# Applying to Mathcamp 2014

## Ready to Apply to Mathcamp?

*We invite applications from every student aged 13 through 18 who is interested in mathematics, regardless of racial, ethnic, religious, or economic background.*

Mathcamp offers two application deadlines. The options are simply for your convenience – we have no preference!

**Early Action:** Submit your application by March 15th; admissions decisions will be announced April 1st.

**Regular Action:** Submit your application by April 15th; admissions decisions will be announced May 1st.

*An application to Mathcamp consists of the following:*

1) Some basic information about **yourself and your math background**. We will ask you to describe the math courses that you've taken at the high-school level or above, along with scores and awards from any math competitions you've done.

2) A **personal statement** about your interest in math and why you want to come to Mathcamp.

3) Your solutions to the 2014 **Qualifying Quiz** (see below).

4) **Two recommendation letters**, academic and personal.

- The first letter should be from a teacher who knows you well, preferably a math teacher. The letter should speak to your mathematical maturity and readiness for Mathcamp.
- The second letter should be from another adult who knows you personally (e.g. an employer, pastor, soccer coach). This letter should address your maturity, independence, social and personal qualities. We are looking for students who will thrive in the atmosphere of freedom and responsibility that characterizes Mathcamp, and who will make a positive contribution to the camp community.

5) The **Request for Financial Aid**, as needed (see below).

*To get started, go to:*

http://www.mathcamp.org/apply

You will have the opportunity to submit your Quiz or recommendation letters online, by postal mail, or by fax. Mathcamp also accepts paper applications; call for information. There is no application fee.

### Contact Us

Email: info14@mathcamp.org

Telephone/Fax: 888-371-4159

On the web: www.mathcamp.org

## Cost and Scholarships

### Full Camp Fee: $4000 US

(This includes tuition, room, board, & extracurriculars.)

***We are deeply committed to enabling every qualified student to attend Mathcamp, regardless of financial circumstances.***

In the past decade, no admitted student has been unable to attend the program for financial reasons. We are committed to meeting 100 percent of demonstrated need for both new and returning students. For those who need assistance covering the cost of transportation to and from camp, we have some travel subsidies available. We work with each and every family to make Mathcamp affordable.

## US & Canadian Families

### Mathcamp is FREE for US & Canadian families with household income below $60,000 US.

Families with incomes above $60,000 who need assistance will be asked to contribute no more than 3% of their income. There is no set formula: we consider each request on an individual, case-by-case basis, taking special circumstances into account (such as siblings in college, medical expenses, unemployment or other changes in situation).

***Admission to Mathcamp is need-blind for US and Canadian students, so your financial need has no impact on your chances of admission.***

## International Families

### International students are also eligible for financial aid, including full scholarships and travel grants.

We *do* take financial circumstances into account for international students during the admissions process because the cost of travel to camp can be very high, and we want to be completely confident that when we admit a student, we are able to meet her or his full financial need. Nonetheless, we admit many international students each year on partial and full scholarships. We also provide travel grants to help new and returning students from all over the world fly to Mathcamp. Please do not let financial need prevent you from applying!

# Mathcamp 2014 Qualifying Quiz

## Instructions

We call it a Quiz, but it's really a challenge: a chance for you to show us how you approach new problems and new concepts in mathematics. What matters to us are not just your final results, but your reasoning. Correct answers on their own will count for very little: you need to justify all your assertions and ***prove*** to us that your solution is correct. (See www.mathcamp.org/proofs.) Sometimes it may take a while to find the right way of approaching a problem. Be patient: there is no time limit on this quiz.

The problems are roughly in increasing order of difficulty, though the later problems often have some easier parts. We don't expect every applicant to solve every problem. However, the more problems you attempt, the better your chances of being admitted, so we strongly recommend that you try all of them! If you are unable to solve a problem completely, send us the results of your efforts: partial solutions, conjectures, methods – everything counts.

Most of the quiz problems require only high-school algebra and geometry (no calculus). This year, you also need to know a tiny bit of elementary number theory. If you are unfamiliar with the $C(n,k)$ notation or are new to modular arithmetic, we encourage you to read our background handout at www.mathcamp.org/background.

None of the problems require a computer; you are welcome to use one if you'd like, but first see a word of warning at www.mathcamp.org/computers.

