CLASS DESCRIPTIONS—WEEK 5, MATHCAMP 2014

Contents

9:10am Classes

The Many Ways to the FTA. (🌶🌶🌶🌶, Alfonso, Tuesday–Friday and Wednesday at 1:10)
The Fundamental Theorem of Algebra (FTA) says that every polynomial (with coefficients in $\mathbb{C}$) has a complex root. There are many ways to prove it, but they all require to go deep into some field of mathematics. And I do mean *some* field, since it appears that advanced knowledge of any subfield of math produces a proof of the FTA. In every meeting of this class I will give you a crash course on a topic, starting from scratch (sort of) that culminates with a proof of the FTA.

- *A crash course on complex analysis* - Let's redo in one hour some of what Kevin did in two weeks and use analysis to prove the FTA!
- *A crash course in algebraic topology* – Let's redo in one hour some of what Jeff did in one week and use the fundamental group to prove the FTA
- *A crash course in Galois theory* – Let's redo in one hour some of what Mark did in two weeks and use it to prove the FTA.
- *A crash course in symmetric polynomials* – Let's redo in one hour what Newton probably did in one hour and use combinatorics to prove the FTA.
- *A crash course in differential geometry* – Let's do in one hour a chunk of a graduate course in Riemannian geometry and use it to prove the FTA.

In case this isn't obvious: each of the five days is independent from the rest. Join us in this whirlwind tour which promises to be Fabulously Tantalizing Awesomeness.
Homework: None
Prerequisites: No prerequisites: yet at the same time I do not promise full understanding, so if you have some knowledge of these topics, it will help.
Related to (but not required for): Math

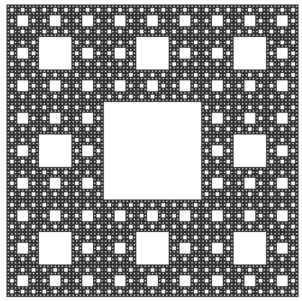
Measure Theory. (🌶🌶🌶, Jeff, Tuesday–Friday)
Every year, more and more students are afflicted with the Riemannianintegrationitis. You may have this if you exhibit any of the following symptoms:

- Have you ever wanted to integrate

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

and found that you just didn't have the right tools to do it?

- Do you ever feel like



needs to have an area assigned to it, but you just aren't in a position to fix this problem?

- Does the fact that the set of integrable functions fail to form a complete metric space keep you awake late at night?

If you answered yes to any of these questions, you may be eligible for *Measure Theory.* Our 4-day program is proven to help relieve and cure some of the these symptoms. 4 out of 5 professional mathematicians suggest using *Measure Theory* to solve life's little difficulties.[1] Talk to your academic advisor today about how *Measure Theory* can improve your life.[2]

Homework: Optional

Prerequisites: You must be able to write an ε-δ proof.

Math Teaching Workshop. (🌶, *Dan Zaharapol*, Tuesday–Friday)
Teaching is not just for professional teachers. Have you ever wanted to present a solution to a class? Show a friend something cool you've learned? Or maybe explain to a non-mathy person what's so interesting about what you do? In this class, we will explore the essential skill of communicating mathematics face-to-face with speech and writing. Great teachers make teaching seem entirely natural; so natural, in fact, that their students often overlook the skills that go into a successful presentation. We'll get into the details of organizing material, board technique, verbal communication, classroom management, pacing, and more. Sound like a lot to handle? Relax – even the best teachers rarely get every detail right. The goal is to give you some awareness of these issues so that you can start improving and take your first steps on the road of teaching. The format will be part lecture and part workshop, in which you will prepare and deliver miniature lectures for the class.

Homework: Required

Prerequisites: None

Combinatorial Trigonometry. (🌶🌶🌶, Kevin, Tuesday–Wednesday)
A *zigzag permutation* is a permutation $a_1, a_2, a_3, \ldots, a_n$ of the numbers $1, 2, 3, \ldots, n$ such that $a_1 < a_2 > a_3 < \ldots$. The number of zigzag permutations for even n are called the zig numbers, and the number of zigzag permutations for odd n are called the zag numbers. Let's define $S(x)$ and $T(x)$ to be exponential generating functions for the zig and zag numbers respectively. Then fun things happen! These functions satisfy identities like $T(x)^2+1 = S(x)^2$, and their derivatives are related to each other in suspicious ways, like $T'(x) = S(x)^2$. Are you starting to guess what's going on? (Spoilers: Read the title!)

