CLASS CLUSTERS FOR MATHCAMP 2016

0. About Clusters

The Mathcamp schedule provides a plethora of options, and we give you the freedom to choose among them as you will.[1] However, you may find the academics this summer more rewarding if you're able to look back in five weeks and reflect that you've learned *a lot* about something. To this end, we've constructed *clusters*: sets of classes with a common theme.

Most clusters do not have prerequisite dependencies within the cluster itself. However, some clusters have a common prerequisite: for example, each class under "Techniques in Graph Theory" assumes that you're familiar with graph theory.[2] We've included prerequisites in this document for your convenience; see also the prerequisite chart.

It is completely fine to only take part of a cluster. Note that fewer than half of the classes this summer are in a cluster; just because a class doesn't live in a cluster doesn't mean that it's not awesome!

1. The Shape of Things

As the old joke goes, a topologist is a mathematician who cannot tell a donut from a coffee cup. The classes in this cluster offer a gentle introduction to a more flexible way of thinking about the shape of curves, surfaces, and other geometrical objects.

Classes in this cluster:

- **Cutting Surfaces into Silly Straws.** ( , Assaf, Week 1)
- **Topology and Geometry of Surfaces.** ( , Jane, Week 3)
- **Knot Theory.** (, Jeff, Week 4)

2. Algebraic Topology

To gain a deeper understanding of which shapes can be transformed into each other, mathematicians have developed a powerful set of tools that associate algebraic structures to each shape. The classes in this cluster describe these connections and their applications.

Classes in this cluster:

- **Extending Inclusion/Exclusion.** ( , Jeff, Week 2)
  Prerequisites: Linear Algebra
- **A Tale of Combs and Hedgehogs.** ( , Alfonso + Chris, Week 3, Superclass)
  Prerequisites: Group Theory
- **The Fundamental Group.** ( , Jane, Week 4)
  Prerequisites: Group Theory

[1] Unfortunately, we don't have any time turners to hand out.

[2] Either by taking the introductory class in Week 1 or through the experience and wisdom gained from a life well-lived.

3. Maps, Graphs, Colors, Walks

Graph theory is the branch of math that ties together questions about coloring maps, drawing shapes without lifting your pencil, playing "Six Degrees of Wikipedia", analyzing the structure of social networks, and others.

Whenever you have a set of objects with connections between them, whether it's a molecule or Facebook, you have a graph. This cluster will start from scratch and teach you the cool things you can do with graphs.

Classes in this cluster:

- **Introduction to Graph Theory.** (🏃, Marisa, Week 1)
- **Almost Planar.** (🏃🏃, Marisa, Week 2)
  Prerequisites: Graph Theory
- **Graph colorings.** (🏃🏃, Mia, Week 3)
  Prerequisites: Graph Theory
- **The Hadwiger-Nelson Problem.** (🏃, Riley, Week 3)
  Prerequisites: Graph Theory
- **Harmonic Functions on Graphs.** (🏃🏃, Yuval, Week 4)
  Prerequisites: Graph Theory

4. Techniques in Graph Theory

Graph theory does not live in a vacuum, and in the past century, mathematicians have used tools from topology, probability, and linear algebra to push graph theory even further. Classes in this cluster will introduce you to some of these tools, and show you how they can be used to solve problems in graph theory.

Classes in this cluster:

- **Graph Minors.** (🏃🏃🏃, Pesto, Week 2)
  Prerequisites: Graph Theory
- **Random Graphs.** (🏃🏃🏃, Misha, Week 3, Superclass)
  Prerequisites: Graph Theory
- **Spectral Graph Theory.** (🏃🏃🏃, Sachi, Week 4, Superclass)
  Prerequisites: Graph Theory, Linear Algebra

5. Algebraic Novelties

Come explore algebra: construct rings with bizarre properties, find groups where every element can be written in terms of the generators but it's impossible to figure out how, and learn how to count how many truly distinct objects there are in a symmetric situation. If you're meeting groups for the first time in week 1, these classes are great followups!

Classes in this cluster:

- **The Word Problem for Groups.** (🏃🏃, Assaf, Week 2)
- **Finitely-Generated Algebras.** (🏃🏃, Susan, Week 3)

- **Burnside's Lemma.** (♪♪, *Alfonso Gracia-Saz*, Week 4)
  Prerequisites: Group Theory

6. Groups

Dive into the study of groups. In this cluster you'll learn about groups in many guises, from matrix groups to groups formed randomly from generators and relations. You'll discover new techniques for understanding the structure of a group, think about what an average group looks like and zoom out until discrete groups look continuous. If you can't get enough of groups, this cluster is for you!

