CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2015

Contents

9:10 Classes

Mathcamp Crash Course. (, Alfonso, Tuesday–Saturday)
This course covers fundamental mathematical concepts and tools that all other Mathcamp courses assume you already know: basic logic, basic set theory, notation, some proof techniques, how to define and write carefully and rigorously, and a few other tidbits. If you are new to advanced mathematics or just want to make sure that you have a firm foundation for the rest of your Mathcamp courses, then this course is *highly* recommended.

Here are some problems to test your knowledge:

(1) Negate the following sentence without using any negative words ("no", "not", etc.): *"If a book in my library has a page with fewer than 30 words, then every word on that page starts with a vowel."*
(2) Given two sets of real numbers A and B, we say that A *dominates* B when for every $a \in A$ there exists $b \in B$ such that $a < b$. Find two, disjoint, non-empty sets A and B such that A dominates B and B dominates A.
(3) Prove that there are infinitely many prime numbers.
(4) Let $f : A \to B$ and $g : B \to C$ be maps of sets. Prove that if $g \circ f$ is injective then f is injective. (This may be obvious, but do you know how to write down the proof concisely and rigorously?)
(5) Define rigorously what it means for a function to be increasing.
(6) Prove that addition modulo 2013 is well-defined.
(7) What is wrong with the following argument (aside from the fact that the claim is false)?

   Claim: On a certain island, there are $n \geq 2$ cities, some of which are connected by roads. If each city is connected by a road to at least one other city, then you can travel from any city to any other city along the roads.

   Proof: We proceed by induction on n. The claim is clearly true for $n = 1$. Now suppose the claim is true for an island with $n = k$ cities. To prove that it's also true for $n = k + 1$, we add another city to this island. This new city is connected by a road to at least one of the old cities, from which you can get to any other old city by the inductive hypothesis. Thus you can travel from the new city to any other city, as well as between any two of the old cities. This proves that the claim holds for $n = k + 1$, so by induction it holds for all n. QED.
(8) Explain what it means to say that the real numbers are uncountable. Then prove it.

If you are not 100% comfortable with most of these questions, then you can probably benefit from this crash course. If you found this list of questions intimidating, then you should *definitely* take this class. It will make the rest of your Mathcamp experience much more enjoyable and productive. And the class itself will be fun too!

Homework: Required

Prerequisites: None.

Linear Algebra. (🌶🌶, Mark, Tuesday–Saturday)
You may have heard that linear algebra involves computations with matrices and vectors, and there is some truth to that — but it sounds much less interesting than the subject really is; what's exciting about linear algebra is not those computations themselves, but

(1) the conceptual ideas behind them, which are elegant and which crop up throughout mathematics, and
(2) the many applications inside and outside mathematics.

In this class we'll deal with questions such as:

- How can you talk about geometric concepts (such as lengths and angles) if you're not in the plane or 3-space, but in higher dimensions?
- What does "dimension" even mean, and if you're inside a space, how can you tell what its dimension is?
- What does rotating a vector, say around the origin, have in common with taking the derivative of a function?
- If after a sunny day the next day has an 80% probability of being sunny and a 20% probability of being rainy, while after a rainy day the next day has a 60% probability of being sunny and a 40% probability of being rainy, and if today is sunny, how can you quickly find the probability that it will be sunny exactly one year from now?

Do come join me in exploring this material. Despite having taught at Mathcamp for many years, I've never taught this class here, so I'm especially excited to find out just how far we can reasonably get in a week!

Homework: Recommended

Prerequisites: None beyond the Mathcamp crash course (even though the blurb refers to taking a derivative, you'll get by if you have no idea what that is)

Related to (but not required for): A (re)Introduction to Polynomials (W1); Multivariable Calculus (W1); Ring Theory (W1); Induced Matchings from Szemeredi's Regularity Lemma (W3); Laurent Phenomenon (W4); Category Theory in Sets (W4)

Required for: Time-Frequency Analysis (W1); Markov Chains to Support your Probabilistic Exploits (W2); Intro Knot Theory (W2); Reflection Groups (W2); The Banach–Tarski Paradox (W2); Lie Algebras (W3); Braid Group (W3); Classifying Spaces (W3); Error-Correcting Codes (W3); Representation Theory (W3–4); Fundamental Theorem of Calculus in Dimension n (W3); Homotopy Theory (W4); Many Facets of Optimization (W4); Advanced Linear Algebra (W4)