Homework: Optional

[1] According to poll of main office, last week.

[2] Common side effect may include doubting the axiom of choice, seeing double spheres, being unable to distinguish functions that differ on only countably many points, overuse of the word generic, mild headaches, nonconstructable sets and infinite dimensional vector spaces.

Prerequisites: Familiarity with generating functions and differential calculus.

Nonzero-Sum Games. (🌶, Pesto, Thursday–Friday)
Sarah wants to take the Chess class and Stephanie wants to take the Go class, but they conflict. Even more than either of them wants to take their preferred class, though, they want to be in the same class, to confuse their teachers. Games like chess and go, with only one winner, have optimal strategies, although they may be hard to find. In Sarah and Stephanie's class selection game, it's not clear what an optimal strategy means. We'll talk about and play games where everyone could win, or everyone could lose, or anything in between. We'll also discuss the relevance of the golden rule and Kant's categorical imperative to game theory.

Homework: Recommended

Prerequisites: None

10:10am Classes

Weird Logics. (🌶🌶, Steve, Wednesday–Thursday)
Consider the sentence "If this statement is true, then Santa Claus exists." Let's call that sentence "P." If P is true, then if P is true, Santa Claus exists; so, since P is true, Santa Claus exists. On the other hand, what if P is false? Well, the only way for an "If A then B"-statement to be false is if A is true and B is false; in this case, since A is "P is true," we have that if P is false then P is true. So clearly P is true and Santa Claus exists. This is a bit of a problem, and has led some people to conclude that natural language is a lot of bunk. But that's no fun. Instead, let's conclude that it is *logic* that is a lot of bunk! And that we should replace it with new, non-bunky logic. In this class, we will de-bunkify logic. Warning: logic is not guaranteed to not be bunk at the end.

Homework: Optional

Prerequisites: None

Perfect Numbers. (🌶, Mark, Friday)
Do you love 6 and 28? The ancient Greeks did, because each of these numbers is the sum of its own divisors, not counting itself. Such integers are called *perfect*, and while a lot is known about them, other things are not: Are there infinitely many? Are there any odd ones? Come hear about what is known, and what perfect numbers have to do with the ongoing search for primes of a particular form, the so-called Mersenne primes - a search that has largely been carried out, with considerable success, by a far-flung network of individual "volunteer" computers.

Homework: None

Prerequisites: None

String Theory. (🌶🌶, Sachi + Tim!, Wednesday–Thursday)
Let's say you want to hang a picture in your room, and you are worried that the 2,000 fans you rented to beat the heat will blow it off the wall, so you want to hang it very securely. You start by wrapping string around some nails, like this:

You figure it will be super-duper secure, because you wrapped it around three nails a lot of times. Don, angry about the electricity bills from running all your fans, comes over and pulls one of your nails out of the wall… and your picture comes crashing to the ground! Actually, no matter which nail he removes, it falls! What happened?

In this class, you'll be playing with string to solve puzzles like this one, and exploring fundamental groups, homology, and monotone boolean functions.

Homework: Optional

Prerequisites: None

Time Travel Algorithms. (🌶🌶🌶, Pesto, Wednesday–Thursday)
Achron is a real-time strategy computer game in which you can make changes to the past. Lost a battle? Go back and try a different strategy, and hope your opponents don't do the same. How do you go from a real-time strategy computer game in which you can't make changes to the past to one in which you can? If you're making a bunch of complicated changes to, say, a spreadsheet, and at some point you realize that you made a mistake early on that affected your later work, you'll probably have to reload and start over. Could you design the spreadsheet program in such a way that you can say "Oh, I meant something else then" and have all your subsequent work update automatically? We'll talk about how to make data structures retroactively changeable, without worrying too much about what the data structures actually are.