Classes in this cluster:

- **Geometric Group Theory.** (♪♪♪♪, Susan, Week 2)
  Prerequisites: Group Theory.
- **Algebraic Groups.** (♪♪♪♪, Don, Week 3, Marathon)
  Prerequisites: Group Theory, Ring Theory, Linear Algebra
- **Representation Theory of Finite Groups.** (♪♪♪♪, Mark, Weeks 3–4)
  Prerequisites: Group Theory, Linear Algebra
- **The Word Problem for Hyperbolic Groups.** (♪♪♪♪, Assaf, Week 4)
  Prerequisites: Group Theory
- **Random Groups.** (♪♪♪, Assaf + Misha, Week 4)
  Prerequisites: Group Theory

7. Rings and Fields

Rings and fields generalize our notion of number: their elements can be added, subtracted, multiplied, and (sometimes) divided. In this cluster, you'll get an idea of why there is no degree 5 version of the quadratic formula. You'll see how factorization into primes breaks, how to measure how badly it breaks, and how to fix it. You'll learn how systems of numbers fit inside each other and how to study them with groups. If you want to expand your understanding of what a number is, check out these classes!

Classes in this cluster:

- **The Democracy of Number Systems.** (♪♪♪, *Clifton Cunningham*, Week 1)
- **Introduction to Ring Theory.** (♪♪♪, *Ari Nieh*, Week 1)
- **K-Theory.** (♪♪♪♪, Don, Week 2)
  Prerequisites: Ring Theory
- **Field Extensions and Galois Theory.** (♪♪♪♪, Mark, Weeks 2–3)
  Prerequisites: Group Theory, Ring Theory, Linear Algebra.
- **Bad Domains, Bad Factorization.** (♪♪♪♪, *Alfonso Gracia-Saz*, Week 3)
  Prerequisites: Ring Theory
- **Algebraic Number Theory.** (♪♪♪♪, David, Week 4, Marathon)
  Prerequisites: Field Extensions and Galois Theory, Bad Domains, Bad Factorization.

8. Games Mathematicians Play

Sometimes game theory is used to find optimal strategies to real-life games. Sometimes we cast real-life situations in game form to find mathematical answers to them. Sometimes we phrase other mathematical questions in terms of games, because mathematicians are silly. In this cluster, you'll see all sorts of ways mathematicians think about games.

Classes in this cluster:

- **Board Game Theory.** (🏃, Assaf, Week 1)
- **Combinatorial Games.** (🏃🏃, Jane, Week 1)
- **Nonzero-Sum Games.** (🏃, Pesto, Week 3)
- **Stupid Games on Infinite Sets.** (🏃🏃, Susan, Week 4)

9. Mathematics and its Applications

One of the most surprising aspects of mathematics is how useful even the most abstract theory can be in tackling real-world problems. This cluster will introduce the interplay between mathematical theory and applications in physics, computer science, and statistics!

Classes in this cluster:

- **Statistical Modeling.** (🏃🏃, Sam, Week 1)
- **Neural Networks.** (🏃, Kevin, Week 2)
- **Functional Programming.** (🏃🏃, Nic, Week 3)
- **Does ESP Exist?** (🏃🏃, Mira, Week 4)
- **Quantum Mechanics.** (🏃🏃🏃, Nic, Week 4)
  Prerequisites: Linear Algebra

10. Summing Series

This cluster will teach you about the things you should and shouldn't do with the symbol

$$\sum$$

as well as some of the more unusual applications of finite and infinite series.

Classes in this cluster:

- **Generating Functions and Partitions.** (🏃🏃, Mark, Week 1)
- **Analytic Number Theory.** (🏃🏃🏃🏃, Sachi, Week 1)
- **Divergent Series.** (🏃🏃, Sachi, Week 2)
- **What Can We Exponentiate?** (🏃🏃🏃, Assaf, Week 3)
  Prerequisites: Group Theory, Linear Algebra
- **dCalculus.** (🏃, Jeff, Week 3)
- **Asymptotics of Generating Functions.** (🏃🏃🏃, Kevin, Week 4)

11. Problem Solving

Classes in this cluster will teach you problem-solving techniques used in math competitions. Expect to solve many math problems along the way!

Classes in this cluster:

- Problem Solving: Triangle Geometry (🍄🍄🍄, Zach, Week 1)
- Problem Solving: Induction (🍄🍄, Misha, Week 2)
- Problem Solving: Symmetry, Parity and Invariants (🍄🍄🍄, *Joshua Zucker*, Week 3)
- Problem Solving: Polynomials (🍄🍄🍄🍄, Pesto, Week 4)