CLASS DESCRIPTIONS—WEEK 5, MATHCAMP 2018

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9:10 Classes

Axiomatic Music Theory part 2: Rhythm. (J-Lo, Tuesday–Wednesday)
Want more music theory? We talked a lot about pitch and intervals and triads and chord progressions in week 3, but very little about another crucial component of any musical piece: *rhythm.* Join a small group and explore how the Euclidean algorithm can be used to derive many traditional rhythmic patterns, find unexpected ways to transform rhythms in ways that preserve the set of time differences between pairs of beats (using a result known as the "hexachordal theorem"), and discover implications of the fact that pitch and rhythm are really the same thing.
Chilis: 🌶
Homework: Recommended
Prerequisites: None (in particular, Axiomatic Music Theory is not required)

Cyclotomic Polynomials and Migotti's Theorem. (Mark, Thursday)
The cyclotomic polynomials form an interesting family of polynomials with integer coefficients, whose roots are complex roots of unity. Looking at the first few of these polynomials leads to a natural conjecture about their coefficients. However, after the first hundred or so cases keep confirming the conjectured pattern, eventually it breaks down. In this class we'll prove a theorem due to Migotti, which sheds some light on what is going on, and in particular on why the conjecture finally fails just when it does.
Chilis: 🌶
Homework: Recommended
Prerequisites: Some experience with complex numbers, preferably including complex roots of unity; some experience with polynomials

Graph Minors. (Pesto, Thursday–Friday)
The graphs K_5 and $K_{3,3}$ are nonplanar. In fact, *Wagner's Theorem* says that a graph is nonplanar if and only if it contains one of those two graphs as a "minor". We'll define minors, prove that theorem, and talk about why K_5 is hardly necessary in that statement. Also, we'll state the most famous unsolved problem in graph theory, a generalization of the Four-Color Theorem, and prove it for some special cases.
Chilis: 🌶🌶🌶

Homework: Recommended
Prerequisites: Graph theory: Understand the first sentence of the blurb.
Related to (but not required for): Intro Graph Theory (W1)

Higher Homotopy Groups. (Larsen, Wednesday–Friday)
In week 4 we will have a class on the fundamental group, also denoted π_1. But if there is a π_1, surely there must be a π_2, and π_{17}! The "higher homotopy groups" are the natural generalization of the π_1, measuring the higher-dimensional complexity of a space. They are notoriously difficult to work with, but have some strange properties, for example they are all abelian.
Chilis:
Homework: Optional
Prerequisites: The Fundamental Group

Irrationality Proofs. (Susan, Tuesday)
I'm sure you can prove that $\sqrt{2}$ is irrational. Let's do some weirder things! In this class, we'll prove the irrationality of $\sqrt{2}+\sqrt{3}$, $\log_5(7)$, e, and π!
Chilis:
Homework: None
Prerequisites: None

One-Day Complex Analysis. (Larsen, Tuesday)
If you poke a hole in the complex numbers, they all stay in one piece—unlike the real line. Thanks to this and other facts, calculus with complex numbers works a lot better than regular calculus. Did you know that if a complex function is differentiable, it is infinitely differentiable? And if it is also bounded, it must be constant? After this one-day crash course, you will see why calculus is what makes complex numbers special.
Chilis:
Homework: Optional
Prerequisites: Calculus

Problem Solving: Tetrahedra. (Misha, Friday)
In the nine years from 1964 to 1972, every IMO competition contained a question with a tetrahedron in it. Since then, no such question has showed up again. In this class, we go back to the halcyon days of yore and solve as many of these problems as we can.
(This is a repeat of the class I taught in 2016.)
Chilis:
Homework: None
Prerequisites: None.

Quadratic reciprocity. (Mark, Tuesday–Wednesday)
Let p and q be distinct primes. What, if anything, is the relation between the answers to the following two questions?
(1) "Is q a square modulo p?"
(2) "Is p a square modulo q?"
In this class you'll find out; the relation is an important and surprising result which took Gauss a year to prove, and for which he eventually gave six different proofs. You'll get to see one particularly

nice proof, part of which is due to one of Gauss's best students, Eisenstein. And next time someone asks you whether 101is a square modulo 9973, you'll be able to answer a lot more quickly, whether or not you use technology!