Homework: Recommended

Prerequisites: None

Constructing a Peano Curve. (🌶🌶, *Conrad Kosowsky*, Friday)
A Peano curve is a function $f : [0,1] \to A \subset \mathbb{R}^2$ where A is a shape with positive area and f is continuous and surjective. If this does not sound pathological, read it again.

Come to this class to learn the black magic of constructing Peano curves! First, I will show how to build it when $A = [0,1]^2$, then when A is any compact, convex subset of $\mathbb{R}^n$, and for the final twist, I will make f infinity-to-one.

Homework: Optional

Prerequisites: None. You don't need to know what compact or convex mean to appreciate this class.

Hyperfinite Approximation. (🌶🌶🌶, Don, Wednesday–Thursday)
Hyperfinite sets are like finite sets, but more so.
Also, they're really good at approximating things.
Homework: None
Prerequisites: None

The Harmonic Series. (🌶🌶, Misha, Friday)
The series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

diverges. How quickly? We will introduce and use some techniques of asymptotic analysis to estimate the sum of the first n terms of this series, with applications to unusual shuffling methods and the stamp collector problem.
Homework: None
Prerequisites: None

Toward a Human-level AI. (🌶, Josh Tenenbaum, Friday)
What will it take to to build a human-level AI, why aren't we there yet, and how long will it be until we are? Let's have a speculative but informed discussion of these questions, driven by your ideas and proposals, but grounded in the latest research in human cognitive science and neuroscience as well as computer science and mathematics relevant to AI. Along the way we'll also talk about some of the most important things scientists have come to understand in the last decade about how human intelligence works, as well as the most important scientific questions about the mind that we don't yet understand.

11:10am Classes

Quadratic Reciprocity. (🌶🌶🌶, Mark, Tuesday–Wednesday)
Let p and q be distinct primes. What, if anything, is the relation between the answers to the following two questions? Question 1: "Is q a square modulo p?" Question 2: "Is p a square modulo q?" In this class you'll find out; the relation is an important and surprising result which took Gauss a year to prove, and for which he eventually gave six different proofs. You'll get to see one particularly nice proof, part of which is due to one of Gauss's best students, Eisenstein. And next time someone asks you whether 101 is a square mod 9973, you'll be able to answer a lot more quickly, whether or not you use technology. If time permits, we'll also explore a different proof that is somewhat shorter and that uses something John Conway pointed out to me: In one sense, the theorem isn't about primes, after all!
Homework: Optional
Prerequisites: A bit of number theory, specifically Fermat's little theorem.

The Nine-Point Circle. (🌶🌶, Mark, Thursday)
There is some beautiful geometry hidden in and around every triangle. In particular, there are several points that can qualify as "centers" of the triangle, but that are different unless the triangle is equilateral. One of those points is the center of a circle that goes through nine related points, so it's not too surprising that it's called the nine-point circle. If you haven't seen this but you like plane Euclidean geometry, you're in for a treat!
Homework: None
Prerequisites: None

The Cayley-Hamilton Theorem. (🌶🌶🌶, Mark, Friday)
Take any square matrix A and look at its characteristic polynomial $f(X) = \det(A - XI)$. (The roots of this polynomial are the eigenvalues of A). Now substitute A into the polynomial; for example, if A is a 4-by-4 matrix such that $f(X) = X^4 - 6X^3 - X^2 + 8X - 17$, compute $A^4 - 6A^3 - A^2 + 8A - 17I$. The answer will always be the zero matrix! In this class we'll use the idea of the "classical adjoint" of a matrix to prove this fundamental fact, which can be used to help analyze linear transformations that can't be diagonalized.

Homework: None

Prerequisites: Linear algebra, including a solid grasp of determinants (the "magic of determinants" class would certainly take care of that).

Related to (but not required for): The Magic of Determinants

Open Problems in Logic. (🌶🌶🌶🌶, Steve, Tuesday–Friday)
What exactly *do* logicians do these days? Well, four things they *don't* do is try to solve the following four problems, because they're too hard:

(1) Consider the set of math problems, partially ordered by relative difficulty. Does the resulting poset have any nontrivial automorphisms? (Computability theory.)
(2) Exactly how hard *is* it to prove that set theory is consistent? (Proof theory.)
(3) What is the largest kind of infinity that can exist before set theory breaks? (Set theory.)
(4) How many different ways can a set of axioms be satisfied...? (Model theory.) And, are we sure...?