Problem Solving: Inequalities. (🌶🌶🌶, Pesto, Tuesday–Saturday)
High-school olympiads usually try to choose problems relying on as little prior knowledge as possible. In inequalities problems, they usually fail completely; training is necessary to solve most and sufficient to solve many of them. We'll go over the common olympiad-style inequalities, and solve problems like the following:

(1) Prove that if a, b, and c are positive and $ab+bc+cd+da=1$, then $\frac{a^3}{b+c+d}+\frac{b^3}{a+c+d}+\frac{c^3}{a+b+d}+\frac{d^3}{a+b+c}\geq\frac{1}{3}$.
(2) [USAMO 2004] Prove that if a, b, and c are positive, then $(a^5-a^2+3)(b^5-b^2+3)(c^5-c^2+3)\geq(a+b+c)^3$

This is a problem-solving class: I'll present a few techniques, but most of the time will be spent having you present solutions to olympiad-style problems you'll've solved as homework the previous day.

Homework: Required

Prerequisites: None.

Metric Spaces. (, Steve, Tuesday–Saturday)
A *metric space* is just a set X of "points" together with a *distance function*, d, which behaves the way distance should: the distance between two points is zero iff they are actually the same point; the distance between x and y is the distance between y and x; and it is never more efficient to go from x to y to z, than to just go from x to z (this is the *triangle inequality*: $d(x,z) \leq d(x,y) + d(y,z)$). For example:

- The set $C_0[0,1]$ of continuous functions from $[0,1]$ to $[0,1]$ forms a metric space, with the distance function $d(f,g) = \max\{|f(x) - g(x)| : x \in [0,1]\}$.
- Another metric we can put on $C_0[0,1]$: $d(f,g) = \int_0^1 |f(x) - g(x)| dx$. There's a bunch more ...
- Given a combinatorial graph G — that is, a set of vertices V together with a set of edges E between them — as long as G is connected, we get a metric on V given by the "shortest path" function.
- Words can be turned into a metric space, where the distance between two words is the number of typos you would need to make to transform one into another.

And so on.

Because they are so general, metric spaces appear everywhere throughout mathematics. In this class, we will begin by studying the general properties metric spaces may have — including compactness, completeness, and connectedness — and how these properties are used. We will then turn to an example of a truly strange metric space: the metric space of compact metric spaces! Time permitting, we will consider another truly bizarre metric space — a line which cannot be cut into two smaller lines ...

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Point-set Topology (W1); Chromatic Numbers (W2); Shortest Distance (W4)

From Geometric Optics to High Speed Underwater Photography. (, *Allan Adams*, Tuesday–Wednesday)
Lecture: Why does squinting help you see better? Why does Architeuthis dux have an eye the size of a dinner plate? (And no, squid can't squint, which is an interesting hint.) Why don't most cell phone cameras bother with an adjustable focus? Geometric optics answers all these questions and explains almost everything there is to say about almost everything that can see. In case you think that's too strong a statement, one famous mathematician used geometric optics to construct a proof of the existence of god! His proof proved flawed, but along the way he developed the principle of least action, so I'll call that a victory.
Lab: Armed with this knowledge — plus an underwater high-speed videocamera from my lab at MIT — we'll run some experiments to answer more questions, like what happens when you pop a balloon underwater? (Hint: It's awesome.)

Homework: Recommended

Prerequisites: Euclidean geometry, trig helpful but not strictly necessary, calculus helpful but by no means necessary.

Related to (but not required for): Special Relativity (W1)

Differentiation under the Integral Sign. (, Kevin, Thursday–Saturday)
Richard Feynman, the eccentric and entertaining physicist, attributes his "great reputation for doing integrals" primarily to one trick – differentiation under the integral sign. In this class, we'll introduce and play with this technique, along with some other useful bits and pieces like half-angle and hyperbolic substitutions.

Homework: Optional

Prerequisites: Single-variable calculus

Related to (but not required for): Summing Series (W2); Lebesgue Measure (W3); Fundamental Theorem of Calculus in Dimension n (W3)

10:10 Classes

Statistical Modelling. (🌶🌶, Sam, Tuesday–Saturday)
Note: this is a course where you'll get to do actual data analysis; during the course you'll need to get your hands dirty working with real data!