If you need clarification on a problem, please email your question to quiz14@mathcamp.org. *You may not consult or get help from anyone else!* You can use books or the Web to look up mathematical definitions, formulas, or standard techniques; just be sure to cite all these sources in your solution. Please do not try to look up the problems themselves: we want to see how well you can do math, not how well you can use Google. **Any deviation from these rules will be considered plagiarism and may permanently disqualify you.**

**Have fun and good luck!**

## Problems

(1) Imagine a chessboard that extends infinitely far in all directions. In the center of every square is a grasshopper.

(a) Show that it is possible for all the grasshoppers to jump simultaneously, so that after the jump there are exactly two grasshoppers in the center of every square. (A grasshopper can jump arbitrarily far; it can also jump straight up and land on the same spot it started on.)

(b) Now suppose that the squares are 1 inch × 1 inch, and a grasshopper can jump at most 100 inches. Is it still possible to end up with two grasshoppers in the center of every square after one jump? If yes, show how it can be done. If no, prove your answer. (Note: Saying that your strategy from part (a) won't work is not a proof that it can't be done, because there might be a different strategy that does work.)

(2) Stephanie is an evil genius; she is building an army of robots and cyborgs. Every day, each of Stephanie's robots builds a copy of itself, while each of her cyborgs builds three new cyborgs and a robot. Once built, the cyborgs and robots never break or die.

(a) If Stephanie wants to double her army each successive day (i.e. to have exactly twice as many robots and twice as many cyborgs as the day before), what ratio of robots and cyborgs should she start with? What if she wants to quadruple her army each day?

(b) More generally, if Stephanie starts with $R$ robots and $C$ cyborgs on day 0, what will be the composition of her army after $n$ days? Prove your answer. (One way to approach this problem is to use part (a), but you're welcome to do it any way you like.)

(3) Let $P_n$ be a regular $n$-sided polygon inscribed in a circle of radius 1. What is the minimum number of circles of radius ½ required to cover $P_n$ completely? (Both $P_n$ and the circles in this problem include the boundary as well as the interior.) Note: in order to prove that something is the minimum, you need to prove both that it works and that nothing smaller works.

(4) Hannah is about to get into a swimming pool in which every lane already has one swimmer in it. Hannah wants to choose a lane in which she would have to encounter the other swimmer as infrequently as possible. All swimmers, including Hannah, swim back and forth at constant speeds, never pausing at the ends of the pool. Hannah swims at speed 1 (one pool length per minute).

(a) Say Hannah chooses a lane with a swimmer who swims at speed $0 < s < 1$. Prove that, if they keep swimming long enough, eventually Hannah and the other swimmer will settle into a pattern where they pass each other (either in the same or in opposite directions, or at the edge of the pool) exactly $N$ times every $M$ minutes, where $M$ and $N$ are relatively prime integers. Find $M$ and $N$. Do they depend on the other swimmer's speed and/or initial position when Hannah enters the pool?

(b) What if Hannah picks a lane with a swimmer of speed $s > 1$?

(c) From Hannah's point of view, what is the ideal speed that another swimmer can have? (Assume Hannah can time her entry into the pool with perfect precision, so she can make the other swimmer's initial position be whatever she wants.)

(5) Let $p$ be a prime. For which integers $n$ is it possible to split the set $\{1,2,\dots,n\}$ into $p$ disjoint subsets such that the sum of the integers in each of the subsets is the same? Prove your answer.

(6) A path from the bottom left to the upper right corner of an $m \times n$ grid is called *monotonic* if on each step it goes either up (U) or to the right (R). Such a path has $m+n$ steps, of which $m$ are U's and $n$ are R's. Thus the total number of monotonic paths is $C(m+n,m)=C(m+n,n)$. We define the *area* of a monotonic path to be the number of squares underneath it. The area can be anywhere between 0 and $mn$. We are interested in areas of monotonic paths modulo a prime $q$. *(If you are new to the C(n,k) notation or to modular arithmetic, visit* http://www.mathcamp.org/background *for a primer.)*

(a) The *forward shift* of a monotonic path $P$ is the path that results when we take the first step of $P$ (either U or R) and move it to the end. For instance, the forward shift of URRURU would be RRURUU. Of course, once we perform $m+n$ forward shifts, we get our original path back. Now suppose $m+n=q$, where $q$ is prime. Let $P_0$ be a monotonic path on an $m \times n$ grid, and let $P_1, P_2, \dots, P_{q-1}$ be successive forward shifts of $P_0$. Show that for each integer $k = 0, 1, \dots, q-1$, exactly one of $P_0, P_1, \dots, P_{q-1}$ has area congruent to $k$ modulo $q$.