This class is really four 1-day classes as opposed to one 4-day class: each day, we'll discuss one problem (in the order listed above); we'll state it precisely, explain why it is interesting and what partial progress has been made. Each day is totally independent - you can come to day 1 only, or days 2 and 4, or whatever! It'll be awesome and terrible! Wheeee!

Homework: None

Prerequisites: Some background in logic or set theory; e.g., what the ZFC axioms are.

Related to (but not required for): Continuum hypothesis

Stupid Games on Uncountable Sets. (🌶🌶, Susan, Tuesday–Wednesday)
Let's play a game. You name a countable ordinal number. And then I name a bigger countable ordinal. We'll keep doing this forever. When we're all done, we'll see who wins. In this class we'll be discussing strategies for winning an infinite game played on ω_1. In particular, we'll talk about how to set up the game so that at any point, neither player has a winning strategy.

Homework: None

Prerequisites: Might help to have seen the ordinals. Talk to Susan if you want a quick intro.

Algebra without (Much) Algebra, or; Graph Theory is Best Theory. (🌶🌶🌶, Paddy, Thursday–Friday)

A GRAPH THEORIST and an ALGEBRAIST are walking to TAU from opposite directions, looking at their class notes.

ALGEBRAIST
Mmm, group theory!

GRAPH THEORIST
Mmm, graph theory!

They COLLIDE.

ALGEBRAIST
You got graphs in my groups!

GRAPH THEORIST
You got groups in my graphs!

NARRATOR
Groups and graphs: two great tastes that go great together!

ALGEBRAIST, GRAPH THEORIST
AAAAAAAA TERRIFYING VOICE FROM THE SKY FLEE FLEE

Somewhat more seriously: in this class, we're going to define Cayley and Schreier graphs, deduce a number of results about these objects, and use our observations to prove that the subgroup of any free group is free! This is a surprisingly non-obvious theorem in algebra, that is incredibly awful to prove without using techniques from other fields – we're going to do it using graphs, using techniques that are both remarkably useful in designing computer-aided algebra algorithms and also satisfying to compute by hand.

Homework: Recommended

Prerequisites: Graph theory, group theory

Related to (but not required for): Graph theory classes, group theory classes

Information Theory and the Redundancy of English. (🌶🌶, Mira, Tuesday–Friday)

NWSFLSH: NGLSH S RDNDNT!! (DN'T TLL YR NGLSH TCHR SD THT)

The redundancy of English (or any other language) is what allows you to decipher the above sentence. It's also what allows you to decipher bad handwriting or to have a conversation in a crowded room. The redundancy is a kind of error-correcting code: even if you miss part of what was said, you can recover the rest.

How redundant is English? There are two ways to interpret this question:

- How much information is conveyed by a single letter of English text, relative to how much could theoreticaly be conveyed? (But what *is* information? How do you measure it?)
- How much can we compress English text? If we encode it using a really clever encoding scheme, can we reduce the length of the message by a factor of 2? 10? 100? (But how will we ever know if our encoding is the cleverest possible one?)

Fortunately, the two interpretations are related. In this class, we will first derive a mathematical definition of information, based on our intuitive notions of what this word *should* mean. Then we'll prove the Noiseless Coding Theorem: the degree to which a piece of text (or any other data stream) can be compressed is governed by the actual amount of information that it contains. We'll also talk about Huffman codes: the optimal way of compressing data if you know enough about its source. (That's a big "if", but it's still a very cool method.) Finally, we'll answer our original question – how redundant is English? – in the way that Claude Shannon originally answered it: by playing a game I call Shannon's Hangman and using it as a way of communicating with our imaginary identical clones!

The class is 4 days long, but you can skip some of the days and still come to the others. Here's how it works:

Day 1 🌶🌶: *Introduction and definition of information.* Required for the rest of the class, unless you've seen some information theory before.

