology in the right sense.

A topology is an object that allows us to talk about limits, convergence, and how close elements are in arbitrary sets. Sort of.

This course will be Moore Method. This means that I will not lecture and you are not allowed to use books, the internet or any other resources. I will be giving you definitions and asking many questions. You will be doing all the work to answer them in the form of daily homework. Class time will be spent discussing your solutions. So you will be the ones proving all the theorems, and sometimes even figuring out what the theorems say in the first place! Be ready to commit daily time to working outside of class if you do not want to get lost immediately.

Because of the nature of this class, **there is homework due on the first day of classes (Tuesday)**. Read through and prepare your answers to (most of) the questions in Chapter 1. If you want to get a sense of the difficulty of the course, please note that Chapter 1 is easier than the rest of it. You can download the course notes from `http://tinyurl.com/MCtopology` and I will also have physical copies with me during opening assembly.

Homework: Required

Prerequisites: You need to be comfortable with quantifiers (∀, ∃). Understanding the ε–δ definition of limit of a function, while not necessary, will help.

Required for: How to Cut a Sandwich

Related to (but not required for): Scandalous Curves (W4)

Problem Solving: The Probabilistic Method. (🌶🌶, Tim!, Tue–Sat)
"When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth." — Sherlock Holmes.

This is a good way to way to solve crimes, and a good way to solve math problems. If you need to prove that some Mathcamp staff member is a spy, calculate the probability that a randomly-chosen staff member is a spy. If the probability is greater than 0, then you can safely conclude that a traitor walks among us (even though you might not know who it is).

Perhaps the most suprising thing about this method is that it is actually useful! In fact, the principle above is all you need to solve this problem:

- Prove that there exists a graph with 1,000,000 vertices such that every set of 40 vertices has a pair of adjacent vertices and a pair of nonadjacent vertices.

One might be worried that a probabilisty-based proof to this problem might not be air-tight because it leaves things to chance, but fear not — even though the proof uses probability, the final result is true with absolute certainty.

In addition to this strategy, we'll see more probability-based approaches to solving problems (including problems whose statements don't reference probability at all). Part of the class time will consist of campers working on problems in groups and presenting solutions.

Homework: Required

Prerequisites: None

Related to (but not required for): Other Problem Solving

Real Analysis. (🌶🌶🌶, Nic, Tue–Sat)
If you've taken a calculus class, it might have occurred to you to wonder what a limit really is. Sure, you can compute them, but what exactly is it that you're computing? The teacher said something about getting "closer and closer," but it's hard to really wrap your brain around a definition like that. How *much* closer do you have to get? When? Where? Why? And I guess a function is "continuous" if I can draw a graph of it and never have to lift my pen off the paper? And while we're at it, what are these "real number" things we keep talking about anyway?

Real analysis is about answering these types of questions and a whole lot more. We'll start going back through the stuff you learned in calculus class, giving meaning to definitions and proofs to theorems, and when we're done you'll have seen what it really means for a function to be continuous, for a sequence or a series to converge, or for a limit to exist.

Homework: Required

Prerequisites: You should be comfortable with proofs and with the basic language of set theory. In particular, you should know what it means to prove something by contradiction, to prove a statement containing the phrase "if and only if", and what it means to write something like $C = \{x : x \in A \text{ and } x \notin B\}$.

Required for: The Intermediate Value Theorem and Chaos; Scandalous Curves

Related to (but not required for): Through the Eyes of a Prime: An Introduction to p-adic Numbers (W1); Complex Analysis (W1); Moore Method Point–Set Topology (W1)

Colloquia

The Laws of Thought. (Noah Goodman, Tuesday)
What mathematics can describe the workings of the human mind? Is there a "calculus of thought"? I will suggest that combining two standard, but ridiculously powerful, ideas from mathematics brings us close to such a calculus: Probability captures rational reasoning under uncertainty, while lambda calculus captures universal computation. The combination – stochastic lambda calculus, as realized in the probabilistic programming language Church – provides a system for modeling many of the subtle inferences that make human thought so remarkable. I will illustrate this with examples of reasoning about team games, reasoning about other people's actions, and more.