(b) Consider all monotonic paths on a $q \times q$ grid, where $q$ is prime. For each integer $k = 0, 1, \dots, q-1$, how many of these $C(2q,q)$ paths have area congruent to $k$ modulo $q$? (Hint: use part (a). It may not seem relevant, but it is.)

(c) More generally, consider all monotonic paths on an $mq \times nq$ grid, where $q$ is prime and $m$ and $n$ are arbitrary positive integers. For each integer $k = 0, 1, \dots, q-1$, how many of these paths have area congruent to $k$ modulo $q$?

(7) A *voting system* is a way of combining the disparate preferences of many individual voters to arrive at a single decision for the group as a whole. When there are two candidates, there is only one reasonable voting system: the candidate who is preferred by the majority of voters wins. But with more than two candidates, things become much trickier. For instance, with three candidates A, B, and C, it is possible that a majority of voters prefers A to B, a majority prefers B to C, and a majority prefers C to A. (See "Voting Paradox" on Wikipedia.) So how should we decide on the winner? There are no easy answers to this question; in fact, you can even prove that, if voter preferences are allowed to be completely arbitrary, there are no good voting systems at all.

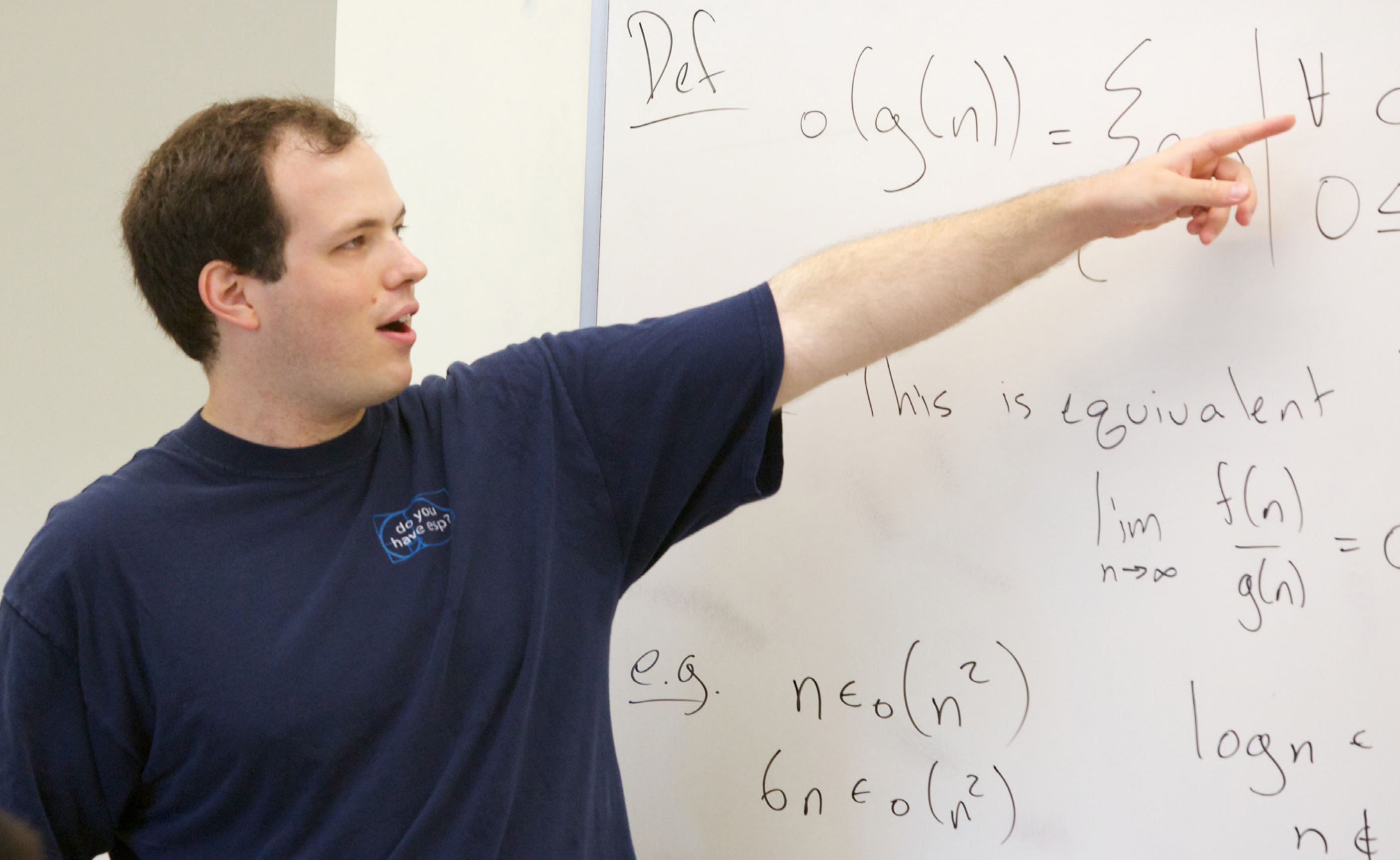
(a) If there are four candidates A, B, C, and D, show that it is possible to have a set of voter preferences such that a majority of voters prefers A to B, a majority prefers B to C, a majority prefers C to D, yet *every single voter* prefers D to A. (The simplest way to show such a thing is simply to write down a set of preferences satisfying these conditions.)