Days 2, 3 🌶🌶🌶: *Noiseless coding and Huffman codes.* The mathematical heart of the class, where we'll prove the Noiseless Coding Theorem.

Day 4 🌶🌶: *Shannon's Hangman and the redundancy of English.* You can come to this class even if you don't come on Days 2 and 3 – you just need the material from Day 1.

Homework: Recommended
Prerequisites: None

1:10pm Classes

The Stable Marriage Problem. (🌶, Alfonso, Tuesday)
N single men and N single women want to pair up and get married. These are their names and preferences:

- Don: Marisa, Ruthi, Lisa, Susan, Angela.
- Jeff: Angela, Marisa, Susan, Ruthi, Lisa.
- Kevin: Angela, Susan, Ruthi, Marisa, Lisa.
- Pesto: Lisa, Ruthi, Susan, Angela, Marisa.
- Tim!: Lisa, Angela, Marisa, Susan, Ruthi.

- Angela: Don, Pesto, Tim!, Kevin, Jeff.
- Lisa: Kevin, Jeff, Don, Tim!, Pesto.
- Marisa: Kevin, Pesto, Tim!, Jeff, Don.
- Ruthi: Kevin, Tim!, Don, Jeff, Pesto.
- Susan: Kevin, Pesto, Jeff, Tim!, Don.

Is it possible to make everybody happy? Obviously not since almost everybody wants to marry Kevin. But is it possible to at least create a *stable* situation? For instance, it is a bad idea for Kevin to marry Ruthi and for Susan to marry Pesto, because then Kevin and Susan would prefer each other rather than staying with their partners, so they will run away together. How can we at least avoid having a run-away couple? Is there more than one way to do it? What is the *best* way to do it? And what if Ruthi and Susan realize that marrying each other is better than marrying Kevin?
Homework: None
Prerequisites: None

The Many Ways to the FTA. (🌶🌶🌶🌶, Jeff, Wednesday)
See 9:10am classes.

Calculus without Calculus. (🌶🌶, Tim!, Thursday–Friday)
If you've taken a calculus class in school, you've surely had to do tons and tons of homework problems. Sometimes, calculus knocks out those problems in no time flat. But other times, the calculus solution looks messy, inelegant, or overpowered. Maybe the answer is nice and clean, but you wouldn't know it from the calculation. Many of these problems can be solved by another approach that doesn't use any calculus, is less messy, and gives more insight into what is going on. In this class, you'll see some of these methods, and solve some problems yourself. Some example problems that we'll solve without calculus:

- You have 40 meters of fence, and you want to build a rectangular pen, where three sides of the pen will be made of fence, and the last side will be a preexisting brick wall. What is the largest area that your pen can enclose?
- Sachi is 5'2" tall and Lisa is 159 cm tall, and they are standing 5 cubits apart. You want to run a string from the top of Sachi's head to the top of Asi's head that touches the ground in the middle. What is the shortest length of string you can use?
- A dog is standing along an inexplicably straight shoreline. The dog's person stands 20 meters way along the shoreline throws a stick 8 meters out into the water. The dog can run along the shoreline at 6.40 meters per second, and can swim at 0.910 meters per second. What is the fastest route that the dog can take to get to the stick?
- Where in a movie theater should you sit so that the screen takes up the largest angle of your vision?

Amaze your friends! Startle your enemies! Annoy your calculus teacher!

Homework: Optional

Prerequisites: Calculus (derivatives) recommended

The Unit Distance Graph Problem And The Axiom Of Choice, or; Graph Theory is Best Theory. (🌶🌶🌶, Paddy, Tuesday)
The **unit distance graph** is the following graph:

- Vertices: the elements of $\mathbb{R}^2$.
- Edges: connect any two vertices whenever their distance in $\mathbb{R}^2$ is exactly 1. For example, $(1, 0)$ and $(\sqrt{2}/2, \sqrt{2}/2)$ are both connected to $(0, 0)$.