Chilis:

Homework: Optional

Prerequisites: Some basic number theory (if you know Fermats Little Theorem, you should be OK)

Wedderburn's Theorem. (Mark, Friday)

Have you seen the quaternions? They form an example of a division ring that isn't a field. (A division ring is a set like a field, but in which multiplication isn't necessarily commutative.) Specifically, the quaternions form a four-dimensional vector space over $\mathbb{R}$, with basis $, i, j, k$ and multiplication rules

$$i^2 = j^2 = k^2 = 1, ij = k, ji = k, jk = i, kj = i, ki = j, ik = j.$$

Have you seen any examples of finite division rings that aren't fields? No, you haven't, and you never will, because Wedderburn proved that any finite division ring is commutative (and thus a field). In this class we'll see a beautiful proof of this theorem, due to Witt, using cyclotomic polynomials (polynomials whose roots are complex roots of unity).

Chilis:

Homework: Recommended

Prerequisites: Some group theory and some ring theory; familiarity with complex roots of unity would help.

Would I ever Lie Group to you? (Apurva, Tuesday–Wednesday)

Lie Groups are groups which are also manifolds. The easiest examples of Lie groups come from Linear Algebra as symmetries of vector spaces. We'll study these Matrix Groups and understand their connections with their geometry and physics.

Chilis:

Homework: Optional

Prerequisites: Linear algebra, Group Theory

Wreath Products. (Viv, Thursday–Friday)

You may have heard of a couple of ways to take products of groups: for example, directly, or semi-directly.

Wreath products are another way! They're also my favorite way. We'll define them and discuss examples, like the lamplighter group or the Rubik's Cube group.

Chilis:

Homework: Recommended

Prerequisites: Group Theory

Yes, you can square the circle. (Misha, Thursday)

Since ancient times it has been an open problem to use compass and straightedge to construct a square and a circle with equal area.

That is, it has been an open problem until it was proved to be impossible in 1882.

But we'll do it anyway: in the hyperbolic plane.

Chilis:

Homework: None

Prerequisites: None

Yes, you can trisect angles. (Ben Dees, Wednesday)
Trisecting arbitrary angles with compass and straightedge alone has been known to be impossible since the development of Galois Theory.

On the other hand, since antiquity we have known how to trisect arbitrary angles, and divide them into 5 pieces (or six, seven, and so on) by use of a construction called a "quadratrix." This isn't a curve we can construct using compass and straightedge, but use of it in mathematical arguments dates back to at least 400 BCE.

In this class, we will learn about what this curve is and about its history, and how to use it to trisect angles. If we have time, we'll discuss other uses of the quadratrix and how we can approximate it with compass and straightedge.

Chilis: 🌶🌶
Homework: None
Prerequisites: None

10:10 Classes

Combinatorial Topology. (Jeff, Tuesday–Wednesday)
So, you want to be a topologist because you love drawing pictures. But, you've never taken point-set topology[1] . How much can you prove about topology?
Turns out, quite a bit. In this class, we'll be developing simplicial complexes, which give combinatorial representations of topological spaces. Then, we'll look at discrete Morse theory, a combinatorial representation of a construction from differential topology. Along the way, we'll draw lots of pictures and diagrams, and get a feel for what topology should do, without messing around with all of those icky open sets.

Chilis: 🌶🌶
Homework: Optional
Prerequisites: None!
Related to (but not required for): Uranus has at least 2 storms (W5)

Cycles in permutations. (Ania, Jessica, Thursday–Friday)
Let's take a random permutation. What is the average number of fixed points in it? What is the average number of cycles? What is the probability that is has exactly one cycle? What is the probability that elements 1 and 2 are in the same cycle? Come to the class to find out and solve riddles about prisoners and planes!

