Contents

## Alan's Projects

**Counterexamples in analysis.** (Alan)

*Description:* When we first learn analysis, one thing we must do is refine our intuition of how functions should behave. Looking at examples and counterexamples is a good way to train our intuition. Actually, trying to construct counterexamples on our own would be even better. Don't get the impression that analysis is all about counterexamples—it's also about learning from these counterexamples and seeing what assumptions need to be replaced to make statements true (e.g., uniform convergence instead of pointwise).

A good source of counterexamples is *Counterexamples in Analysis* by Gelbaum and Olmsted. We can use the book as a reference, but you should try to come up with counterexamples on your own!

*Expected Input:* You spend some time investigating whether various statements are true or false.

*Expected Output:* You have a better understanding of analysis!

*Difficulty:* 

*Prerequisites:* Single-variable calculus, know the epsilon-delta definition of a limit

**Discrete calculus.** (Alan)

*Description:* In calculus, we usually study functions whose domains are the real numbers. Can we still do calculus if we restrict the domain to the integers? In some sense, yes. You can define a "derivative" as $f'(x) = f(x+1) - f(x)$. It turns out there are analogues of many things you learn in calculus (e.g., product rule, fundamental theorem of calculus, Taylor's theorem). How many can you find? Most of the proofs are much less technical than the corresponding proofs in calculus.

*Expected Input:* Spend some time discovering, proving, and applying theorems.

*Expected Output:* Some knowledge of discrete calculus!

*Difficulty:* 

*Prerequisites:* None. Calculus is not required (but how this project proceeds depends on how much calculus you know)

**Functional analysis.** (Alan)

*Description:* Functional analysis can be thought of as linear algebra in infinite-dimensional vector spaces. When you study such spaces, you often have to work with infinite sums, so not surprisingly, analysis plays a more important role when you're dealing with infinite-dimensional spaces.

In this project you'll investigate and learn how finite- and infinite-dimensional spaces are different. A good reference for this topic is *Introductory Functional Analysis with Applications* by Kreyszig.

*Expected Input:* Spend some time reading about functional analysis.

*Expected Output:* Learn some things about functional analysis!

*Difficulty:* 

*Prerequisites:* Some analysis and some linear algebra

## Alan & Linus's Projects

**Prove your favorite puzzles are hard.** (Alan & Linus)

*Description:* Puzzles/games like Sudoku and Super Mario Bros are known to be "NP-hard." This means that if you had a fast algorithm to determine if an arbitrary Sudoku puzzle is possible, then you could use it to (e.g.) break all cryptography.

For this project, you'll pick one or more puzzles/games that are interesting to you, and attempt to figure out whether they are NP-hard.

Optional coding part: demonstrate your power: write a computer program that inputs a composite number, and outputs a puzzle such that solving the puzzle requires factorizing the number.

*Expected Input:* At least several hours a week.

*Expected Output:* NP-completeness proofs. Possibly a computer program to demonstrate at the Project Fair.

*Difficulty:* 🌶🌶🌶

*Prerequisites:* Technically none. But if you don't know what P and NP are, you'll need some time to learn.

## Apurva's Projects

**Do a math.** (Apurva)

*Description:* I am happy to mentor a reading project where you read a book/paper/notes related to topology/algebra/category theory. You should only do this if you have been dying to learn something on your own and need someone to help you with it.

*Expected Input:* Self study + typing notes on Overleaf or Github

*Expected Output:* A nice set of shareable notes.

*Difficulty:* 🌶🌶🌶🌶

*Prerequisites:* You should know/be willing to learn LaTeX.

**Topological Data Analysis.** (Apurva)

*Description:* Topological data analysis is a relatively new topological tool for analyzing data. The slogan is that topological data analysis studies the "shape of data" but really it is just linear algebra packaged nicely. This is an exploratory project. I know the theory behind TDA but I have never done any coding related to it. We will be exploring this together and I do not know how successful this will be. You'll start by reading some basic notes/papers about TDA.

*Expected Input:* Reading math papers in the first half and then python coding in the second half.

*Expected Output:* Unclear. It really depends on how much progress we make.

*Difficulty:* 🌶🌶🌶 – 🌶🌶🌶🌶

*Prerequisites:* You should be *really* comfortable with basic Linear Algebra and things like: rank-nullity theorem, computing ranks and nullity of matrices, direct sums of vector spaces, etc.

You do *not* need to know any topology. The theory is really just linear algebra.

You should know some programming language, preferably python.

## Apurva & Maya's Projects

**So you think you can Lean.** (Apurva & Maya)

*Description:* Code some cool math in Lean. If you want to just do a chill project you can solve mathcamp qualifying quiz problems in Lean. If you want a non-chill project you can create some new math in Lean, see https://leanprover-community.github.io/100.html and https://leanprover-community.github.io/undergrad_todo.html.

The only difference between the two options is the amount of time you would want to commit (think chilis). They will both be challenging and (hopefully) fun.

*Expected Input:* You'll have to code the proofs (ofc) and also comment on each other's proofs. This will be a collaborative project.

*Expected Output:* Some really cool proofs.

*Difficulty:* 🌶🌶 – 🌶🌶🌶

*Prerequisites:* Lean. And also, you'll need to have Lean and VSCode installed on your computer.

## Ben's Projects

**Dynamical Systems: Draw Your Own Fractal.** (Ben)

*Description:* Dynamical systems are something everyone secretly knows about, because they're what happens when you pick up a calculator, type in a number, and then start pressing a key repeatedly. If you press any number and then keep hitting "cosine," you'll approach $0.739\ldots$ pretty rapidly; this is the unique fixed point of cosine.

But way more complicated stuff can happen, and dynamical systems are one of the natural settings to model a lot of systems in the real world. We want to use them for something else, though: drawing fractals.

In this project you'll work on learning some stuff about dynamical systems, and also work on learning how to make images using some software, to try to draw some of the pretty fractal pictures that we sometimes see!

*Expected Input:* Campers in the project will read about dynamical systems and work with some mathematical software to create images

*Expected Output:* Some pretty fractal images that campers can show off at the project fair or use as a computer background

*Difficulty:* 🌶🌶–🌶🌶🌶

*Prerequisites:* None! Knowing what continuity means might be helpful.