Here's an open research problem: find the chromatic number of this graph! In this class, we're not going to answer this problem (though we will find the strongest known bounds currently known to mathematics.) Instead, we're going to focus on the following incredible conjecture: the chromatic number of this graph? It probably depends on the axiom of choice. Vote for this class to see why!

Homework: Recommended

Prerequisites: Graph theory.

Related to (but not required for): Graph theory classes, being paranoid about the axiom of choice in all contexts

A Math Wrangle. (🌶🌶 or 🌶🌶🌶, Mira, Wednesday)
Mathcamp students have been invited to participate in a Math Wrangle at MathFest (the big math conference happening in Portland) on the morning of Saturday, August 9. In preparation, we're running a Math Wrangle of our own.

A Math Wrangle is a type of math competition popular in Russia, which is just beginning to take root in the US. In a Math Wrangle, two teams of three players have an hour to work on a set of 8 proof-based problems ahead of time. During the Wrangle itself, the two teams take turns challenging each other to present their solutions in front of a live audience and a panel of judges. The team that has been challenged may choose to *accept* the challenge or to *return* it. If the challenge is returned, the point-value of the problem goes up (form 7 to 17), and the team that issued the original challenge must present their own solution. In any case, after a solution is presented by one team, the other team has a chance to present a rebuttal, where they show how to correct or improve on their opponents' solution.

There are two ways you can participate in the Mathcamp Math Wrangle: as a competitor or as an audience member.

- If you want to be an audience member (and we definitely need an audience!), just show up.
- **If you want to be a competitor, you must sign up on the schedule board by Saturday 11:59 pm.** (But if you play in the Mathcamp Math Wrangle, you do *not* have to go to MathFest.) You will need to indicate whether you prefer 🌶🌶 or 🌶🌶🌶 problems.

If you want to do the Math Wrangle at MathFest, you must *attend the Mathcamp Wrangle, either as audience or competitor.* If we have a lot of people wanting to compete, then we'll assign some to do it at MathFest and some at Mathcamp.

Homework: Recommended

Prerequisites: None

Hyperplane Arrangements. (🌶🌶🌶, Kevin, Thursday–Friday)
Suppose I want to cut a watermelon into pieces. With 0 cuts, I can make 1 piece: the entire watermelon. With 1 straight cut, I can make 2 pieces. With 2 cuts, I can make 4, and with 3 cuts, I can make 8. I'm sure you see the pattern by now: with k cuts, we can make $\frac{1}{6}(n^3+5n+6)$ pieces. These cuts are an example of a *hyperplane arrangement*, which is simply a collection of k hyperplanes in n-dimensional space. We'll learn some powerful techniques for counting the number of regions that a hyperplane arrangement determines, and we'll maybe even see some connections to graph theory. Throughout, posets will abound!

Homework: Optional

Prerequisites: None

A Game You Can't Play, but Would Win if You Could. (🌶, Mira, Tuesday)
Once upon a time, in the Kingdom of Aleph, lived a cruel King who decided to put his 100 wise men to a test. He had 100 identical rooms constructed in his palace. In each room the king placed an infinite sequence of boxes; in each box he put a real number. The sequence of numbers was exactly the same in each room, but otherwise completely arbitrary.

The king told the wise men that when they were ready, each of them would be locked in one of the 100 rooms. Each of them would be allowed to open all but one of the boxes in the room. (This, of course, would take an infinite amount of time, but in the Kingdom of Aleph, they're pretty cavalier about infinities.) Finally, each wise man would be required to name the number in the box that he did *not* open. If more than one wise man names the wrong number, they would lose their jobs and their lives.

There is no reason for anyone to hurry in the Kingdom of Aleph, and the king gives the wise men an infinite amount of time to work out a strategy. Do they have any hope of making it through his cruel test alive? What should they do?