(b) Suppose there are $n$ candidates $C_1, C_2, \dots, C_n$, and for every pair $C_i, C_j$ (with $i \neq j$), I tell you which one should get a majority in a two-way election. Show that it is always possible to construct a set of voter preferences that satisfy these pairwise conditions.

(c) Extra credit: How many voters did you need in your proof in part (b)? Can you find a proof that uses fewer? For a given number $n$ of candidates, what is the smallest number of voters that is guaranteed to work? (Note: We don't know the solution to this problem. We can get the number to be pretty small, but we can't prove that our solution is the smallest. Kudos to you if you can prove it – or if you can prove us wrong by coming up with something smaller!)

*Problem #6 is due to Bill Kuszmaul, MC '12–'13; all other problems by the Mathcamp staff.*

*The Mathematics Foundation of America invites you to apply to the twenty-second annual*




# Academics

## A Variety of Choices

The Mathcamp schedule is full of activities at every level, from introductory to the most advanced:

- Courses lasting anywhere from a day to a month
- Lectures and seminars by distinguished visitors
- Math contests and problem-solving sessions
- Hands-on workshops and individual projects

*You can learn more at:*
http://www.mathcamp.org/academics

## Classes

Course offerings vary from year to year, depending on the interests of the students and faculty. Some of the topics taught in previous years have included:

**Discrete Mathematics:** Combinatorics • Generating functions • Partitions • Graph theory • Ramsey theory • Finite geometries • Polytopes and Polyhedra • Combinatorial game theory • Probability

**Algebra and Number Theory:** Primes and factorization algorithms • Congruences and quadratic reciprocity • Linear algebra • Groups, rings, and fields • Galois theory • Representation theory • p-adic numbers

**Geometry and Topology:** Euclidean and non-Euclidean (hyperbolic, spherical, projective) geometries • Geometric transformations • Combinatorial topology • Algebraic geometry • Knot theory • Brouwer Fixed-Point Theorem

**Calculus and Analysis:** Fourier analysis • Complex analysis • Real analysis • Measure theory • Dynamical systems • Non-standard analysis

**Computer Science:** Cryptography • Algorithms • Complexity • Information theory • P vs. NP

**Logic and Foundations:** Cardinals and ordinals • Gödel's Incompleteness Theorem • The Banach-Tarski Paradox • Model theory • Category theory

**Connections to Science:** Relativity and quantum mechanics • Dimensional physics • Voting theory • Bayesian statistics • Neural networks • Mathematical biology • Cognitive science

**Discussions:** History and philosophy of mathematics • Math education • "How to Give a Math Talk" • College, Graduate School and Beyond

**Problem Solving:** Proof techniques • Elementary and advanced methods • Contest problems of various levels of difficulty • Weekly "Math Relays" and team competitions

*"One cannot compare my ideas of what `I'm interested in math' meant before and after Mathcamp."*
– Asaf Reich (Vancouver, BC, Canada)

*"There was no pressure: the incentive to learn came from within."*
– Keigo Kawaji (Toronto, ON, Canada)

*"When I applied, I was really scared to go to a five week camp. What if I were bored or lonely? Having come to Mathcamp, I don't think it's possible to be bored here. I can't imagine a more inclusive, active, or interesting community. I wish it were more than five weeks!"*
– Jackie Bredenberg (Bloomfield Hills, MI, USA)

## The Freedom to Choose

Mathcamp does not have a set curriculum or a list of requirements. We encourage the faculty to teach what they are most passionate about, and we let the students choose what they are interested in learning. With the help of an academic advisor, you will design a program of study that reflects your own interests and goals. You can take any classes you want, and even the number of classes that you attend each day is up to you: you can use your time to review what you've learned, talk to one of your professors, work on problems, do a project, or just take a break. For many students, the freedom to take charge of their own education is one of the aspects of Mathcamp that they value most.