Chilis: 🌶🌶–🌶🌶🌶
Homework: Optional
Prerequisites: None.

Elliptic functions. (Mark, Tuesday–Friday)
Complex analysis, meet elliptic curves! Actually, you don't need to know anything about elliptic curves to take this class, but they will show up along the way. Meanwhile, if you like periodic functions, such as *cos* and *sin*, then you should like elliptic functions even better: They have two independent (complex) periods, as well as a variety of nice properties that are relatively easy to prove using some complex analysis. Despite the name, which is a kind of historical accident (it all started with arc length along an ellipse, which comes up in the study of planetary motion; this led to so-called elliptic integrals, and elliptic functions were first encountered as inverse functions of those integrals), elliptic functions don't

[1] Disclaimer: I have never taken point set topology

have much to do with ellipses. Instead, they are closely related to cubic curves, and also to modular forms. If time permits, we'll use some of this material to prove the remarkable fact that

$$\sigma_7(n) = \sigma_3(n) + 120 \sum_{k=1}^{n-1} \sigma_3(k)\sigma_3(nk),$$

where $\sigma_i(k)$ is the sum of the i-th powers of the divisors of k. (For example, for $n = 5$ this comes down to

$$1 + 5^7 = 1 + 5^3 + 120[1(1^3 + 2^3 + 4^3) + (1^3 + 2^3)(1^3 + 3^3) + (1^3 + 3^3)(1^3 + 2^3) + (1^3 + 2^3 + 4^3)1],$$

which you are welcome to check if you run out of things to do.)

Chilis:

Homework: Recommended

Prerequisites: Functions of a complex variable, in particular Liouvilles Theorem.

Related to (but not required for): Accidental Mathematics (W2)

How to Juggle. (Viv, Tuesday–Wednesday)

In this class, you will learn to juggle...in theory.

We'll discuss juggling sequences, a mathematical model for juggling that revolutionized the juggling world!

Chilis:

Homework: None

Prerequisites: None!

Special Relativity. (Nic Ford, Tuesday–Friday)

Around the beginning of the twentieth century, physics was undergoing some drastic changes. The brand-new theory of electromagnetism made very accurate predictions, but it forced physicists to come to grips with a strange new truth: there is no such thing as absolute space, and there is no such thing as absolute time. Depending on their relative velocities, different observers can disagree about the length of a meterstick, or how long it takes for a clock to tick off one second.

In this class, we'll talk about the observations that forced physicists to change their ideas about space and time, and how the groundwork of physics has to be rebuilt to accommodate these observations. We will see how, as Minkowski said, "Space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind union of the two will preserve an independent reality." At the end, we'll also briefly look at how to revise the classical definitions of momentum and energy and see why we should believe that $E = mc^2$.

Chilis:

Homework: Optional

Prerequisites: none

Uranus has at least 2 storms. (Jeff, Thursday–Friday)

True Life Story of Mathematics: At UC Berkeley, we're required to take a qualifying oral exam in our second year. The exam is moderated by 3 mathematicians, and an outside member not from the mathematics department whose job is to make sure the 3 faculty members on your committee are treating you fairly. They're also suppose to lob you some easy questions, like "what made you study this field," or "what is your favorite theorem?"

One of my subjects was differential topology, which is what I mostly study now. My outside committee member asked me the following wonderful question:

"Describe a result from your field in a way which anybody can understand."

I feel like differential geometry is full of these types of results, so I brought out my favorite result: that a generic smooth function on the sphere has 2 critical points. I tried to package this in a way which was simpler to state.

"On any planet, at any given moment, there are at least 2 storms."

I thought this was a clever way of stating the theorem. By replacing the construction of gradient vector field with wind, and by classifying critical points of a vector field as a storm, I figured I had discovered some elegant way of getting at the crux of the field. To which, the professor (who was a tenured member of the astronomy department) looked at me and stated.

"Uranus has no storm systems."

I ended up passing my qual, but not before the 3 mathematicians members of the committee spent 10 minutes trying to convince this astronomer that Uranus did indeed have 2 storms.