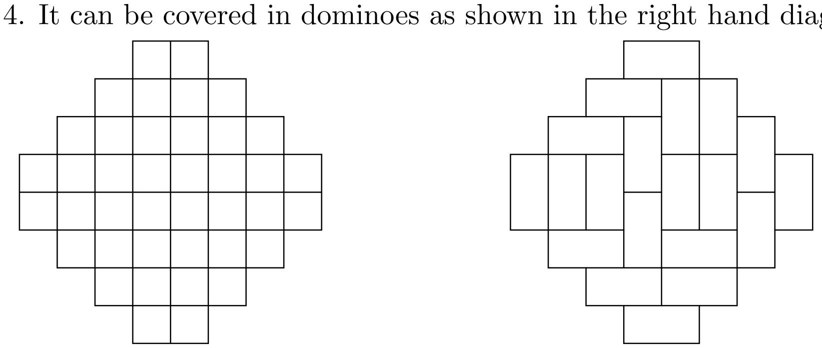
Infinitesimals. (🌶🌶, Don, Wednesday–Friday)
When calculus first came into the world, those who created it did so using a fairly controversial tool: infinitesimal values. Infinitesimals were useful, because there's no need to define limits when you can work directly with a notion of "incredibly close." When infinitesimals are allowed, derivatives can be computed directly, and dx is an actual value instead of a vague concept. Ultimately, the debate over infinitesimals was won by their opponents, who claimed that these things could not be made rigorous, and thus had no place in mathematics. The debate lay dormant for centuries, until a few mathematicians realized that, even though $\mathbb{R}$ contains no infinitesimals, there are reasonable systems of mathematics that do. In this class, we'll be constructing an alternative to standard calculus, in which infinitesimals create no contradictions. In order to get there, we'll have to throw out a few axioms - you know, minor things like $x \vee \neg x$ being a tautology - but once we've done so, we'll be in a beautiful place, where every function is smooth, and the Banach-Tarski paradox is no more.
Homework: None
Prerequisites: Calculus is suggested, but not required

Wedderburn's Cute Little Theorem. (🌶🌶🌶, Susan's Nose Function, Tuesday)
Wedderburn's Little Theorem states that any finite domain is a field. So you want a finite domain with zero divisors? Too bad! Finite noncommutative division rings? Not gonna happen! The proof of this adorable theorem will be presented by an appropriately adorable entity: Susan's nose fuction.
Homework: Recommended
Prerequisites: Definition of group, ring, field, and domain.

BIG Numbers! (🌶, Julian, Wednesday)
We'll be learning the original proof of van der Waerden's theorem: if we colour the natural numbers with c colours, there are monochromatic arithmetic progressions of arbitrary (finite) length.

BIG WARNING: This class will involve big numbers. I mean, really huge numbers. Astonishingly colossal numbers. In fact, numbers so large that this class comes with a health warning: Do not attend this class if you are of a nervous disposition or are scared of large numbers!
Homework: None
Prerequisites: No fear of big numbers

Aztec Diamonds. (🌶🌶, Julian, Thursday–Friday)
An Aztec diamond is a diamond shape made of squares, as shown in the left hand diagram for a diamond of order 4. It can be covered in dominoes as shown in the right hand diagram.



How many ways are there of covering such a shape with dominoes? And what would a random such tiling look like on average? These seemingly hard questions have been investigated in the past twenty years and have revealed some beautiful mathematics (as well as astonishing pictures).

In this class, we will explore some of the hidden secrets of the Aztecs; there will be a few proofs and sketch proofs, but the emphasis will be on the beautiful known results.

Homework: None
Prerequisites: None

2:10pm Classes

The Magic of Determinants. (🌶🌶, Mark, Tuesday–Thursday)
Determinants, which are numbers associated to square matrices, have many useful properties; for example, they are needed to give a general formula for the inverse of a matrix. Unfortunately, determinants are often defined in a very *ad hoc* way (using Laplace expansion) which may obscure what is really happening. This class will give a "better" theoretical framework as well as some geometric intuition, and I'll try to at least give an outline of the proofs of all the main computational properties of determinants, such as the Laplace expansion.
Homework: Optional
Prerequisites: Matrix multiplication, and the idea of a linear transformation. (No previous knowledge of determinants is required.)

The Art Gallery Problem. (🌶🌶🌶, *Yonah Borns-Weil*, Friday)
What do we do when people try to steal from the largest art museum in Northern New England? We place guards! But how many guards do we need in order to ensure sure that they can see everything, so no priceless works can be stolen?