## Projects

Every student at Mathcamp has the opportunity to do a project, supervised by one of the mentors or faculty. Projects range in scope from creative applications of simple techniques to advanced problems connected to faculty research. Project topics in previous years have included:

- Periodicity of Fibonacci numbers mod $n$
- Information theory and psychology
- Knight tours on an $m$-by-$n$ chessboard
- Cellular automata
- Games on abelian groups
- Constructing the regular 17-gon
- Admissible covers of algebraic curves
- Setless sets in SET, in dimension 4 and beyond
- Computer-generated counterpoint
- The elasticity equation of string
- Digital signal processing
- Light paths in universes with alternate physics
- Playing 20 Questions with a Liar
- Dirichlet's Theorem on Arithmetic Progressions
- Teach your own one-hour Mathcamp class!
- Non-Orientable Knitting

## Spotlight on a Class

**Elections: Influence and Stability (2013)** • Does your vote matter? In a majority election, your vote can only change the outcome if all the other votes are split evenly. But what about other voting systems, like the electoral college? Which system gives each voter more influence over the outcome of the election? Is there any voting system that can do better than both electoral college and straight majority? In this class, we'll assign cold, hard scores to different voting systems (for elections with two candidates) – "influence" will be one kind of score, and another will be "noise stability" – and we'll see which voting systems do best. It turns out that there are simple, beautiful formulas for these quantities – in terms of Fourier coefficients! Voting systems will lead us to study Fourier analysis on the boolean cube (no calculus is required) and a proof of Arrow's Impossibility Theorem. Warning: Your faith in democracy may be restored after this class.

*"New understandings of number theory, topology, and real analysis were not the only things I took from Mathcamp. I learned how much I have left to learn, and thanks to my new friends and mentors, I couldn't be more excited about the world of math."*
– Rachel Hong (San Jose, CA, USA)

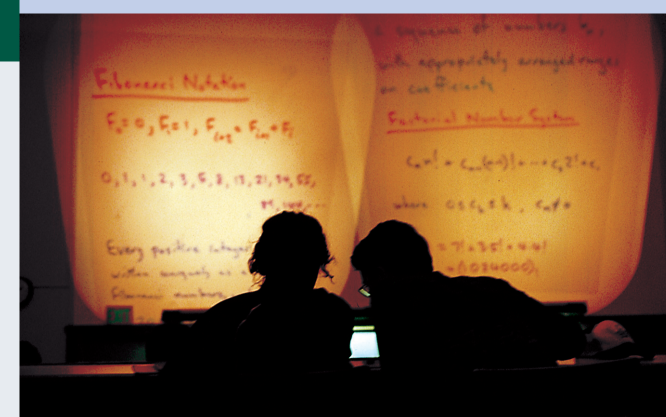
# Discover Mathcamp!

*"Out of nothing I have created a strange new universe."*
– Janos Bolyai, co-discoverer of hyperbolic geometry

## Mathcamp is a chance to...

- Live and breathe mathematics: fascinating, deep, difficult, fun, mysterious, abstract, interconnected (and sometimes useful).
- Gain mathematical knowledge, skills and confidence – whether for a possible career in math or science, for math competitions, or just for yourself.
- Set and pursue your own goals: choose your classes, do a project, learn what you want to learn.
- Study with mathematicians who are passionate about their subject, from internationally known researchers to graduate students at the start of their careers, all eager to share their knowledge and enthusiasm.
- Make friends with students from around the world and discover how much fun it is to be around people who think math is cool.