**Measure Theory Reading Project.** (Ben)

*Description:* In the nineteenth century, problems pertaining to sequences and series of functions became increasingly prominent. You may have encountered some of these problems in the analysis classes at this camp! By the end of the century, the field was in such a state that Henri Poincaré complained of the "mass of bizarre functions" and of "monsters."

One of the tools to defend against these monsters is the theory of measure theory, which generalizes the notion of the "length" of a line segment to a broader class of sets, and then produces a notion of an integral that behaves far better with limits. Thus, Lebesgue's theory of integration resolves a lot of the issues that arose in the prior century!

*Expected Input:* Reading the source material and working on the exercises, problems, and so forth.

*Expected Output:* An understanding of measure theory, hopefully including several of the theorems on passing limits through integrals

*Difficulty:* 🌶🌶🌶🌶

*Prerequisites:* Some familiarity with analysis.

**Space-Filling Curve Art.** (Ben)

*Description:* One of the reasons I'm teaching a course on Hilbert's construction of a space-filling curve is that I think it's quite pretty. In this project, campers will create some pieces of a space-filling curve (perhaps drawn, but really anything in the appropriate shape would do). Any amount of detail is fine; hopefully campers will put their own personal twist on the pieces. At the end, we'll patch them together in some way at whatever level of detail we can manage!

*Expected Input:* Some time, some effort, some small amount of artistic inspiration

*Expected Output:* 1 very fun-looking squiggly curve, made up of many parts

*Difficulty:* 🌶

*Prerequisites:* None. In particular, the mathematical content of my Space-Filling Curve class is not necessary at all; this is just about some shapes.

## Ben and Lizka's Projects

**Make videos to explain or show off math.** (Ben and Lizka)

*Description:* Do you watch 3blue1brown, Vihart, or Numberphile? Have you ever wondered what it's like to make explainer videos about math? Here, we will try to make one ourselves! Pick a topic to learn enough about to explain, think about how to summarize it for consumption, and start filming!

*Expected Input:* Some time to figure out the math, some time to film, a fair bit of time to edit and put everything together.

*Expected Output:* Videos explaining awesome math things!

*Difficulty:* 🌶🌶–🌶🌶🌶🌶

*Prerequisites:* None, but if you don't know anything about video editing, be prepared to spend a lot of time learning

**Mathcamp Newspaper.** (Ben and Lizka)

*Description:* HEADLINE, HEADLINE, READ ALL ABOUT IT!! MATHCAMP NEWSPAPER RETURNING? THE DAILY VARIETY: WILL YOU JOIN?

Mathcamp used to have a newspaper. With your help, it can have a newspaper again! Report on all of the exciting events happening around you and help all the other campers keep up with what's going on!

*Expected Input:* Some time per week for reporting and writing.

*Expected Output:* The return of The Daily Variety to Mathcamp!

*Difficulty:* 🌶🌶🌶

*Prerequisites:* None.

**Mathcampus: Conclusive Sagas and Plots.** (Ben and Lizka)

*Description:* Little is known of Mathcampus's history, or its full geography. If you would like to imagine what lies over the river, or what mythical figures are responsible for the formal garden, or speculate about any of the Mathcampus's other inhabitants, you can do that here! Draw an elaborate map! Learn[1] its extensive history and post historical plaques! Get into contentious debates about the geography, history, and mystery of Mathcampus!

*Expected Input:* Time, effort, and creativity.

*Expected Output:* An Official map or history of Mathcampus. (Or, even better, multiple Official maps and/or histories of Mathcampus, which ardently disagree.)

*Difficulty:* 🌶🌶

*Prerequisites:* None.

**Writing Mathematically Inspired Fiction.** (Ben and Lizka)

*Description:* Have you read "Flatland"? Have you wondered what life would be like in Flatland, or a world governed by physical laws quite unlike our own? Perhaps a world in which time is discrete, or in which velocity can become unbounded, or even infinite, allowing for teleportation—except perhaps between points that are, themselves, infinitely far apart?

If these ideas inspire any stories for you, or sketches of stories, or if you have other weird mathy ideas for stories, that's what this project is about! Write a short story! It can be whimsical, serious, comical, tragic, or anything in between, as long as it's some kind of a story with a mathematical twist.

[1] i.e. fabricate

*Expected Input:* Time, effort, and creativity.

*Expected Output:* A story that you can summarize and share with the other campers!

*Difficulty:* 🌶🌶

*Prerequisites:* None.

## Dennis & Milan's Projects

**More cats, more fun!** (Dennis & Milan)

*Description:* Cats are fun. Let's do some more! This project is a continuation of the first week class; we'll be trying to work through as much of Lawvere and Samuel's *Conceptual Mathematics* as we can; my personal goal being to make it to adjoint functors. We'll have to first discuss morphisms of categories (functors), morphisms of functors (natural transformations), following a famous adage: "categories were defined in order to talk about functors. Functors were defined in order to talk about natural transformations". In our journey we'll see lots of sets, preorders, graphs, and the like as examples. For example, we'll see graphs described as a certain functor into the category of sets!

*Expected Input:* 2–5 hours a week reading/digesting

*Expected Output:* Learn about adjoint functors!

*Difficulty:* 🌶🌶

*Prerequisites:* W1 category class, or come talk to me!

## Emily's Projects

**More hyperplanes.** (Emily)

*Description:* A continuation of my week one course, but at a spicier level. We will pick back up with the correspondence between faces and flats. Then we will move on to other parts of hyperplane arrangements, such as cones, lunes, and galleries, and explore maps and equations related to these. We could also discuss some proofs that we neglected during week one.

*Expected Input:* 2–5 hours per week reading and trying exercises.

*Expected Output:* A deeper love for hyperplanes.

*Difficulty:* 🌶🌶–🌶🌶🌶

*Prerequisites:* W1 Hyperplane arrangements.

**The 17 wallpaper patterns.** (Emily)

*Description:* Wallpaper consists of a two-dimensional pattern that is repeated to tile the plane. This pattern can have different types of symmetry, such as translational, rotational, and mirror symmetry. When we consider all the different types of symmetry a pattern can have at one time, there exist exactly 17 different possible wallpaper patterns. Our goal is to create our own artwork based on these 17 patterns, using whatever medium you would like!

*Expected Input:* Varies greatly and is dependent on your creative process.

*Expected Output:* Either one or multiple pieces of art.