We'll start by introducing a framework for statistical modelling, and we'll illustrate that framework with one of the most powerful tools in the statistician's toolkit: the linear model. We'll cover fundamental statistical principles, including: fitting, analyzing, and interpreting linear models, hypothesis testing, and extensions of the linear model. By the end of the week, my hope is that you'll be comfortable with these statistical methods from a mathematical and intuitive perspective, and that you'll also be comfortable with your own ability to analyze data you care about.

Homework: Required

Prerequisites: You should feel good about basic matrix/vector notation and operations (inversion and transposition) and a few ideas from probability (if you're OK with the sentences "X and Y are independent" and "X is normally distributed with mean zero and variance σ^2," you should be set. If you aren't quite sure you have the prereqs, but you're not too put off by those sentences, talk to me and we should be able to work something out!).

Related to (but not required for): Markov Chains to Support your Probabilistic Exploits (W2); Network and Combinatorial Optimization (W3)

Measure and Martin's Axiom. (🌶🌶🌶🌶, Susan, Tuesday–Saturday)
If we want to develop a notion of "length" on the real line, there are some properties we know it ought to satisfy. For instance, the length of the interval (a, b) ought to be $b - a$. And the length of a single-point set ought to be zero.

In this class, we will be interested in the behavior of the sets with size zero. Any countable union of measure-zero sets has measure zero. However, we can take the union of continuum-many measure-zero sets and obtain a set that does not have measure zero. (For example, the union of all single-point sets $\{x\}$ with $0 \leq x \leq 1$ is the unit interval, which has measure 1.)

So what happens if we take the union of an uncountable—but not continuum-sized—collection of measure-zero sets? This question turns out to be independent from standard ZFC set theory. We'll explore one possible answer that arises in a universe where we've added an extra-set-theoretic axiom called Martin's Axiom to the mix.

This class should be fun for campers who want to mess around with really big numbers ($\aleph_1$) and really small numbers ($\epsilon > 0$) at the same time.

Homework: Recommended

Prerequisites: understanding of basic facts about cardinality

Related to (but not required for): The Banach–Tarski Paradox (W2); Lebesgue Measure (W3); Ultrafilters (W4)

Multivariable Calculus. (🌶🌶🌶, Mark, Tuesday–Saturday)
In real life, interesting quantities usually depend on several variables (such as the coordinates of a point, the time, the temperature, the number of campers in the room, the real and imaginary parts of

a complex number, ...). Because of this, "ordinary" (single-variable) calculus often isn't enough to solve practical problems. In this class, we'll quickly go through the basics of calculus for functions of several variables. As time permits, we'll look at some cool applications, such as: If you're in the desert and you want to cool off as quickly as possible, how do you decide what direction to go in? What is the total area under a bell curve? What force fields are consistent with conservation of energy?
Homework: Recommended
Prerequisites: Single-variable calculus (differentiation and integration)
Related to (but not required for): Linear Algebra (W1)
Required for: Functions of a Complex Variable (W2–3); Fundamental Theorem of Calculus in Dimension n (W3); Shortest Distance (W4)

A (re)Introduction to Polynomials. (🌶🌶, *Adam Marcus*, Tuesday–Saturday)
This course will be a (re)introduction to polynomials. Why a course on boring polynomials, you ask? Because polynomials appear in a lot of places. Places like convex optimization, algebraic geometry, complex analysis, combinatorics, as well as a whole theory built around, well, just polynomials. This makes them *very* versatile creatures, and as a result, some really famous unsolved problems have been solved (or re-solved in an absurdly easy way) recently using them. Problems involving things like distinct distances in the plane (geometry/combinatorics), matrix paving (functional analysis), matchings (graph theory), permutations (combinatorics), just to name a few.