In this class, we'll prove that Uranus has at least 2 storms, and many other things about differential manifolds, by using the tools of *Morse Homology*, which provides incredibly strong inequalities for the number of critical points on a manifold.

Chilis:

Homework: Recommended

Prerequisites: Linear algebra is a must. Comfort with multi-variable calculus, or faith in pictures will be helpful. This is the full-fat, homological version of the discrete Morse theory class we covered in Combinatorial topology, so attending that class will give a nice framing for this story, but is not strictly necessary.

Related to (but not required for): Combinatorial Topology (W5)

11:10 Classes

Continued Fraction Expansions and e. (Susan, Wednesday–Friday)

The continued fraction expansion of e is

$$1+\cfrac{1}{0+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{\ddots}}}}}}}}}}}}}}$$

Okay, but seriously, though, why?!?! Turns out we can find a simple, beautiful answer if we're willing to do a little integration. Or maybe a bit more than a little? No previous experience with continued fractions necessary. Come ready to get your hands dirty—it's gonna be a good time!

Chilis:

Homework: Recommended

Prerequisites: None.

How to Get Away from Dead Guys. (Pesto, Tuesday)

To get his PhD at MIT, Adam gave a talk on how get away from dead guys if you are in some area A and can run at rate 1, and they have to stay out of A but can run at a rate $z > 1$; you get away if you get to the edge of A with no dead guy at the same spot, but the dead guys try to stop you and be at the same spot on the edge of A. I'll tell you how to get away for some z, how the dead guys can stop you for some z, and why it's hard to say who wins for some more z.

Also, like a dead guy who can't talk well, I'll stay in the game of four.

Chilis:

Homework: None

Prerequisites: If you don't know what the statement "The problem of finding a set of k vertices in a graph such that every edge contains at least one is NP hard" means, ask before class.

Information Theory and the Redundancy of English. (Mira, Tuesday–Friday)

NWSFLSH: NGLSH S RDNDNT!! (BT DN'T TLL YR NGLSH TCHR SD THT)

The redundancy of English (or any other language) is what allows you to decipher the above sentence. It's also what allows you to decipher bad handwriting or to have a conversation in a crowded room. The redundancy is a kind of error-correcting code: even if you miss part of what was said, you can recover the rest.

How redundant is English? There are two ways to interpret this question:

- How much information is conveyed by a single letter of English text, relative to how much could theoretically be conveyed? (But what is information? How do you measure it?)
- How much can we compress English text? If we encode it using a really clever encoding scheme, can we reduce the length of the message by a factor of 2? 10? 100? (But how will we ever know if our encoding is the cleverest possible one?)

Fortunately, the two interpretations are related. In this class, we will first derive a mathematical definition of information, based on our intuitive notions of what this word should mean. Then we'll prove the Noiseless Coding Theorem: the degree to which a piece of text (or any other data stream) can be compressed is governed by the actual amount of information that it contains. We'll also talk about Huffman codes: the optimal way of compressing data if you know enough about its source. (That's a big "if", but it's still a very cool method.)

Finally, we'll answer our original question — how redundant is English? — in the way that Claude Shannon, the father of information theory, originally answered it: by playing a game I call Shannon's Hangman and using it as a way of communicating with our imaginary identical clones!

The class is 4 days long, but you can skip some of the days and still come to the others. Here's how it works:

Day 1: Introduction and definition of information. (Required for the rest of the class.)

Days 2, 3: Noiseless coding and Huffman codes. (The mathematical heart of the class, where we'll prove the Noiseless Coding Theorem.)

Day 4: Shannon's Hangman and the redundancy of English. (You can come to this class even if you don't come on Days 2 and 3 — you just need the material from Day 1.)