*Difficulty:* 🌶🌶–🌶🌶🌶🌶

*Prerequisites:* None.

**Using math in music composition.** (Emily)

*Description:* Math and music are highly intertwined, and what better way to explore this than through writing our own music! First we will introduce a few ways in which rhythms and melodies can be described mathematically; specifically, we will discuss cyclic rhythms, perfectly balanced and well-formed rhythms, and what it means to add and subtract scales and chords. From there, it's up to you to decide which concepts speak to you the most, and use them to compose your own piece of music. We will be using a music composition software, so this can be for any instrument(s) you want. We will also discuss music theory as needed/requested.

*Expected Input:* Varies greatly and is dependent on your creative process.

*Expected Output:* A piece of music written using some music composition software (like Musescore), and a brief write-up describing how math was used in the writing process.

*Difficulty:* 🌶🌶–🌶🌶🌶🌶

*Prerequisites:* Some music theory; playing an instrument will most likely give you enough theory to work with. Depending on the exact math you want to use, some basic linear algebra would be useful.

## Eric's Projects

**Build a 3D crossword.** (Eric)

*Description:* We'll try to build a $5 \times 5 \times 5$ crossword puzzle. Maybe it won't work? I'm sure this has been done before but playing around with it will be fun. Plus there are plenty of other form factors we can experiment with. Along the way we could read up a little bit on the information theory of crosswords and try give some heuristics on what sizes/dimensions we should be able to do it for.

*Expected Input:* 2–3 hours a week. I expect most of the actual work will be asynchronous and collaborative, with occasional meetings to plan.

*Expected Output:* Some small 3D crosswords or (likely harder) some evidence ruling out such things from a specific dictionary and sizes.

*Difficulty:* 🌶🌶

*Prerequisites:* A little bit of programming knowledge will likely be helpful as we might want to do some dictionary searching beyond what a crossword construction program offers.

**Kronecker–Weber.** (Eric)

*Description:* Reading project aiming to prove the Kronecker–Weber theorem that any abelian Galois extension $F/\mathbb{Q}$ is contained in a cyclotomic field. We'll need to develop some amount of the theory of local fields, number fields, and their Galois theory in order to get there. This is ambitious, but there are also plenty of interesting things we learn along the way, so more of a "journey is more important than the destination" type of reading project.

*Expected Input:* Meet once or twice a week to discuss material and plan out the next steps in reading. Doing the readings and working on exercises. Depending on background participants might need to invest a fair amount of time into putting together all the pieces.

*Expected Output:* Participants learn a proof of the Kronecker–Weber theorem.

*Difficulty:* 🌶🌶🌶🌶

*Prerequisites:* Some familiarity with ring theory, algebraic number theory, Galois theory are all helpful.

***p*-adic Numbers.** (Eric)

*Description:* Reading course focused on learning about $p$-adic numbers. The $p$-adic numbers are an extension and generalization of modular arithmetic which show up all over the place, especially in number theory. We'll work through the several definitions of $p$-adic numbers, showing why they are equivalent; this will also let us explore the topology of the $p$-adics along the way. There are a few accessible applications that we could aim for along the way, my favourite being the Skolem–Mahler–Lech theorem about linear recurrence sequences which has a wonderful proof using $p$-adics even though it doesn't look like it has anything to do with them.

*Expected Input:* We'll meet maybe once or twice a week at TAU to chat about the material and talk through questions. You'll have some reading you can do and problems to work on to help you learn the material.

*Expected Output:* Participants will learn about $p$-adic numbers.

*Difficulty:*

*Prerequisites:* Modular arithmetic. Having seen a small amount of metric spaces before could be useful but is not essential.

## Eric & Maya's Projects

**Remote Bach.** (Eric & Maya)

*Description:* Bach's contrapuntal music is particularly well-suited for splicing together different voices/recordings. We'll pick a few pieces, dissect the parts, then record the parts individually and splice them together.

*Expected Input:* 3 phases. 1: preparing pieces which we can do asynchronously and with a couple of short meetings. 2: recording which might need some central coordination but is otherwise totally asynchronous. 3: splicing recordings which can be done asynchronously and is a few hours of work.

*Expected Output:* Spliced video/audio recordings of a group of campers playing Bach together.

*Difficulty:*

*Prerequisites:* Willingness to record yourself singing/playing an instrument. Proficiency with said instrument preferred. Some familiarity with recording software (nothing fancier than garage band) and simple video editing software would be helpful.

## Eric & Tim!'s Projects

**Change Ringing.** (Eric & Tim!)

*Description:* In 16th(?) century England, people figured out that they could make church bells louder by hanging them on wheels. Eventually they got bored of just ringing their bells back and forth and wanted to play music. But too bad, they can't. The bells just physically can't do it. What they could do, though, is play all the bells, and then play them all again in a slightly different order. Their goal was to play all the possible permutations of the bells without repetition (an *extent*). This exercise continues to this day; there are over 5000 bell towers in England (and approx. 50 in North America) that are equipped for "ringing the changes."

We don't have access to a bell tower or a set of handbells this year, but fortunately, and unlike many other types of music, we can do change ringing online. If you join this project you'll learn change ringing with a small group of campers. We'll teach you a little bit of the math involved as well, but our focus will be on actually learning change ringing.

*Expected Input:* Attend practices, think about it outside of practice for $< 1/2$ an hour a week.

*Expected Output:* A change ringing performance in some format? For their own enjoyment there are a few milestones that participants can aim for while learning (calling a touch, ringing an extent, learning some interesting methods).

*Difficulty:*

*Prerequisites:* None.

## Katharine's Projects

**Cohomology of spaces.** (Katharine)

*Description:* Cohomology is a sequence of groups associated to a topological space. (This is also true of homology, and the construction is much the same, but includes the extra step of dualizing. In the context of groups, dualizing means we replace a group, $G$, with the collection of homomorphisms from $G$ into some other chosen group.) Much like homology, cohomology provides a way of detecting information about topological spaces. Unlike homology groups, cohomology groups fit together to form a ring.

My idea is to read at least some of Chapter 3 of Hatcher's *Algebraic Topology,* but the resource we pick really depends on past exposure to algebraic topology. So, if you're interested, definitely talk to me!