OK, so maybe not *so* boring. But they require very little background and can be extremely powerful. And a lot of it is pretty new (like developed in the last 10 years) with some of the most powerful stuff being *extremely* new (like developed in the last 2 years). That makes for lots of open questions that would make interesting research projects (if, you know, you were looking for such things).
Homework: Optional
Prerequisites: These will be useful, but not necessary: derivatives, generating functions, induction, basic linear algebra, complex numbers
Related to (but not required for): Linear Algebra (W1); Abel's Theorem (W1–5); Ring Theory (W1); Induced Matchings from Szemeredi's Regularity Lemma (W3)

Introduction to Groups. (🌶🌶🌶, Mira, Tuesday–Saturday)
In how many ways can you paint the vertices of a cube with one of three colors? (We consider two colorings the same if they can be obtained from each other by rotating the cube – so the answer is not just 2^8.) How can you analyze the symmetries of geometric figures, or the workings of a Rubik's cube? How do physicists predict the existence of certain elementary particles before setting up expensive experiments to test those predictions? Why can't fifth-degree polynomial equations, like $x^5+3x+17=0$, be solved using anything like the quadratic formula, although fourth-degree equations can?

The answers to all these questions depend on group theory (although most of them are beyond the scope of this class). Knowledge of some group theory is at least helpful, and often crucial, for many other parts of mathematics. This course will cover the basics of group theory through the First Isomorphism Theorem, and also hopefully have time for some fun applications and examples.

(If you can state and prove the First Isomorphism Theorem in group theory, then you do not need to take this course.)
Homework: Required
Prerequisites: None.
Related to (but not required for): Classifying Spaces (W3); Hyperbolic Geometry (W4)
Required for: Abel's Theorem (W1–5); Fundamental Group (W2); Galois Cohomology (W2); Reflection Groups (W2); The Banach–Tarski Paradox (W2); Braid Group (W3); Representation Theory (W3–4); Homotopy Theory (W4); Tiling Problems (W4); Galois Theory (W4)

11:10 Classes

Algorithms. (🌶🌶, *Michelle Bodnar*, Tuesday–Saturday)
What is the fastest possible algorithm to sort a collection of objects? I have a list of things to do, each with a deadline, and each with a penalty for not finishing. What's the best order in which to carry out these tasks? An employment agency sends you a list of candidates to interview for a job, but interviewing and hiring temporary employees is costly. What strategy can you use to minimize these costs? What strategy can a malicious agency use to maximize them? In this class, you'll learn some of the main ideas underlying algorithms that solve these problems and many more. In addition, we'll talk about how to prove correctness and optimality of algorithms, and what we can prove about an algorithm that doesn't always give the right answer.

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Algorithms in Number Theory (W1); Introduction to Complexity (W2); Automated Proofs in Geometry (W3)

Abel's Theorem (Week 1 of 5). (🌶🌶🌶, Alfonso + Mira + Julian, Tuesday–Saturday)
How do you solve a polynomial equation of degree n?

- The linear ($n = 1$) case is easy: the solution to $ax + c = 0$ is just $x = \frac{-c}{a}$.
- The quadratic formula ($n = 2$) was known to the ancient Babylonians.
- Gerolamo Cardano discovered the cubic formula ($n = 3$) in the 1540's.
- Very soon afterwards, Cardano's student Ludovico Ferrari figured out the quartic formula ($n = 4$).

Then everyone started looking for the quintic formula ($n = 5$). They looked and looked, for almost three centuries — until in 1824, the great Norwegian mathematician Niels Abel proved that there is *no general formula* for solving a polynomial equation of degree 5. This is the theorem for which our class is named and which you will prove over the course of the summer (although not by the same method that Abel used).

Notice how we said "*you* will prove"? That's because the course will be taught by the Moore Method. This means that we will not lecture and you are not allowed to use books, the internet or any other resources (though you are encouraged to collaborate with each other). We will be giving you definitions and asking many questions. You will be doing all the work to answer them in the form of daily homework. Class time will be spent presenting and discussing your solutions. So you will be the ones proving all the theorems!

This is a four-week (maybe even a five-week) course, which, by Mathcamp standards, is a big commitment. You also have to be prepared to commit significant time daily to working on homework. In return, by the end of the course, you will have gained an in-depth understanding of at least two important fields of mathematics (group theory and Riemann Surfaces); a lot of experience writing and presenting proofs; and the deep satisfaction of discovering new mathematics for yourself.

Because of the nature of this class, there is homework due on the first day of classes (Tuesday). Read through and prepare your answers to the first few questions. You can download the notes from `http://www.math.toronto.edu/~alfonso/MC/abel.html` and we will also have physical copies with us during opening assembly.