Chilis:

Homework: Recommended

Prerequisites: None

Latin Squares and Finite Geometries. (Marisa, Tuesday–Wednesday)

In 1782, Euler conjectured an answer to the following yes/no question: is it possible to arrange six *regiments* consisting of six officers each of different *ranks* in a 6×6 square so that no rank or regiment will be repeated in any row or column? We know the answer now, but surprisingly, the question remained open until 1901. In this class, we'll be exploring combinatorial design questions like this one through the lenses of Latin Squares (like Sudoku puzzles) and Finite Geometries (like the Fano plane).

Chilis:
Homework: Optional
Prerequisites: None

Minus Choice, Still Paradoxes. (J-Lo, Thursday–Friday)
The Banach-Tarski paradox says that a ball can be broken into finitely many pieces, and using only translations and rotations, can be rearranged into two balls, each the same volume as the original. The "Axiom of Choice" is a crucial part of this construction, and historically some people have used Banach-Tarski as an argument against this axiom.

But it turns out that paradoxical decompositions (breaking something into finitely many pieces and rearranging them into two copies of the original) exist even without the Axiom of Choice! This class will discuss some of these constructions, which are all based in group theory, and see what specific role Choice plays in the Banach-Tarski case.
Chilis:
Homework: None
Prerequisites: Group Theory

Nonzero-sum games. (Pesto, Wednesday)
We'll play and talk about combinations of (Iterated) (Community) (Hidden) Prisoner's Dilemmas (with punishment). Ask me at TAU about their relationship to theories of ethics.
Chilis:
Homework: Optional
Prerequisites: None

Problem-Solving: The Just Do It method. (Linus, at least, Tuesday)
You are navigating a 50-by-50 maze on your computer when suddenly your monitor shuts off. You don't remember where the maze's walls are. You only remember that you are in the top-left corner, the exit is in the bottom-right corner, and there is some path from you to the exit.

Is there some sequence of arrow key presses you can do that guarantees that you exit the maze at some point?

Learn the "Just Do It" method, a problem-solving technique in combinatorics that solves problems like this one by revealing that they are actually trivial.
Chilis:
Homework: None
Prerequisites: None

What is a "Sylow" anyways? (Ben Dees, Thursday–Friday)
Recently, in 2004, the Classification of Finite Simple Groups was completed, finishing off decades of work spread out over hundreds of articles. We won't be able to prove this theorem in class (among other things, I don't know if anyone in the world knows all of the details), but we can look at one of the useful tools involved in the classification!

The Sylow Theorems are a few results arising from group actions, proven by the Norwegian mathematician Peter Sylow. These theorems provide a way to show that all groups of order 15 are cyclic, that there are no nonabelian simple groups of order less than 60, among other consequences.

What are these theorems? How can we use them? What are simple groups? What are a few other facts about groups that come in handy when looking for simple groups? What are some fun facts about Sylow, and how do you pronounce his name? These are some of the questions we'll investigate in this class.

Chilis: 🌶🌶🌶

Homework: Optional

Prerequisites: Group Theory

1:10 Classes

Calculus without Calculus. (Tim!, Tuesday–Wednesday)
If you've taken a calculus class in school, you've surely had to do tons and tons of homework problems. Sometimes, calculus knocks out those problems in no time flat. But other times, the calculus solution looks messy, inelegant, or overpowered. Maybe the answer is nice and clean, but you wouldn't know it from the calculation. Many of these problems can be solved by another approach that doesn't use any calculus, is less messy, and gives more insight into what is going on. In this class, you'll see some of these methods, and solve some problems yourself. Some example problems that we'll solve without calculus:

- Jessica is 5 cubits tall and Larsen is 3.9 cubits tall, and they are standing 3 cubits apart. You want to run a string from the top of Jessica's head to the top of Larsen's head that touches the ground in the middle. What is the shortest length of string you can use?
- Ania rides a bike around an ellipical track, with axes of length 100 meters and 150 meters. The front and back wheels (which are 1 meter apart) each trace out a path. What's the area between the two paths?
- A dog is standing along an inexplicably straight shoreline. The dog's person stands 20 meters way along the shoreline throws a stick 8 meters out into the water. The dog can run along the shoreline at 6.40 meters per second, and can swim at 0.910 meters per second. What is the fastest route that the dog can take to get to the stick?
- Where in a movie theater should you sit so that the screen takes up the largest angle of your vision?
- What's the area between the curves $f(x) = x^3/9$ and $g(x) = x^2 - 2x$?