The Chebyshev Prime Number Theorem. (Mark Sapir, Wednesday)
There is nothing more basic and important in mathematics than prime numbers, and their distribution in the set of natural numbers is one of the main mysteries. I will give a completely elementary proof of the celebrated theorem by Chebyshev that the number of primes below any natural number n is approximately $\frac{n}{\log n}$ (up to a constant multiple). This 19^{th} century result was the first major breakthrough in the study of prime numbers since Euclid's proof that the set of primes is infinite. It led to many other important results such as the Prime Number Theorem and the Riemann hypothesis.

The story of ranks and cranks of partitions. (Holly, Thursday)
Beginning with simple counting and addition, we explore a function called the partition function which has some surprising arithmetic properties. Underlying these properties are combinatorial objects of interest: the rank and crank of a partition. We will discuss the history of the rank and crank, how they have furthered our understanding of partitions, and some recent work about their relationship to each other.

Hydras. (Susan, Friday)
The Lernean Hydra was a legendary monster with many heads, poisonous breath, and an all-around bad attitude. The hero Heracles was sent to kill the beast, but found that whenever he cut off one of its heads, two would grow back in its place. What's a hero to do? We will attempt to slay a different kind of Hydra. In the Hydra game, we start with a rooted tree (our Hydra), and in each turn, we remove a "head". On the n^{th} turn, n new Hydra heads will grow back in its place. Heracles's story has a happy ending—he was able to kill the Lernean Hydra with an extremely clever plan of attack. What sort of cleverness do we need to kill our Hydra? Come and find out!

Visitor Bios

Yvonne Lai. Mathematical interests: Hyperbolic geometry, geometric group theory (my advisor would say geometric geometry), slicing and dicing geometric objects

Hobbies: Biking, spelling things like a Canadian, playing sand volleyball badly

Fun Fact: Did you know there is a sport called kayak polo? It is like water polo, except that all players are on single person kayaks. My now hometown Lincoln has a club for it and I tried it out this year!

Nic Ford. Nic recently received his Ph.D. from the University of Michigan and is now working as a software developer at Jane Street Capital (which is hosting several former campers as interns this summer!). When he was still doing math research, it was in combinatorial algebraic geometry, specifically Schubert calculus, which he still thinks is pretty great. He was a Mathcamp mentor for four years, and very much regrets not being able to stay for the full summer this time.

Daoji Huang. I'm about to enter my 2nd year of PhD in math at Cornell University. While I have special feelings towards algebra, I enjoy learning almost all kinds of math and listening to other people talking about their math. Non-math things I care a lot about include coffee, doing pottery, the game of go, and trees.

Mark Sapir. Mark Sapir is a math professor at Vanderbilt University, interested in algebra (specifically, algorithmic, geometric and probabilistic invariants of groups, semigroups and algebras). He earned his PhD from Ural State University in the USSR. He's the creator of http://www.math.vanderbilt.edu/ msapir , *wise* words from a math professor. His older daughter Jenya is a Mathcamp alumna and newly-minted math PhD and his son Yasha is visiting Mathcamp with him.

Noah Goodman. Once upon a time, Noah Goodman was a math graduate student at UT Austin, studying geometric topology. He came to Mathcamp as a mentor from 1999 to 2001, where he met Mira's husband Josh Tenenbaum (whom you will also get to meet in Week 2). Josh introduced Noah to cognitive science, the mathematical study of the mind, and turned him over to the dark side: after getting his PhD, Noah left pure math and came to work in Josh's lab as a postdoc (but not before spending a couple of years as a real-estate developer in Chicago). Now Noah is a professor of psychology, computer science, and linguistics at Stanford, doing research on computational models of cognition and the integration of probability and logic. He's only around for a day this year (just giving a colloquium, not teaching any classes), but he loves hanging out with Mathcampers. Come and talk to him soon, before he disappears!

Holly Swisher. Holly Swisher studies number theory and combinatorics. She is particularly interested in cool things like partition theory, modular and mock modular forms, and hypergeometric series. Partition theory deals with counting ways to add up positive integers to get a specific positive integer. Sounds easy, right? Not always! This topic includes the study of various partition functions, their combinatorial properties, congruences, asymptotics, and connections with modular forms. (Modular forms were involved in the famous proof by Wiles et al of Fermat's Last Theorem through their connections with elliptic curves!)

Matt Wright. Matt recently finished grad school (studying logic, with a focus on computability theory) and has now moved on to being a software engineer at Dropbox. A mentor last summer and a camper many, many summers ago, Matt is really excited to be back visiting Mathcamp!