Homework: Required

Prerequisites: Group theory up through the First Isomorphism Theorem. The Week 1 Group Theory course is sufficient and it is okay to take it at the same time as this course; we'll need more group theory, but we'll develop it as part of the class.

Related to (but not required for): A (re)Introduction to Polynomials (W1); Galois Theory (W4). Most people associate the proof of the unsolvability of the quintic with Galois, not Abel, because Galois proved a stronger result and developed a whole new general approach to these kinds of probems. Our approach in this course will be related to Galois', and yet different: while he worked with algebraic extensions of $\mathbb{Q}$, we will be working with geometric extensions of the complex plane. So if you've seen Galois Theory before, you'll get a new perspective on it in this course. And if you decide to take Nancy's Galois Theory class in Week 3, then this course will give you an interesting perspective on what she does there. The approaches are similar in many ways, but there should be synergy rather than redundancy.

Classifying Symmetry. (🌶🌶, *Frank Farris*, Tuesday–Thursday)
This course uses mathematical art produced by Frank Farris to study the concept of symmetry, with an emphasis on classifying patterns. The main mathematical vocabulary is the concept of a group. If this is your first time studying groups, the patterns in this class provide the best first examples; if you are familiar with the concept, you'll still learn new things about it. Each day will include a presentation about a new topic and then a workshop session where you put the concepts into action by classifying patterns. You should emerge from this course seeing the world differently. (Personally, I find myself pausing movies to classify the wallpaper patterns; very distracting.)

On day one, we establish basic vocabulary about symmetry and explain why the symmetries of a pattern necessarily form a group. The pattern types for the first day are rosette and frieze patterns. On the second day, we study wallpaper patterns. Although we will not prove the wonderful result that there are exactly 17 (isomorphism classes of) wallpaper groups, the result will be highlighted by vivid examples. On the last day, we study color-reversing symmetries, which naturally introduce the concept of normal subgroups. Time permitting, we will also glimpse polyhedral and non-Euclidean patterns.

The homework will involve classification of patterns; for those who wish, there will be several puzzles each day in the area of transformation geometry.

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Aperiodic Tiling (W2)

Fun with Compactness in Logic. (🌶🌶, *Matt Wright*, Friday–Saturday)
Mathematical proofs are finite—and that's a good thing, because if they weren't, we wouldn't be able to write them down or present them in (finitely long) Mathcamp classes! But that simple observation leads to a surprising theorem of logic called the compactness theorem, which lets us take finite structures and in some cases use them to say things about infinite ones. This has some strange consequences! We'll use the compactness theorem to show that there are structures that satisfy all of the axioms of the natural numbers and yet look pretty wildly different, and we'll look at some simple things we can't express in logic.

Homework: Recommended

Prerequisites: None.

Point-set Topology. (🌶🌶🌶, Nancy, Tuesday–Saturday)
Topology lurks behind the scenes in many subjects of mathematics including analysis, geometry and algebra. While these subjects can be learned on their own, having a knowledge of topology gives a deeper understanding of the underlying concepts and themes.

In a sense, a topology is a mathematical generalization of "closeness". For example, a space with a notion of distance (aka metric) is an example of a topological space. But we use the word "close" to mean all sorts of things. For example, my mother and I are very close, but we live hundreds of miles apart. To describe this type of closeness with a topology, we would say that my mom and I are elements of an open set.

Why do we care about "closeness" anyway? Believe it or not, we can use information about closeness of points to determine the shape of the space. Is my space connected (in one piece)? Is my space compact (will it fit in your suitcase)?

Maybe you thought you knew what the world around you looks like, but by the end of this class you will finally be able to understand why a donut and a coffee mug really are the same!

Homework: Required

Prerequisites: None.

Related to (but not required for): Metric Spaces (W1)

Required for: Fundamental Group (W2); Classifying Spaces (W3); Homotopy Theory (W4)

Special Relativity. (🌶🌶, *Nic Ford*, Tuesday–Saturday)
Around the beginning of the twentieth century, physics was undergoing some drastic changes. The brand-new theory of electromagnetism made very accurate predictions, but it forced physicists to come to grips with a strange new truth: there is no such thing as absolute space, and there is no such thing as absolute time. Depending on their relative velocities, different observers can disagree about the length of a meterstick, or how long it takes for a clock to tick off one second.