*"Mathcamp was the first place where I really understood the beauty and intricacies of abstract mathematics."*
– Paul Hlebowitsch (Iowa City, IA, USA)



*"Mathcamp was definitely the most fun I've ever had."*
– Avichal Garg (Cincinatti, OH, USA)

## Lewis & Clark College

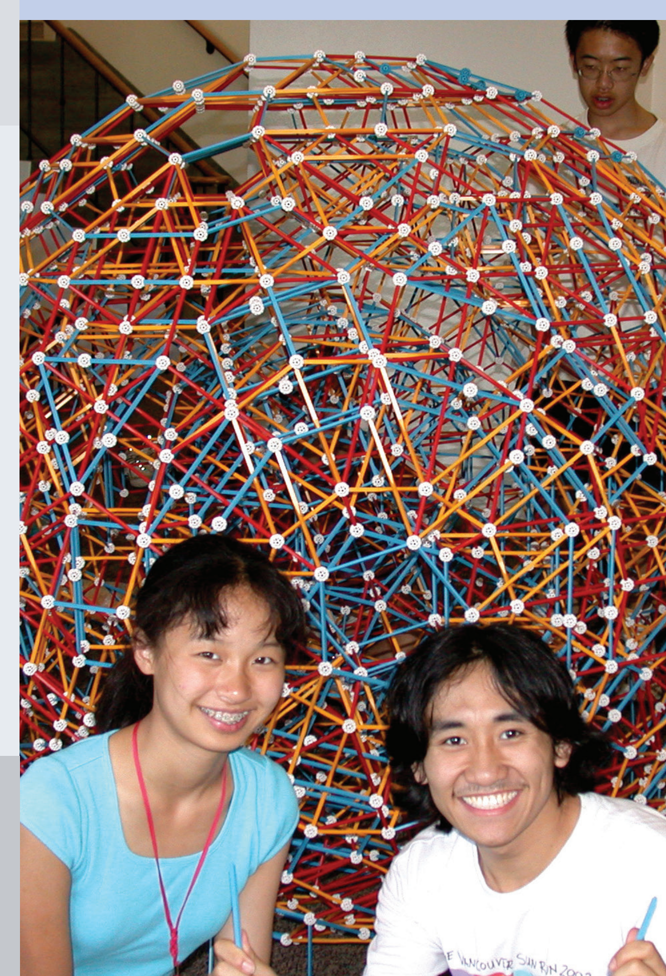
*Courtesey of Lewis & Clark.*

## Site of Mathcamp 2014

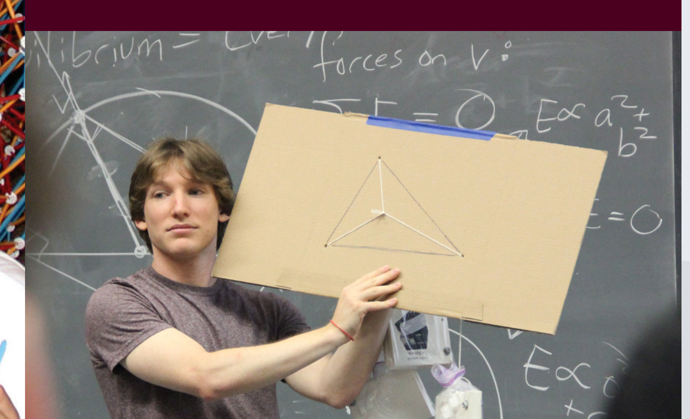
*"Mathcamp isn't really a camp. It's more of a five-week long festival – a congregation of people who celebrate math, enjoy math, learn math and essentially live math. Through it all I've discovered cool theorems that I wouldn't have understood before and cool people I didn't know existed. I've learnt that I actually know close to nothing about the weird and wonderful subject that is mathematics, and that I will probably pursue it for the rest of my life. Math on, Mathcamp!"*
– Yongquan Lu (Singapore)

*"It's not often that you find a place that is exciting to the mind and liberating to the spirit. Mathcamp is both."*
– Greg Burnham (Memphis, TN, USA)

*"I've changed so much in my two years here. I think about math in a new, deeper way. I approach problems differently. I've gained perseverence and learned to ask for help without shame and give it with joy."*
– Hallie Glickman-Hoch (Brooklyn, NY, USA)



*"Go, just go! Trust me!"*
– Jian Xu (Toronto, ON, Canada)



# People and More

## Our Staff

The staff includes Faculty (professors and professionals in math and related fields), Mentors (math graduate students), and Junior Counselors ("JCs" – undergraduate students, all of them camp alumni).

Courses at Mathcamp are taught by Faculty and Mentors; each instructor designs her or his own curriculum, picking the course topics freely from among their favorite kinds of math. In addition, we bring several visiting speakers each week who give guest lectures on math and its applications. Outside the classroom, JCs run the non-academic side of camp (from field trips to birthday cakes to frisbee games).