*Expected Input:* 4–6 hours a week of reading/exercises

*Expected Output:* Understanding of various constructions of cohomology

*Difficulty:*

*Prerequisites:* Group theory, ring theory, some exposure to homology

**Intro to (Mathematical) Knitting.** (Katharine)

*Description:* Learn to knit and make some cool mathematical objects. I don't have the video setup to teach you to knit entirely over Zoom, but I am willing to sign up to be your knitting helpline! I'll help you pick a project, curate a list of videos to demonstrate the steps, and respond to calls for help.

*Expected Input:* However much time you want to spend watching some videos

*Expected Output:* Some cool mathematical objects. I'm thinking Möbius bands (from ring-sized to scarf-sized), knots, and rectangular things (e.g. scarves or washcloths) that encode sequences would all be doable first projects.

*Difficulty:*

*Prerequisites:* None.

**Math history.** (Katharine)

*Description:* A research project into the history of math in a particular culture, history of a mathematical idea, or a particular person in the history of math. I'm pretty open to whatever you'd like to do here, but I can make suggestions based on your interests for topics you might like. It can be very math-content-heavy (let's try to understand proofs for the Pythagorean theorem from lots of different times and places) or more focused on the people and cultures surrounding the math.

*Expected Input:* Totally up to you!

*Expected Output:* Totally up to you! Make a poster, write a poem, give a dramatic reading, write an essay, give a talk…

*Difficulty:*

*Prerequisites:* None.

**Puzzles with algebra.** (Katharine)

*Description:* We'll use the methods of abstract algebra to analyze puzzles like the 15-puzzle (the one with sliding tiles) and the Hungarian rings game (look up an online simulator, it's really fun!).

*Expected Input:* 3–4 hours a week thinking about puzzles

*Expected Output:* Winning puzzle skills!

*Difficulty:*

*Prerequisites:* Group theory, some linear algebra might be helpful

**Quotient Groups.** (Katharine)

*Description:* If you've seen some group theory, you've probably run across quotient groups (even if you didn't know it). For example, addition modulo $n$ on the integers 0 through $n-1$ is a quotient of the integers under addition. This means we've stuck together some elements of a group (in this case, 0 and all multiples of $n$), to get a new, smaller group.

We'll read a few lectures by Frédérique Oggier to learn about how quotient groups are constructed. If we have time, we may also do some additional reading on the group isomorphism theorems.

*Expected Input:* About 4 hours a week of reading and doing exercises

*Expected Output:* An intuitive understanding of quotient groups and some practice proving things about them

*Difficulty:* 🌶🌶

*Prerequisites:* Some group theory (definitions of a group and of group homomorphisms and isomorphisms)

## Kayla's Projects

**Construction of Classical Lie Groups and Lie Algebras.** (Kayla)

*Description:* I envision this project as an add-on to my week 3 course and believe it could be done in a week or two. We are working through classifying simple Lie algebras in my course and one could ask how we actually construct these objects. Campers would work through the construction of these Lie groups and Lie algebras in types A and D.

*Expected Input:* Independently reading the construction in types A and D

*Expected Output:* A richer understanding of the classification of simple Lie algebras, and maybe a talk to campers that were in the Lie Theory course

*Difficulty:* 🌶🌶🌶

*Prerequisites:* My Lie theory course or a camper who is familiar with basics of Lie theory

**Intro to Cluster Algebras.** (Kayla)

*Description:* I would like to read through Lauren Williams's "Cluster Algebras: An Introduction" which is a really great exposition about cluster algebras and their connection to other fields e.g. Teichmuller theory, discrete dynamical systems. This is a relatively short read, so we can also look into other cluster algebra papers and topics e.g. snake graphs and cluster expansion formulas, bases for cluster algebras, etc.

*Expected Input:* Campers read 5–8 pages a week and work through relevant exercises. I also think it would be nice for them to prepare presentations for me / others in the group about material.

*Expected Output:* Knowing about cluster algebras and potentially understanding open problems in this field

*Difficulty:* 🌶🌶🌶

*Prerequisites:* Some algebra background e.g. knowing what an algebra is, generators and relations

**Posets and the Möbius Function.** (Kayla)

*Description:* Give an example-heavy introduction to posets and lattices in algebraic combinatorics following https://arxiv.org/pdf/1409.2562.pdf section 4.

*Expected Output:* Get a sense of some structures of interest to people in algebraic combinatorics

*Difficulty:* 🌶

*Prerequisites:* None.

**Symmetric Functions.** (Kayla)

*Description:* I would like to explore the algebra of symmetric functions with the emphasis on bases for this algebra. Once we define these bases, we can ask how we can change from one basis to another and explore combinatorial interpretations of the change of basis matrices / positivity. This would tie in well with my and Emily's "Combinatorics of Tableaux" class and we could further explore Schur polynomials highlighting reasons that they are "god-given." To up the chili level on this, I could also see us defining the Hopf algebra structure and exploring Hopf algebras in combinatorics!

*Expected Output:* Understanding symmetric function theory and giving more connections to my and Emily's week 2 class

*Difficulty:* 🌶🌶

*Prerequisites:* Definition of basis and understanding change of basis (could learn this through the project)

## Kevin's Projects

**Project Euler in Google Sheets.** (Kevin)

*Description:* In the words of its own website (https://projecteuler.net/), "Project Euler is a series of challenging mathematical/computer programming problems that will require more than just mathematical insights to solve. Although mathematics will help you arrive at elegant and efficient methods, the use of a computer and programming skills will be required to solve most problems." These problems start out simple—the first problem involves adding up a bunch of multiples of 3 or 5—but rapidly become more complicated. And they're, you know, designed to be solved with the help of a real programming language.

But I like Google Sheets a lot (:.

*Expected Input:* Up to you! There are effectively infinitely many Project Euler problems, but the simpler ones won't take too much time.

*Expected Output:* If you play with enough of the problems, you'll become a spreadsheet wizard!

*Difficulty:* 🌶🌶–🌶🌶🌶

*Prerequisites:* None. If you're not comfortable writing spreadsheet formulas, that's okay as long as you're willing to look things up in the documentation and learn. (It's similar to, but in some ways quite different from, standard programming; you should have an interest in that sort of thing.)