In this class, we'll talk about the observations that forced physicists to change their ideas about space and time, and how the groundwork of physics has to be rebuilt to accommodate these observations. We will see how, as Minkowski said, "space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind union of the two will preserve an independent reality." Along the way, we will also have to revise the classical notions of momentum and energy, allowing us to derive the famous relation $E = mc^2$. If there's time at the end, we might discuss some other topics in modern physics.

Homework: Recommended

Prerequisites: high school physics

Related to (but not required for): From Geometric Optics to High Speed Underwater Photography (W1)

1:10 Classes

Non-classical Constructions. (🌶🌶, Chris, Tuesday–Saturday)
Compasses are for the weak! I never liked using them. The ancient Greeks loved doing geometric constructions with straight-edge and compass. I, on the other hand, can get by pretty well with just a straight-edge and so can you!

Straight-edge and compass constructions were first studied around 600 BC. Astonishingly, we can construct many things with those tools, such as 36° degree angles, regular 17-gons, the golden ratio ϕ and much more. It is equally surprising that there are things that cannot be constructed by straight-edge and compass, such as the regular 7-gon or the trisection of an angle. It took over 2000 years, but by the early 19th century all this was well studied and understood.

Now I ask the question: But what if I don't have a compass? We will explore these non-classical constructions using only a straight-edge and there is still a surprising amount of things that can be constructed. Indeed, if we give ourselves just one circle in the plane, we can do everything only using a straight-edge that the ancient Greeks still needed a compass for. In this class we will not only give a proof for this fact, but we will also understand algebraically why just a straight-edge alone is not enough. In the homework you will discover many non-classical constructions (such as finding parallels, perpendiculars, . . .) yourself, some of which are vital to the proofs done in class.

Homework: Required

Prerequisites: Basic (high school) geometry

Related to (but not required for): Automated Proofs in Geometry (W3); Apollonian Circle Packings (W4); Galois Theory (W4)

Infinitesimals. (🌶🌶🌶, Don, Tuesday–Saturday)
The early days of calculus made liberal use of a controversial tool: infinitesimal values. There's no need to define limits when you can work directly with a notion of "incredibly close." However, leading minds and religious authorities alike put forth bans on the use of infinitesimals, and ultimately, the debate over infinitesimals was won by these opponents. The debate lay dormant for centuries, until a few mathematicians realized that, even though $\mathbb{R}$ contains no infinitesimals, there are reasonable systems of mathematics that do.

In this class, we'll be looking at a couple of ways to do math in which infinitesimals create no contradictions. In one, as long as we have powerful enough axioms, we can just add infinitesimals to $\mathbb{R}$. In another, by weakening logic itself, we'll get amazing results, like every function being smooth — but that is quite a high price to pay.

Homework: Recommended

Prerequisites: None (Calculus suggested)

Related to (but not required for): Ordinal Arithmetic (W3); Ultrafilters (W4)

Time-Frequency Analysis. (🌶🌶🌶🌶, Jeff, Tuesday–Saturday)
When you put your ear up to a shell, you hear the tumbling of the ocean around you. The sound that you hear is not caused by a miniature ocean confined to a shell, rather, the shell's shape and composition distort the white noise into the sound of churning water.
One of the goals of signals analysis is to understand how systems like the shell distort input signals. For instance, if we know how the shell sounds when we strike it gently with a hammer, could we figure out what an actual ocean would sound like if it was in the shell? How would a sine wave sound like if it were playing in the shell? What about a one thousandth scale version of the Beatles singing *Love me Do?*
In this class, we will draw from our human and physical intuition to develop mathematical tools including Fourier transforms, distributions, and time-frequency representations. Th–4is will allow us to take derivatives of things like $y = |x|$ at $x = 0$, and solve nonhomogeneous ordinary differential equations. We will then turn this machinery the other way around to study how we perceive reality by building audio filters, exploring the Uncertainty Principle, and studying lossy AV compression.

Homework: Recommended

Prerequisites: Calculus, Linear algebra.