Amaze your friends! Startle your enemies! Annoy your calculus teacher!

Chilis: 🌶🌶

Homework: Recommended

Prerequisites: Some calculus will be useful for context, but we won't actually use calculus (that's the point).

Dirichlet Series. (Nicholas, Friday)
Prime numbers have been among the central objects of study in number theory since the time of the ancient Greeks. While Euclid's proof of the infinitude of primes is well known, his proof tells us little about how dense the primes actually are in the natural numbers. As it turns out, the answer to this question is intimately related to the zeros of the Riemann zeta function. In fact, the Riemann Hypothesis is equivalent to the statement

$$\pi(x) = \frac{x}{\log x} + O(x^{\frac{1}{2}} \log x),$$

where $\pi(x)$ is the number of primes less than or equal to x.

In this class we will take an introductory look at this function and others like it, which are known as Dirichlet Series. Find out how functions of a complex variable are able to encode information related

to number theory, and, if time permits, see the role of Dirichlet Series in proving a weaker result which states that $\pi(x)$ and $\frac{x}{\log x}$ are asymptotically equal.
Chilis:
Homework: None
Prerequisites: Comfort with infinite series and some experience with functions of a complex variable.

Dirichlet's Theorem on Primes a mod b. (Kevin, Tuesday–Thursday)
Did you know there are infinitely many primes? Did you know there are infinitely many primes congruent to a mod b, as long as a and b are relatively prime? We will study the world of L-functions, which generalize the Riemann zeta function, and use them to prove this fact.
Chilis:
Homework: Optional
Prerequisites: Familiarity with complex numbers. Some knowledge of groups and basic complex analysis (such as the word "pole") is helpful.

Galois Theory and Number Theory. (Aaron, Viv, Tuesday–Thursday)
Can you find an example of a polynomial which is irreducible over the integers but reducible mod every prime?

In this class, we'll see what this question has to do with Galois theory. We'll also investigate many other relations between Galois theory and number theory.

Possible further topics include:

(1) Galois groups of finite fields
(2) Cyclotomic extensions
(3) Kummer Extensions
(4) Extensions of local fields (and the infamous secret society of p-adics)
(5) Inseparable extensions
(6) Purely inseparable extensions
(7) Perfect fields

Chilis:
Homework: Recommended
Prerequisites: Galois Theory, some background in algebraic number theory may be helpful but it's not necessary that you took the week 1 class on algebraic number theory

Infinitesimaller Calculus: Sequences, Integrals, and Democracy. (Tim!, Thursday–Friday)
In Infinitesimal Calculus, we constructed the hyperreal numbers, and started to use them to construct pretty proofs about continuous functions and derivatives, ending with the Extreme Value Theorem. But there is so much more calculus to prettify!

We'll handle sequences and their limits with ease, we'll define integrals the way you've *always* wanted to, and we'll bask in the Fundamental Theorem of Calculus.

And hey, since we've been talking about hyperreal numbers in terms of voting, we're actually pretty well prepped to do some voting theory! We'll use ultrafilters to prove Arrow's Impossibility Theorem, which says that the only fair voting system is a dictatorship. But instead of hyperreal numbers, we'll use hyper-something else!
Chilis:
Homework: Recommended
Prerequisites: Infinitesimal Calculus

The Art of Live-TeXing. (Aaron, Friday)
Have you wanted to type math notes in real time but were too intimidated by the prospect? If so, this is the class for you. I'll describe some tips and tricks to make live-TeXing a feasible, and even enjoyable, experience. Students are encouraged to bring their laptops to class.

Chilis:

Homework: None

Prerequisites: Minimal prior experience with latex would be helpful.