**Reading Enumerative Combinatorics.** (Kevin)

*Description:* Richard Stanley hosts the entire first volume of his Enumerative Combinatorics book on his website (http://www-math.mit.edu/~rstan/ec/). Emily and Kayla have proposed projects overlapping with a lot of Chapter 2 (about posets and hyperplane arrangements), but if something catches your eye in the other chapters, we can dive in!

*Expected Input:* Varies; there's tons more in this book than can be covered in one summer (and so many exercises!), but I think a couple hours of outside reading a week is the minimum.

*Expected Output:* Cool combinatorics knowledge!

*Difficulty:* 🌶🌶🌶

*Prerequisites:* None, though students with existing familiarity in some enumerative combinatorics might be able to get further.

## Linus's Projects

**HyperRogue.** (Linus)

*Description:* HyperRogue is a computer game that takes place on the hyperbolic plane. Play the game and learn about the hyperbolic plane.

Even if you don't decide to officially do this project, you can still play the game and learn from it. There is both a free version and a paid version.

*Expected Input:* You can put as much or as little time into this as you want.

*Difficulty:* 🌶

*Prerequisites:* None.

## Lizka's Projects

**Impossible Drawings.** (Lizka)

*Description:* In the spirit of M. C. Escher and the Mathcamp logo (a variation on the Penrose Triangle), let's draw some impossible things. This can range from working on exciting realizations of known impossible objects (like your take on the Penrose Stairs) to designing your own impossible things.

*Expected Input:* Time (amount up to participants), effort (up to participants), and creativity

*Expected Output:* Impossible drawings, and maybe even new impossible objects!

*Difficulty:* 🌶🌶

*Prerequisites:* A willingness to draw, paint, or produce art in some form

**Math art (models and materials).** (Lizka)

*Description:* Do you wish you could spend more time exploring math and playing with materials that could almost be called art supplies (do pipe cleaners count? How about fruit by the foot?)? Pick a mathematical object—like a torus, a family of knots or links, a hyperbolic plane tiling, some kind of polyhedron, etc.—and figure out a way to make a model of it in real space, using a material you have chosen. Maybe you'll discover that some materials work poorly for certain objects. (Why is that?) Maybe you'll produce a fantastic toothpick sculpture of a group's Cayley graph.

*Expected Input:* Heavily depends on preferences; ranges from 1 hour per week to ∼3 hours per day of work

*Expected Output:* A model (or several models) of a chosen mathematical object or concept, ideas about why, say, a torus dislikes paper so much.

*Difficulty:* 🌶–🌶🌶🌶🌶

*Prerequisites:* Willingness to explore math and work with your hands

## Maya's Projects

**Advanced Mathematical Knitting.** (Maya)

*Description:* Study knitting & designing knitting projects from a more mathematical perspective. Discuss some knitting puzzles / design challenges. Build off of this to design your own mathematical knitting project (and start work on it, if desired).

*Expected Input:* Spend a few hours per week working on puzzles and then discuss them at TAU. Spend time working on design and/or construction.

*Expected Output:* More familiarity with design strategies and challenges with knitting. Designing and partial construction of a mathematical knitting project.

*Difficulty:* 🌶🌶🌶

*Prerequisites:* Should be able to knit a rectangle with a knit side and a purl side

## Maya and Lizka's Projects

**Escher-like tilings and hyperbolic fun with computers.** (Maya and Lizka)

*Description:* M. C. Escher is famous for his prints, especially the creative plane tilings and metamorphoses. He also worked with the mathematician Coxeter to produce hyperbolic tilings, which gave him a lot of trouble. For this project, you'll create some tiles, and use something Escher never had—computers!—to help you create awesome hyperbolic tilings. For some examples, see: http://www.josleys.com/show_gallery.php?galid=325.

*Expected Input:* Some time spent to draw a tile (or a few!), lots of time to programmatically stitch the tiles together.

*Expected Output:* Pretty hyperbolic tilings!

*Difficulty:* 🌶🌶🌶🌶

*Prerequisites:* Some programming experience, preferably in Python, and a willingness to draw something.

## Milan and Joanna's Projects

**Design a Dominion Expansion.** (Milan and Joanna)

*Description:* Dominion is popular deck-building card game at Mathcamp. In this project, you will design your own Dominion expansion, that future generations of Mathcampers will have a chance to enjoy later!

*Expected Input:* 2–7 hours a week brainstorming, playtesting, naming cards/creating theme

*Expected Output:* A Dominion expansion!

*Difficulty:* 🌶

*Prerequisites:* Some familiarity with Dominion

## Mira's Projects

**High-achieving girls in math: what do the data say?** (Mira)

*Description:* Over the past decade, economists Glenn Ellison and Ashley Swanson have written a series of papers using data from the AMC and the AIME to try to understand the dynamics—especially, but not exclusively, the gender dynamics—of high math achievement in high school. In other words, these papers are basically about you guys!

Want to know what these economists are saying about you? Let's read the papers together, try to understand the results (they actually use quite a bit of math in some of their statistical analyses), and see if Mathcampers' personal experiences can help shed some light on the issue. Maybe we can find an angle that the economists have overlooked! If so, we can write Glenn Ellison (who was a visitor at Mathcamp several times and whose daughter Anna was a Mathcamper and JC) and see if he can send us some of their data to analyze.

*Expected Input:* Reading, thinking, talking

*Expected Output:* A better understanding of what might be driving the gender-gap in high math achievement; also, a better understanding of what analysis of data by two really good economists looks like!

*Difficulty:* 🌶🌶

*Prerequisites:* Knowing some basic statistics is helpful but not required.

**Modeling COVID19.** (Mira)

*Description:* Learn about the SEIR model (the basic epidemiological model of infectious disease) and experiment with fitting it to COVID19 data.

You can also try designing some simple agent-based models of COVID19 spread and think about how to make them mesh with the SEIR model.

WARNING: I don't know very much at all about this subject! We'd be learning about it together.

*Expected Input:* Reading about the SEIR model, getting some data, programming your own model.

*Expected Output:* Some toy COVID19 models and a better understanding of what goes into modeling the spread of the virus.

*Difficulty:*

*Prerequisites:* Programming (e.g. in Python); calculus desirable but not strictly required.

**Probabilistic programming.** (Mira)

*Description:* This project will follow the Web book *Probabilistic Models of Cognition*. The book is built around demos and problems in the probabilistic programming language WebPPL, which you will learn along the way. The book is short and accessible enough that you should be able to get through much of it during Mathcamp. (Also, one of the authors of the book, Josh Tenenbaum, will be visiting Mathcamp in Week 5.)