Ring Theory. (🌶🌶, Sachi, Tuesday–Saturday)
When pressed for a one sentence definition, many mathematicians will describe a ring as "a set of things that you can add and multiply together". However, this hardly scratches the surface of what rings do and why they're everywhere. In this class, you'll learn not just how rings are places where you can add and multiply, but how we use rings to describe analogues of familiar objects like the integers, functions, matrices, and even geometric shapes like the parabola and hyperbola. We will

study prime numbers and their cousins, understand how to build square roots and imaginary numbers from polynomials, and see why points on a line have an alter-ego as maximal ideals of a ring. Come learn why rings are such a fundamental part of modern mathematics!

Homework: Required

Prerequisites: None.

Related to (but not required for): Linear Algebra (W1); A (re)Introduction to Polynomials (W1); Representation Theory (W3–4); Tropical Curves (W3); Number Theory Polynomials (W4); Galois Theory (W4)

Required for: Advanced Linear Algebra (W4)

Algorithms in Number Theory. (🌶🌶, Misha, Tuesday–Saturday)
Modular arithmetic is a key tool in number theory that can be used to find answers to all sorts of problems. Are there integer solutions to $13x + 17y = 1$? Is $23^{43} + 43^{23}$ divisible by 66? Is the number 47 147 113 459 prime? We will learn how to answer all of these questions, and more.

In this class, we will learn about all those things from an algorithmic point of view. Unsatisfied with merely knowing that solutions exist, we will find recipes for finding these solutions.

With the aid of computers, we will go even further. You probably don't want to check if a 100-digit number is prime by hand, but you will learn to teach your computer to do it. (No previous programming knowledge is required.)

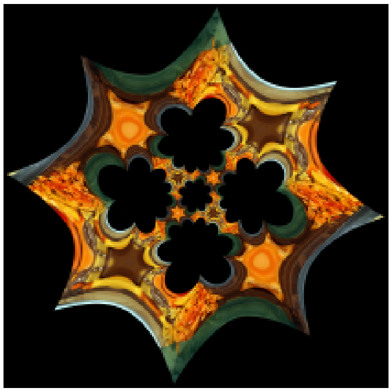
Homework: Recommended

Prerequisites: None.

Related to (but not required for): Algorithms (W1); Galois Cohomology (W2); Absolute Values (W3); Number Theory Polynomials (W4)

Colloquia

Polyhedral Symmetry in the Plane? (*Frank Farris*, Tuesday)
Can you see the tetrahedron in the image provided?



To do so, you have to break some conventions about symmetry. Under traditional definitions, the only thing you can do to that image to leave it completely unchanged is turn it upside downa 180 degree rotation. In this talk, we expand the definition of symmetry to go beyond the usual rosettes, friezes, and wallpaper patterns to find polyhedral symmetry in the plane. We combine a little group theory, a little complex analysis, and several other mathematical ingredients in the service of mathematics and art.

Pad Thai with Electrons. (*Allan Adams*, Wednesday)
The world is so deeply weird, so staggeringly strange, that people studying it one hundred years ago invented a whole new language to describe its lemony ways: Quantum Mechanics. In this colloquium, we'll introduce a couple of the most counterintuitive aspects of reality.

An Intro to "Data Science". (*Adam Marcus*, Thursday)
Data science is all over the media these days. But what is it? And what do "data scientists" do? I will give an introduction to data science (a.k.a. machine learning) and discuss some of the more interesting mathematical problems. Topics will include what a career in data science might entail and how to prepare for such a career. No background knowledge will be required, and I hope to leave plenty of time for questions.

Making Cantor Super Proud. (Steve, Friday)
How big is infinity? In 1874, Georg Cantor discovered something awesome: some infinities are bigger than others! In particular, there are more real numbers than natural numbers - there is no injection from $\mathbb{N}$ to $\mathbb{R}$.

New question: how bigger is infinity? We know $\mathbb{R}$ is bigger than $\mathbb{N}$, but how much bigger? Cantor thought that the answer was, "barely": he conjectured that there is no set of reals larger than $\mathbb{N}$ but smaller than $\mathbb{R}$. This became known as the "Continuum Hypothesis".

Cantor — and others — spent a long time trying to prove or disprove CH. SPOILER: that's impossible! In 1940, Kurt Goedel showed that we cannot *disprove* CH from the usual set-theoretic axioms; in 1963, Paul Cohen showed that we cannot *prove* CH from the usual set-theoretic axioms.