The Stable Marriage Problem. (Marisa, Shiyue, Friday)

N single men and N single women want to pair up and get married. These are their names and preferences:

- Jeff: Susan > Shiyue > Jessica > Viv > Ania
- Ben: Ania > Susan > Viv > Shiyue > Jessica
- Agustin: Ania > Viv > Shiyue > Susan > Jessica
- Pesto: Jessica > Shiyue > Viv > Ania > Susan
- Tim! : Jessica > Ania > Susan > Viv > Shiyue

- Ania: Jeff > Pesto > Tim! > Agustin > Ben
- Jessica: Agustin > Ben > Jeff > Tim! > Pesto
- Susan: Agustin > Pesto > Tim! > Ben > Jeff
- Shiyue: Agustin > Tim! > Jeff > Ben > Pesto
- Viv: Agustin > Pesto > Ben > Tim! > Jeff

Is it possible to make everybody happy? Obviously not since almost everybody wants to marry Agustin. But is it possible to at least create a stable situation? For instance, it is a bad idea for Agustin to marry Shiyue and for Viv to marry Pesto, because then Agustin and Viv would prefer each other rather than staying with their partners, so they will run away together. How can we at least avoid having a run-away couple? Is there more than one way to do it? What is the best way to do it? And what if Shiyue and Viv decide that marrying each other is better than marrying Agustin?

Chilis:

Homework: None

Prerequisites: None

Tropical Plane Curves. (Shiyue, Tuesday–Thursday)
Happy Halloween! We saw some creature named tropical geometry going around TAU. But what is that? Tropical Geometry is an emerging subfield of algebraic geometry. Technically speaking, it provides a modern degeneration technique to replace algebraic varieties with combinatorial objects. Classical algebraic geometers study the interplay between polynomials and their zeros. But zeros could be hard to study. In 1990s, people discovered that transforming all our polynomials into tropical semiring, which is $\mathbb{R} \cup \{\infty\}$ with the usual addition and multiplication replaced with taking the min/max and addition, will turn polynomials into piecewise linear functions and their zeros into polyhedra. Now we can do combinatorics to tackle these algebraic geometry problems! This course will cover basic arithmetic in tropical semiring, tropical plane curves, Bezout's Theorem for tropical plane curves, etc.

Chilis:

Homework: Recommended

Prerequisites: Some ring theory will be useful

2:10 Classes

A Slice of PIE. (Brian Reinhart, Thursday)
If you've ever wondered about the sizes of things that aren't infinite, you might have run into the Principle of Inclusion-Exclusion:

$$\left|\bigcup_{A\in\mathcal{F}} A\right| = \sum_{S\subseteq\mathcal{F}} (-1)^{|S|-1} \left|\bigcap_{A\in S} A\right|$$

This is a formula which tells you the size of a union of a bunch of sets based on the sizes of their intersections. But... we use a lot of information about the sets, and end up with one very small piece of information. Shouldn't we be able to figure out more stuff if we know the sizes of ALL possible intersections?

In this class, we'll be taking a look at how we can modify inclusion-exclusion to be more general. First, we'll see how we can find the size of something called the "symmetric difference" of all of the sets. We'll also see how this is like cutting the union in half, and find out what happens when we cut into more than two pieces. Along the way, we'll see some generating functions and complex numbers, and get some insight into why Inclusion-Exclusion is so useful.

Chilis:

Homework: Optional

Prerequisites: You should be familiar enough with Inclusion-Exclusion to know how it applies to this problem: How many 3-digit numbers have an even first digit or a last digit divisible by 3 (or both)?

Related to (but not required for): Partially Ordered Sets (W3); Generating Functions, Catalan Numbers, and Partitions (W3)

Combinatorial Game Theory. (Laura, Wednesday–Thursday)
We'll look at some examples of two player impartial games and the winning strategies, and learn about how to figure out the winning positions for a game and assign nimbers to positions to figure out how to win!

Chilis:

Homework: Recommended

Prerequisites: None

Curse of Dimensionality. (Dyusha, Michelle, Wednesday)
As those of you who went to Po's lecture already know, oranges can be deceiving.