The book focuses on applications of probabilistic programming to the study of human cognition—so if you're interested in psychology or cognitive science, this is obviously a great project for you. But in fact, the mathematical tools and ideas described in the book are much more broadly applicable: probabilistic programming is a new and promising approach in statistics and machine learning more generally, and the book provides one of the most user-friendly introductions to the subject that you will find anywhere. So even if you are not particularly interested in cognition but want to explore the frontiers of modern statistics, this project may be of interest to you. (Once you learn the basics in the book, we can discuss other directions you can take it. At that point, we would probably want to switch from WebPPL to another probabilistic programming language, such as Stan; for that, you would need to already know how to program in Python or R.)

*Note: This project has a lot of overlap with the "Probabilistic Models" course I taught last year, but is not identical to that course. If you took the course and liked it, you will be able to go much faster, but there's definitely more to learn!*

*Expected Input:* Time reading on your own and playing around with WebPPL

*Expected Output:* Learning probabilistic programming.

*Difficulty:*

*Prerequisites:* None.

**Surreal Numbers.** (Mira)

*Description: You find a fragment of an ancient manuscript that begins...*

> *In the beginning, everything was void and JH Conway began to create numbers. Conway said, "Let there be two rules that bring forth numbers great and small. This shall be the first rule: every number corresponds to two sets of previously created numbers such that no member of the left set is greater than or equivalent to any member of the right set. And the second rule shall be this: one number is less than or equivalent to to another number if and only if no member of the first number's left set is greater than or equivalent to the second number and no member of the second number's right set is less than or equivalent to the first number." And Conway examined these two rules, and behold, they were very good!*
>
> *And the first number was created from the void left set and the void right set. Conway called this number zero and said that it should be a sign to separate positive numbers from negative numbers. Conway proved that zero is less than or equivalent to zero, and he saw that it was good. And the evening and the morning were the day of zero.*
>
> *One the next day, two more numbers were created, one with zero as its left set and one with zero as its right set. And Conway called the former number "one" and the latter he called "minus one". And he proved that minus one is less than but not equivalent to zero and that zero is less than but not equivalent to one. And the evening*

*There are many things to figure out here. Can you actually prove that zero is less than but not equivalent to one? What happens on the next day, and the next, and the next? Eventually you might end up finding more manuscript fragments with addition and multiplication... maybe even infinity.*

The great mathematician and friend of Mathcamp, John Horton Conway, died of COVID-19 earlier this year; I am running this project in his memory.

One of Conway's most famous mathematical inventions were the surreal numbers: a way of enlarging the set of real numbers by considering numbers as combinatorial games. In this project, you will rediscover the surreal numbers for yourself, following the whimsical approach of Donald Knuth, as above. It's an incredibly beautiful theory and a great chance to practice developing a whole new mathematical framework from scratch.

*Expected Input:* Writing lots of proofs.

*Expected Output:* The surreal numbers!

*Difficulty:* 🌶🌶🌶

*Prerequisites:* None.

**Using statistics to study society.** (Mira)

*Description:* Interested in doing some statistical analysis of social science data (political, sociological, historical)? Come talk to me! Whether you know a lot of statistics or none at all, we can look for a topic that you find interesting, find some data sets for you to look at, set you up with some relevant statistical tools, and see what happens.

*Expected Input:* Formulating a question that you're interested in, looking for relevant data, developing a statistical model

*Expected Output:* Greater insight into both statistics and society

*Difficulty:* 🌶🌶

*Prerequisites:* At least a little programming experience, in any language. (You may find it useful to learn some R for this project; that should not be hard if you already know something about programming, but probably not realistic if you're starting completely from scratch.)

## Misha's Projects

**Modular Origami.** (Misha)

*Description:* All you need is (lots of square sheets of) paper!

Modular origami is a fun hobby to take up that doesn't require staring at a computer screen. Like all origami, it involves folding sheets of paper without cutting them. Unlike most origami, you do this lots of times (usually the individual units are fairly simple) and then assemble into one large structure!

Modular origami creations are often more mathematical than usual; you can make polyhedra or fancier objects with the symmetries of a polyhedron.

*Expected Input:* You can make one reasonably ambitious thing in 1–3 hours of work, which is easily split over any number of days. I can help you pick things that take more or less time.

*Expected Output:* Some origami creations.

*Difficulty:* 🌶🌶

*Prerequisites:* Ideally, some origami paper.

**Raymond Smullyan: The Project.** (Misha)

*Description:* "If I am a bear, then Ben thinks he is a vampire."

You often encounter statements like these in logic puzzles. Raymond Smullyan is an author who wrote lots and lots of whole books of logic puzzles of this flavor, with knights that always tell the truth and knaves that always lie and lots of other exciting variants.

This is a reading project in which you'll pick one of those books, go through it, and solve the logic puzzles. The chili count varies with the book you choose, but 2 is probably the median.

*Expected Input:* Entirely up to you.

*Expected Output:* Fun with puzzles!

*Difficulty:* 🌶🌶

*Prerequisites:* None.

**nayllumS dnomyaR: The Project.** (Misha)

*Description:* "If I am a bear, then Ben thinks he is a vampire."

You often encounter statements like these in logic puzzles. Except not exactly like this, because very few people write logic puzzles about Mathcamp staff who may or may not be bears. Clearly, this needs to be urgently fixed.

In this project, you'll take the standard framework of logic puzzles (with knights who always lie and knaves who always tell the truth) and give it a twist you come up with. Then, you'll write your own logic puzzles exploring the rules you decided on.

*Expected Input:* Discussions a few times a week, at TAU or in some other format.

*Expected Output:* Logic puzzles!

*Difficulty:* 🌶🌶

*Prerequisites:* None.

**vZome.** (Misha)

*Description:* At in-person Mathcamp, in a room called the Zome lounge, there are drawers and drawers of a construction toy called Zometool which lets you build fancy polyhedra out of colorful sticks connected by white balls.

vZome is a Java application that lets you do the same thing from the comfort of your computer. You can find it online at

https://vzome.com/home/index/vzome-7/

(You can also find this link pinned to the `#zome` channel on Slack.)