More specifically, given a (not necessarily convex) n-sided polygon, we want to place "guards" at its vertices so that any point inside the polygon is within the unobstructed view of a guard, who can see in all directions but cannot move. The Art Gallery Problem (now a theorem) gives a formula minimizing the number of guards necessary, and is well-known for having a fantastically short proof that makes the theorem seem obvious. We will show that proof, as well as the proof for the related Fortress Problem, which is instead concerned with seeing the *outside* of polygons. If we have time, I will also present other related results and generalizations, some of which are still open.

All the math presented here is relatively recent; the standard Art Gallery Theorem, which started the chain of results, was only proved in 1975. So if you think that all new math uses words like "homological algebra" or "algebraic variety," come to this class to see how much more simple stuff there still is to figure out.
Homework: None
Prerequisites: Knowing what a graph is.
Related to (but not required for): Graph theory classes

Memorylessness: The Exponential Distribution. (🌶🌶, Misha, Tuesday–Friday)
Every second that Paddy spends in the main lounge, there is a small but constant chance that he will be drawn on by campers. How long will Paddy have to wait for this to happen? The exponential distribution answers this question. In this class, we will use it to find shortest paths in graphs, analyze the load on a server, and find the mean lifetime of a supply of radioactive uranium.
Homework: Recommended
Prerequisites: None

Facing the Music. (🌶→🌶🌶🌶, Aaron and J-Lo, Tuesday–Friday)
Math and music, friends forever! They go together like...

- Scales and rational approximations. (Why do we use a 12-note scale?)
- Rhythms and the Euclidean algorithm. (What do samba, bossa nova, and Pink Floyd's "Money" have in common?)

🌶🌶 Chord progressions and group theory. (Why do so many songs use the same four chords?)
🌶🌶🌶 Timbre and harmonic functions. (What does a sphere sound like?)

This class will be a survey of connections between math and music. Each day we'll talk about a musical concept and then introduce a mathematical idea that some insight into it. Each day will be more or less self-contained.

Homework: None

Prerequisites: None

Schemes. (🌶🌶🌶🌶, Ruthi, Tuesday–Friday)

Define a commutative ring for me. Go ahead, I dare you. Did you write something down about binary operators that obey certain axioms and so on? I'm here to tell you that this is the *wrong* way to think about commutative rings. The right definition is

A commutative ring is a collection of functions on a space.

Maybe by now you're used to think about polynomials as elements of a ring. But that's not how you first saw them, is it? When you first saw a polynomial, you were probably told it was a function! But what about the integers? They're a ring too, aren't they? What space are they functions on? The answer is: the integers are functions on a *scheme* where its points are the primes. In this class, we'll dissect what this means and why this is the right thing to be thinking about.

Homework: None

Prerequisites: Ring theory.

Colloquia

The Video Game Engine in Your Mind. (Josh Tenenbaum, Tuesday)

What is common sense, and how can we describe it in mathematics? There is a sense in which even a young child has a basic common-sense understanding of how the world works that goes beyond what any formal system, any artificial intelligence (AI) or other machine system, has ever shown. I will argue that we can begin to characterize this common-sense understanding using the same mathematical abstractions that video game designers have adopted to simulate – approximately but extremely efficiently! – the basic ways in which physical objects move and interact, and intentional agents interact with the physical world and with each other. This mathematics helps us build better scientific models of human thought, and also engineer more human-like AI systems.

What is *Your* Axiom of Choice? (Staff, Wednesday)

The Axiom of Choice. What does it state? How do we prove it? Why do we need it? Why it is controversial? The Mathcamp Staff proudly present controversy, cuteness, and choice.

- **Act 1:** Russell's paradox is discovered. The Zermelo-Fraenkel Axioms boldly enter — cardinals are compared — and the Axiom of Choice claims to be natural.
- **Act 2:** The Well-Ordering Principle, the Axiom of Choice, and Zorn's Lemma have a cute-off.
- **Act 3:** The Banach-Tarski paradox; the Axioms defend their worth. Zermelo and Fraenkel assert their independence!