The staff live in the dorms and socialize informally with the students, sharing hiking trips and Scrabble games. Like campers, the staff often return year after year to Mathcamp.

## Faculty

**Mira Bernstein (MC 1997 – present)** is the Executive Director of Mathcamp. Her recent Mathcamp courses have included Information Theory and Machine Learning.

**Mark Krusemeyer (MC 1996 – present)** is a Professor at Carleton College. His recent Mathcamp courses have included Representation Theory and Complex Analysis.

**Susan Durst (MC 2008 – present)** is a Postdoc at the University of Arizona. Her recent Mathcamp courses have included Knot Theory and The Continuum Hypothesis.

**Alfonso Gracia-Saz (MC 2004 – present)** is a Lecturer at the University of Toronto. His recent Mathcamp courses have included Combinatorial Games and Metric Spaces.

## Visiting Speakers

**John H. Conway (Princeton)** • One of the most creative thinkers of our time, John Conway has made groundbreaking contributions to such diverse fields as knot theory, geometry of high dimensions, group theory, transfinite arithmetic, and the theory of mathematical games. Outside of the mathematical community, he is perhaps best known as the inventor of the "Game of Life."

**Po-Shen Loh (Carnegie Mellon)** • Po-Shen Loh is the national lead coach of the USA International Math Olympiad team. He studies questions that lie at the intersection of two branches of mathematics: combinatorics (the study of of discrete systems) and probability theory.

**Josh Tenenbaum (MIT)** • Josh Tenenbaum is a professor of Cognitive Science and a member of the MIT Computer Science and Artificial Intelligence Lab. In his research, he builds mathematical models of human and machine learning, reasoning, and perception. He thinks humans are a lot smarter than most psychologists give them credit for.

*"Mathcamp took every limitation I thought I had—social, academic, and personal—and shattered it."*
– Andrew Kim (Dover, MA, USA)

## Our Students

We never cease to be amazed at the variety of talents and passions our students bring to the program! While everyone at camp shares a love of mathematics, their other interests run the gamut. Each year's camp is a collection of 120 students who are musicians and writers, jugglers and dancers, athletes and actors, artists, board game players, hikers, programmers, students of science and philosophy – all sharing their interests and experiences with each other.

Most of the students at camp come from North America, but many come from overseas. Students have come to camp from Bulgaria, Egypt, India, Japan, Lithuania, Luxembourg, Macedonia, Paraguay, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Korea, Spain, Switzerland, Tanzania, Thailand, Turkey, and many other places around the globe.

It is a testament to our students' maturity and independence that they can be serious about doing math while still finding so many different ways to have fun. Many camp activities are organized entirely by campers, and students routinely cite each others' company as one of the best aspects of camp.

## Beyond Math

Mathematical activities are scheduled for five days a week; whatever math happens on the other two days is purely informal. The weekend is reserved for relaxation and the incredible number of activities that quickly fill the schedule. All of these activities are optional, and students can choose simply to spend time with friends or curl up with a book.

Field trips in the past have included hiking, sea kayaking, whitewater rafting, amusement parks, and museums. Lots of activities happen on campus, too: there are rehearsals for the choir and the contemporary a cappella group, salsa dancing workshops, improv, and bread baking (and subsequent eating). There is an annual team "puzzle hunt" competition, a talent show, and ice cream made with liquid nitrogen. Campers also organize many events themselves—from sports and music to chess and bridge tournaments—and each year, a group of students creates the camp yearbook.

## A Note To Parents

***Student safety and enjoyment are Mathcamp's first priorities.*** Students will be housed in secure campus dormitories, with male and female students in designated sections of the same building. Each student is assigned a Mentor or JC as their residential advisor; RAs live on the same hall as their advisees and look out for them on a day-to-day basis. In case of a medical problem, we have a camp nurse at camp or on call, and the hospital is minutes away. Students will have access to college athletic facilities and computers. Every effort will be made to enable students who so desire to attend weekly religious services of their faith. Mathcamp is committed to an atmosphere of mutual tolerance, responsibility, and respect, and we are proud of our past record of creating such an atmosphere.

*"Coming to Mathcamp has given me a community with which to interact, not just five weeks a year, but all year round."*
– Eric Wofsey (St Louis, MO, USA)