In this project, you'll build things in vZome! I have some ideas to start with, but you can also do your own thing.

*Expected Input:* Varies greatly depending on how many ideas you have, and how ambitious they are! You can build something really cool in a single afternoon, but you may want to build multiple cool things, or bigger ones.

*Expected Output:* Some vZome creations that can be virtually displayed at an online Project Fair.

*Difficulty:*

*Prerequisites:* None.

## Neeraja's Projects

**Philosophy of math.** (Neeraja)

*Description:* We'll read a text in (broadly speaking) the philosophy of math, probably either Ludwig Wittgenstein's *Tractatus Logico-Philosophicus*, or Imre Lakatos' *Proofs and Refutations*. If you are interested in doing a reading project in the philosophy of math but know a different text that you'd like to read, please tell me and we might possibly be able to arrange this project around that as well.

The goal of the project is to understand our chosen text as well as we can by (i) reading the text with care, (ii) meeting at least once a week to discuss the questions being handled in it, and (iii) place the text in context by looking at (extracts of) other relevant texts that preceded it.

*Expected Input:* At least 2 hours of reading per week + at least 1 hour of discussion

*Expected Output:* A poster or a short presentation

*Difficulty:*

*Prerequisites:* None.

## Pesto's Projects

**Constructing Linguistics Olympiad Problems.** (Pesto)

*Description:* The Linguistics Olympiads are linguistics-themed contests of pure logic puzzles, like the one at the end of this blurb. How does one construct such a puzzle? Could you?

Match the following Kurdish[2] sentences to their translations and translate "H'irç' min dibîne."

| | |
|---|---|
| A. You see Bear. | 1. Ez h'irç'e dibînim |
| B. You see me. | 2. Tu dir'evî |
| C. Bear runs. | 3. Tu min dibînî |
| D. You run. | 4. H'irç' dir'eve |
| E. I see Bear. | 5. Ez dir'evim |
| F. I run. | 6. Tu h'irç'e dibînî |

*Expected Input:* At least 4 hours solving problems so you have a sense of what sort of problems you might construct.

*Expected Output:* Maybe a relays problem? Maybe, if you're sworn to secrecy, a national olympiad problem for some other country?

*Difficulty:*

*Prerequisites:* None.

[2]Puzzle by N. Bailey.

**Graph Minors Research Project.** (Pesto)

*Description:* In Pesto's undergraduate thesis, he made one conjecture that he couldn't solve, but that he thinks might be approachable with no more than an hour's worth of graph theory background teaching. The statement:

**Conjecture.** *For all positive integers $k$ and all 2-connected simple graphs $G$ with $|V(G)| \geq k+3$ and $\frac{|E(G)|-1}{|V(G)|-2} > \frac{3k-1}{k}$, there exists a 2-connected simple graph $H$ such that $G \geq_3 H$, $|V(H)| \geq k+2$, and $|E(H)| - 3|V(H)| \geq -6$, where $\geq_3$ is minor containment.*

*Expected Input:* At least an hour to learn enough to understand the problems.

*Expected Output:* Minimum (1 hour): a bit of understanding of graph theory. Medium (16 hours of thought, times or divided by 4): a solution to an easier version of the conjecture that Pesto solved. Maximum (even if you put as much time as possible in and do well, you might not get anywhere): prove (or disprove) the conjecture.

*Difficulty:*

*Prerequisites:* None.

**The Worst Election Ever.** (Pesto)

*Description:* You can have a plurality-voting election in which one candidate is more popular than each other candidate, but doesn't win. In "Instant Runoff Voting" (used in, e.g., Australia), making your ranking of a candidate higher can make them lose. These and similar paradoxes are usually illustrated by sample elections in which they occur: for instance, if

- 48% of voters prefer Gore to Nader to Bush,
- 49% prefer Bush to Gore to Nader, and
- 3% prefer Nader to Gore to Bush,

then Gore would beat Bush and Gore would beat Nader, but Gore loses the 3-way election.

Can we make one such example election for which every one of the most famous paradoxes would occur in all of the most famous voting systems vulnerable to it, or are there sets of (paradox, voting system) pairs that can't all happen simultaneously? Is there a systematic way to combine examples of some of the paradoxes, so we don't have to create one big counterexample all at once?

*Expected Input:* The more time you put in, the worse the election can get.

*Expected Output:* An exquisitely awful election.

*Difficulty:*

*Prerequisites:* None.

**Wyt Queens.** (Pesto)

*Description:* A queen is on a point $(x, y)$ of a quarter-infinite chessboard. You and an opponent take turns making queen's moves toward the origin (left, down, or diagonally left-and-down). For each $(x, y)$, who wins, with optimal play?

*Expected Input:* 2–8 hours

*Expected Output:* An answer to the question in the description and hopefully a proof.

*Difficulty:*

*Prerequisites:* None.

## Susan's Projects

**Balance Puzzles.** (Susan)

*Description:* Our jumping-off point for this project is the following question:

You have twelve coins. One of them is counterfeit—it weighs a different amount from the other coins. You would like to find the counterfeit coin. To do so, you must use an old-fashioned balance scale to compare the weights of two sets of coins. What is the smallest number of weigh-ings that you can use to determine which coin is fake, and whether it is heavier or lighter than the other coins?

Your goal is to solve this puzzle, and then find ways to frame and solve more general versions. What if there are twenty-five coins? What's the largest number of coins you can do with five weigh-ings? What if you already know that the counterfeit coin is heavier? What if you don't care whether it's heavier or lighter? You choose the direction you want to take things!

*Expected Output:* Some summary of their results—either in document form or, if they'd like, a video explainer.

*Difficulty:* 🌶

*Prerequisites:* None.

## Tim!'s Projects

**Evasiveness.** (Tim!)

*Description:* This is a reading project on *evasiveness*, a subtopic of complexity theory in theoretical computer science.

One way to measure the complexity of a problem (like "Does this graph have a Hamiltonian cycle?") is by its time complexity—roughly, how long it takes a computer to solve it. Another important way to measure complexity is query complexity—roughly, how many questions you need to ask about the graph to answer the problem. The graph properties with maximum query complexity are called evasive. A conjecture about the complexity of graph properties has eluded mathematicians for over forty years: the conjecture is that a huge class of graph properties—specifically, all those that are nontrivial and monotone—are evasive.