Since our very birth, all of us, and all of our progeny unto eternity, have lived and will forever live in a three dimensional world. For humans (even Po), an orange is just a healthy snack, but for n-dimensional beings (where n is large), life is not nearly as simple. Even essentials such as peeling an orange to reveal juicy flesh are no longer viable. For instance, training a neural network to discern good poetry from bad is simple in three or four dimensions, but optimizing thousands, if not millions, of factors, in a high-dimensional space is quite the challenge. Similarly, clever strategies to find the k nearest neighbors from a given point in 2 dimensions quickly fall apart when dimensions grow larger.

This funky behavior in large dimensions is known as the *Curse of Dimensionality*. In this class, we'll dive into several examples of things that work just fine in low dimensions, discuss how the curse arises, and propose some strategies to combat it.

Chilis:

Homework: None

Prerequisites: If you've heard about neural networks and vector spaces, that would be helpful but not at all required

King Chicken Theorems. (Marisa, Thursday)

Chickens are incredibly cruel creatures. Whenever you put a bunch of them together, they will form a pecking order. Perhaps "order" is an exaggeration: the chickens will go around pecking whichever chickens they deem to be weaker than themselves. Imagine you're a farmer, and you're mapping out the behavior of your chickens. You would like to assign blame to the meanest chicken. Is it always possible to identify the meanest chicken? Can there be two equally mean chickens? Are there pecking orders in which all the chickens are equally mean?

Chilis: 🌶

Homework: None

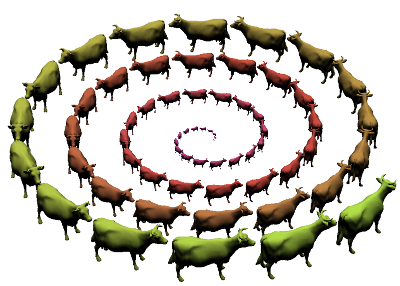
Prerequisites: None

Mathematica Workshop. (Misha, Wednesday–Thursday)

All campers get a free 1-year license for Mathematica from Wolfram.[2] But do you know how to use it?

In this class, you will learn how to do things in Mathematica, including:

- Drawing pictures such as the one below:



- Solving crossword puzzles.
- Various tricks I've learned for dealing with hard combinatorial problems.
- Using the incredibly extensive help function.

There will be time for you to work on doing your own cool things in Mathematica, which I will try to help you with.

Chilis: 🌶🌶

Homework: Optional

Prerequisites: None.

The Maths of Peppa Pig. (Linus, Wednesday)

Let's examine some of the advanced mathematical topics featured in the short film series Peppa Pig. This class will be a disjointed, ill-structured compilation of random mathematical thoughts I have while watching Peppa Pig next week.

Chilis: 🌶🌶🌶🌶

[2]Also, if you're going off to college, many universities provide free Mathematica licenses for their students.

Homework: None
Prerequisites: Get ready for algorithms and maybe topology.

The Quantum Spring. (Apurva, Wednesday–Thursday)
How do groups and algebras show up in physics? What does the set of 2×2 trace 0 matrices have anything to do with the quantum mechanics? How do Linear Operators create and annihilate particles? Why is Linear Algebra the answer to all your prayers?
Chilis: 🌶🌶–🌶🌶🌶
Homework: Optional
Prerequisites: Linear Algebra, specifically, Eigenvalues and Eigenvectors

Colloquium

Future of Mathcamp. (*The Staff*, Tuesday)
This is an event for all of us to come together as a community and discuss our favorite moments at Mathcamp, as well as suggestions for future years. Be part of a creative process that will build a better camp!

Future of You. (*The Staff*, Wednesday)
Come learn about your future! In this colloquium we'll tell you about the differences between liberal arts colleges and state universities, about study abroad and options for taking time off before college, about what life in college is like and what factors to consider when making a choice of school. Everyone is welcome, whether you've already decided on a school or haven't even begun thinking about it.