OOPS.

It turns out, though, that Cantor had the right idea: every "reasonably describable" set of real numbers is either countable, or has the same size as $\mathbb{R}$. In fact, "reasonably describable" sets of real numbers are well-behaved in lots of ways. This was the beginning of the field known as *descriptive set theory*, which seeks to answer the following questions:

- What properties do "reasonably describable" sets have?
- What does "reasonably describable" mean?
- I like games!
- Something something large cardinals?

We will look at different aspects of descriptive set theory and the continuum hypothesis; time permitting, we'll talk about some current directions in set theory.

Vistor Bios

Adam Marcus. Adam splits his time between being Chief Scientist at Crisply, a machine learning startup that he helped to cofound, and doing academic research. In the former, he mixes statistics, computer science, and optimization in attempt to build a "meta-learner" (something that can learn how to learn). In the latter, he likes looking for the right answers in the wrong places.

Frank Farris. Frank Farris considers himself to have reinvented wallpaper patterns. Instead of stamping them out from repeated copies of a motif, he uses functions called complex waveforms that have just the right symmetry. This is the subject of his new book from Princeton University Press, "Creating Symmetry, The Artful Mathematics of Wallpaper Patterns". The underlying mathematics combines Fourier analysis, group theory, and partial differential equations. There's also a liberal sprinkling of visual art, since the functions call up pixels from photographs in order to create the patterns. It's math, it's art, and it's fun.

Allan Adams. Allan Adams works on quantum versions of algebraic and differential geometry that play a fundamental role in string theory, and uses black holes in 5 spacetime dimensions to study high-temperature superconductors in the usual 4. He believes that everyone should understand quantum mechanics - which is as beautiful and strange as it is true - and looks forward to discussing it at Mathcamp.

Michelle Bodnar. Michelle Bodnar is a 3rd year graduate student at the University of California, San Diego studying combinatorics and algorithms. She enjoys drinking coffee, playing the flute, cooking, hiking, and reading math books not assigned for a class. This is Michelle's first time at Mathcamp!

Julian Gilbey. Julian Gilbey is developing teaching resources for 11th and 12th grade math(s) classes for the Cambridge Mathematics Education Project (that's Cambridge, UK, by the way!). He's a Mathcamp old-timer, having first been a mentor in 2000. His mathematical interests are varied, and include algebra, combinatorics, fractals, statistics, and anything else fun that happens to come his way. His other pastimes are somewhat eclectic - feel free to ask him about them!

Nic Ford. Nic is a former four-time Mathcamp mentor who now works for Jane Street Capital in New York City (where four former campers are now interns!). Before that, he got his PhD at Michigan, where he studied algebraic geometry. He is also really into biking, board games, rock climbing, and video game design.

Matt Wright. After finishing grad school (studying logic, with a focus on computability theory), Matt has moved on to being a software engineer at Dropbox. A mentor in 2013, a visitor last year, and a camper many, many summers ago, Matt is really excited to be visiting Mathcamp again!

David Roe. I've been coming to mathcamp since 1999, my first year as a student. I study number theory, representation theory and–4 p-adic computation (currently at UBC in Vancouver, soon at the University of Pittsburgh). I love hiking and board games and hope to enjoy these at camp!

Youlian Simidjiyski. Youlian is a former Mathcamper ('06, '07) and JC ('08, '09, '11) who now lives in Seattle and works as a software engineer for Microsoft. While he has grown to appreciate the real world more and more as of late, he still thinks Mathcamp is the happiest place on Earth and is thrilled that he gets to see you all this summer. If you're interested in talking about practical software engineering, philosophy of life, or if you just learned something incredible in class that you want to geek out over, find the bearded dude with the shoulder-length black hair* and we'll chat.

*Depending on who attends camp this summer, this may or may not be a unique identifier. So it goes.

Yvonne Lai. When not at camp, Yvonne is an assistant professor at the University of Nebraska-Lincoln. Mathematically, she was born a geometric group theorist/hyperbolic geometer. Since then her life has taken some turns, including a a second post-doctoral position to retrain in mathematics education research. Her current focus is in the area of "mathematical knowledge for teaching"; she investigates the knowledge of mathematics that is needed to teach well, particularly that which is different from that which students are meant to learn.