You'll trace the story of this conjecture by reading papers. The story starts in 1973, and from there it branches out—you can choose which way to go. There's a surprising appearance of topology in 1984, that has persisted through much of the more recent work. There are interesting results from within the past decade. Along the way, you'll see scorpion graphs, clever counting, collapsible simplicial complexes, transitive permutation groups, and hypergraph properties.

*Expected Input:* You will read papers and work through some of the math in those papers yourself. There's essentially no upper bound on how long you can spend reading these (there are a lot of interesting papers), but to get something meaningful out of it, you should plan to spend at least 10 hours.

*Expected Output:* Knowledge about evasiveness, and perhaps some ideas on how to explore this topic yourself.

*Difficulty:* 🌶🌶🌶–🌶🌶🌶🌶

*Prerequisites:* Group theory, graph theory.

**Fourier Analysis of Boolean Functions.** (Tim!)

*Description:* This is a reading project on *Fourier Analysis of Boolean Functions*. There is a wonderful book on this topic, written by Ryan O'Donnell, which is available free online at http://analysisofbooleanfunctions.net/. The book goes into many more applications, as well as more of the theory.

You can sign up for this project whether or not you took the Week 1 class, but if you took the class, you can expect to get further (and if you didn't, see the class blurb on page 6 of https://www.mathcamp.org/static/yearly/2020/academics/Week1Blurbs.pdf).

Please don't tell Yuval that I used the word "analysis" in this project proposal.

*Expected Input:* There are enough interesting topics in the book to spend any amount of time on. If you took the class and want to learn more about elections, plan to spend at least 6 hours, or if you want to learn about a new topic, plan to spend at least 10 hours. If you did not take the class, plan to spend at least 8 hours.

*Expected Output:* More understanding of Fourier analysis of boolean functions.

*Difficulty:* 🌶🌶🌶

*Prerequisites:* None.

**Guess Who?: The hardest game that you thought was easy.** (Tim!)

*Description:* I've become a bit obsessed with the board game *Guess Who?*. It's a simple children's guessing game—you ask yes/no questions to try to determine your opponent's secret character from among 24 possibilities, and whoever guesses correctly with fewer questions wins.

My obsession started after watching this video by YouTube star (and former NASA scientist) Mark Rober, entitled *"BEST Guess Who Strategy- 96% WIN record using MATH"*: https://www.youtube.com/watch?v=FRlbNOno5VA. Lots of other people on the internet have thought of this strategy as well, and perhaps you, too, assumed this was the best strategy.

The only problem is that it just really isn't. This strategy can be totally destroyed. The actual best strategy is much more complicated, but has such a clear pattern to it that it will leave you asking "why?"

Hopefully, in this project, you will figure out the best strategy, and also why the answer has its strange but distinct pattern. I know the answer for the original game, which has 24 characters, as well as the version with $N$ characters for a few different values of $N$. I don't know the answer in general, but I have a few ideas on how to approach it that we could work on together.

Expect to explore concepts like "decision trees," "entropy," "matrix games," "convex optimization," and "relaxations."

*Expected Input:* Expect to put in at least 6 hours of work outside of our meetings to get to a satisfying ending point. Plan on 15 hours or more to be able to answer the "why" question and to get into working on the problem for general $N$.

*Expected Output:* A (possibly digital) poster showing off your findings.

*Difficulty:* 🌶🌶🌶

*Prerequisites:* Some programming experience is useful.

## Yuval's Projects

**My favorite math book.** (Yuval)

*Description:* My favorite math book is Matoušek's *Thirty-three miniatures*. It's a collection of short gems, each of which is a slick application of linear algebra to a question that seems to have no linear algebra anywhere in it. These applications are usually combinatorial, though there's some geometry, computer science, and algebra thrown into the mix as well.

In this project, you will read a Miniature or two every week (either alone or in a small group), and then we'll meet to discuss them. Which precise sections we read would depend on your interest!

*Expected Input:* A couple hours a week for reading and thinking.

*Expected Output:* Knowing some cool math! If you'd like, you can also pick your favorite topic and make a poster about it or present it in some other way.

*Difficulty:* 🌶🌶🌶

*Prerequisites:* Linear algebra, especially the definition of a basis of an arbitrary vector space. Specific sections might have some other prereqs, but if you know only linear algebra, you could still get a lot out of this project.

**Reading the Elements.** (Yuval)

*Description:* We'll read through some parts of Euclid's *Elements* (in English translation). The goal is to try to understand what ancient Greek math "looks like"—how did they structure their proofs, how did the diagrams come into it, etc. We'll also discuss the ways in which their proofs are like ours, and the ways in which they're so markedly different. Ideally, campers would come away with a good sense of how ancient Greek mathematicians thought about math.

If someone is very interested, we could also read some things that aren't the *Elements*. Obvious options include Archimedes and Apollonius, though if someone is excited, we could also branch out and read some non-Greek math for comparison (e.g. the *Nine Chapters* for people interested in Chinese mathematics). I know much less about non-Greek ancient math, but I'd be excited to learn!

*Expected Input:* A few hours a week to read some ancient math. Especially at the beginning, we'll probably also have some group meetings to work through some things together.

*Expected Output:* Learning more about ancient math! If you'd like, you can also make a poster or some other presentation about one of the topics. Alternately, you can try writing your own favorite proof in ancient Greek style!

*Difficulty:*

*Prerequisites:* None.

## The staff's Projects

**Teach a week 5 class.** (The staff)

*Description:* Do you have an idea for a class you'd like to teach? Find a staff member and describe your idea—be sure to have an idea of the outline and the punchline of your potential course. Feel free to approach any staff person who you think is a good mathematical fit (though keep in mind that not all staff will be comfortable supervising a teaching project).

*Expected Input:* In the first few weeks you'll have to spend a good amount of time figuring out exactly what you want to include in your 50 minutes of time for your class—that's shorter than you might expect! You'll also spend some time outlining the class and preparing it as well as giving two practice talks before the real class. You will give a practice talk for your staff sponsor some time in Week 3, and after receiving their feedback you'll give at least one more practice talk some time in Week 4. Assuming that goes well, you'll be able to teach in Week 5.

*Expected Output:* A possible spot in the Week 5 class schedule!

*Difficulty:*

*Prerequisites:* Depends on what you want to teach.