CLASS CLUSTERS FOR MATHCAMP 2020

0. About Clusters

The Mathcamp schedule provides a plethora of options, and we give you the freedom to choose among them as you will.[1] However, you may find the academics this summer more rewarding if you're able to look back in five weeks and reflect that you've learned *a lot* about something. To this end, we've constructed *clusters*: sets of classes with a common theme.

It is completely fine to only take part of a cluster. Note that fewer than half of the classes this summer are in a cluster; just because a class doesn't live in a cluster doesn't mean that it's not awesome!

1. Algebra and geometry

What is $y = x^2$? It's an equation, so we think of it as living in the world of algebra. But it's also a parabola, so maybe it's geometry? These classes will explore the connection between equations, functions, and geometric objects, and how going back and forth between the various worlds can help us understand each of them better.

Classes in this cluster:

- **Cubic curves** (🏃🏃🏃, Mark, Week 1)
- **Introduction to ring theory** (🏃🏃🏃, Eric, Week 2)
- **Geometry of lattices** (🏃🏃🏃, *J-Lo*, Week 3)
- **Solving equations with origami** (🏃🏃, Eric, Week 4)

2. Counting things

In these classes, you'll learn a lot of concepts that really count.

Classes in this cluster:

- **Determinantal formulas** (🏃🏃, Kayla, Week 1)
- **Hyperplane arrangements** (🏃, Emily, Week 1)
- **Combinatorics of tableaux** (🏃🏃🏃, Emily & Kayla, Week 2)
- **Regular expressions and generating functions** (🏃🏃, Linus, Week 3)
- **Extremal set theory: intersecting families** (🏃, Neeraja, Week 3)

[1] Unfortunately, we don't have any time turners to hand out.

3. Graph theory

Graph theory is the branch of math that ties together questions about coloring maps, drawing shapes without lifting your pencil, playing "Six Degrees of Wikipedia", analyzing the structure of social networks, and more.

Whenever you have a set of objects with connections between them, whether it's a molecule or a Facebook, you have a graph. These classes will teach you some cool things you can do with them.

Classes in this cluster:

- **Introduction to graph theory** (🏃🏃, Misha, Week 1)
- **Graphs on surfaces** (🏃🏃, Marisa, Week 2)
- **Ramanujan graphs and number theory** (🏃🏃🏃🏃, *Dan Gulotta*, Week 2)
- **Conflict-free graph coloring** (🏃🏃, Pesto, Week 2)
- **Spectral graph theory** (🏃🏃, Ania, Week 3)
- **Brooks' theorem blues** (🏃🏃🏃, Misha, Week 4)
- **Extremal graph theory** (🏃🏃🏃, Mia, Week 4)

4. Group theory

These classes will show you what groups are, and how they can be understood both for their own sake and for understanding other things. Lots of objects in mathematics and in the real world have group structures, and being able to exploit these is the power of group theory.

Classes in this cluster:

- **An inquiry-based approach to group theory** (🏃🏃, Katharine, Week 1)
- **The Rubik's cube group** (🏃🏃, Alan & Dennis, Week 2)
- **Representation theory** (🏃🏃🏃🏃, TBA, Weeks 3–4)
- **Grammatical group generation** (🏃, Eric, Week 3)

5. Math and computers

The study of telling computers how to solve problems began with Euclid over 2000 years ago. Much later, when computers appeared, people began to finally see the point.

In this cluster, you'll see both how math can help us understand computers, and how computers can help us understand math.

Classes in this cluster:

- **Teaching math to computers** (🏃🏃🏃, Apurva, Week 1)
- **Fourier something something boolean functions** (🏃🏃🏃, Tim!, Week 1)
- **Modeling computation** (🏃🏃, Mia, Week 2)
- **Complexity theory** (🏃🏃, Linus, Week 4)

6. Number theory

In this cluster, you'll explore properties of the integers from many different perspectives. Maybe the right way to think about the integers is by reducing them modulo a prime? Or maybe you should embed them in a bigger set of complex "integers"? Or maybe they really live in the p-adics?...

Classes in this cluster:

- **Introduction to number theory** (🏃🏃🏃, Mark, Week 2)
- **Congruences of Bernoulli numbers and zeta values** (🏃🏃🏃🏃, Eric, Week 3)
- **(Relatively) prime complex numbers** (🏃🏃🏃, Emily, Week 4)
- **Fair squares (mod p)** (🏃🏃, Maya, Week 4)

7. Probability and statistics

What do you get when you cross probability theory with real-life data? You get statistics, a field of study with many beautiful ideas and applications to almost every aspect of daily life.

Classes in this cluster:

- **The bell curve** (🏃🏃🏃, Mira, Week 1)
- **Causal inference** (🏃, Mira, Week 2)
- **Markov chains and random walks** (🏃🏃🏃, Misha, Week 2)
- **Information theory** (🏃🏃🏃, Mira, Week 3)

8. Real analysis

Calculus is great, but what exactly *is* a derivative? An integral? In this cluster, you'll learn all about how to make calculus rigorous, and how to use it to do some pretty amazing things.

Classes in this cluster:

- **Integration on manifolds** (🏃🏃🏃🏃, Neeraja, Week 1)
- **Introduction to analysis** (🏃🏃, Alan, Week 1)
- **Hilbert's space-filling curve** (🏃🏃, Ben, Week 2)
- **Weierstrass approximation** (🏃🏃, Neeraja, Week 2)
- **Cantor, Fourier, and the first uncountable ordinal** (🏃🏃🏃🏃, Ben, Week 2)
- **Bairely complete** (🏃🏃🏃, Ben, Week 3)
- **Fourier analysis** (🏃🏃, Alan, Week 3)
- **The Kakeya needle problem** (🏃🏃, Alan, Week 4)
- **Uncertainty principle** (🏃🏃🏃🏃, Neeraja, Week 4)

9. Topology

As the old joke goes, a topologist is a mathematician who cannot tell a donut from a coffee cup. The classes in this cluster will introduce you to a more flexible way of thinking about the shapes of various objects.

Classes in this cluster:

- **Clopen for business: an inquiry-based approach to point-set topology** (🐾🐾🐾, Katharine, Week 2)
- **FUNdamental groups and friends: an introduction to topological invariants** (🐾🐾🐾, Katharine, Week 3)
- **So you like them triangles?** (🐾🐾🐾, Dennis, Week 4)