CLASS THEMES FOR MATHCAMP 2021

Contents

About Themes

The Mathcamp schedule provides a huge selection of options, and we give you the freedom to choose among them as you please.[1] You may want to learn a little bit about everything, or you may want to learn *a lot* about one or two things, or anything in between! Regardless of your approach to academics, it might think to help you to think about classes in terms of *themes*: sets of classes that have a common topic, technique, or structure to study.

How to use this document: we do *not* recommend you try to take all the classes in a single theme (in some cases this is impossible). The goal of themes is just to point out similarities between classes: if you were excited by a topic in one class, you can look up what themes it's a part of; you might want to consider exploring other topics with the same theme!

[1] Unfortunately, we don't have any time turners to hand out.

0. Smörgåsbord

These classes aren't tied together by a common topic — in fact, it's exactly the opposite! If you take any of these courses, you will be able to **explore multiple different topics within the same class block**. This is either because the course itself covers a wide range of topics, or because there are built-in opportunities to switch between multiple classes in a single week.

Classes in this theme:

- **Better sleep through modeling** (🌶, *Olivia Walch*, week 1. Also in Math in real life)
- **Better sheep through modeling** (🌶, J-Lo, week 1. Also in Games, Math in real life)
- **Topics in number theory** (🌶🌶, Misha, week 1. Also in Number theory)
- **Mathcamp crash course** (🌶, Assaf, week 1)
- **A pair of fractal curves** (🌶🌶, Ben, week 2. Also in Discrete analysis)
- **The pirate game** (🌶, Ben, week 2. Also in Games)
- **Euclidean geometry beyond Euclid** (🌶🌶, *Yuval Wigderson*, week 2. Also in Discrete analysis, Rigid shapes)
- **Algorithms on your phone** (🌶🌶, *Agustin Garcia*, week 2. Also in CS & algorithms, Math in real life)
- **The 17 worlds of planar ants** (🌶🌶, *Dror Bar-Natan*, week 4. Also in Symmetries)
- **The probabilistic method** (🌶🌶, Mia, week 4. Also in Discrete analysis, Graph theory)
- **The derivative as a linear transformation** (🌶🌶, Alan, week 4. Also in Analysis)
- **The inverse and implicit function theorems** (🌶🌶🌶, Alan, week 4. Also in Analysis)
- **Trail mix** (🌶🌶→🌶🌶🌶, Mark, week 4)
- **The fundamental theorem of algebra and its many proofs** (🌶🌶🌶🌶, Jorge, week 4. Also in Analysis)

1. Algebraic combinatorics

Combinatorics is about counting things; algebraic combinatorics is specifically about using algebraic techniques to count things. In these classes, you will define some new objects (groups, formal power series, etc.) and use these definitions to prove exact formulas for how many things there are. Sometimes you can even go the other way, and use combinatorics to prove new things about algebra!

Classes in this theme:

- **A combinatorial proof of the Jacobi triple product identity** (🌶🌶🌶🌶, Gabrielle, week 1)
- **Representations of symmetric groups** (🌶🌶🌶🌶, Samantha, week 2. Also in Algebraic structures)
- **Combinatorial species** (🌶🌶🌶, Linus, week 2)

2. Algebraic structures

What do the Euclidean plane, the integers, and permutations have in common? The classes in this theme help you answer questions of this nature. They're all about finding common properties underlying many important mathematical objects of study, and proving facts about *any* objects that happen to share these properties. You'll explore the ways that these objects behave like you'd expect, and also the ways they can be truly bizarre and new and exciting.

Classes in this theme:

- **Introduction to group theory** (🌶🌶, Samantha, week 1)
- **Introduction to linear algebra** (🌶🌶, Emily, week 1)
- **Model theory** (🌶🌶🌶, Aaron, week 2)
- **Representations of symmetric groups** (🌶🌶🌶🌶, Samantha, week 2. Also in Algebraic combinatorics)
- **Introduction to ring theory** (🌶🌶🌶, Susan, week 2)
- **Finite fields and how to find them** (🌶🌶🌶, Viv, week 3)
- **Lights, camera, group actions!** (🌶🌶🌶, Emily, week 3. Also in Symmetries)
- **Surreal numbers** (🌶🌶, Aaron, week 3. Also in Games)
- **Noncommutative ring theory** (🌶🌶🌶🌶, Susan, weeks 3 & 4)
- **Archers at the ready!** (🌶🌶🌶🌶, Zoe, week 4)
- **Nonunique factorization in the Chicken McNugget monoid** (🌶🌶, Gabrielle, week 4)

3. Analysis

Analysis is all about limits: describing what happens to functions when you make very very very very small changes to the inputs. In these classes, you'll think about limits in all sorts of contexts: real numbers, complex numbers, higher dimensions, and applications to other areas of math.

Classes in this theme:

- **Multivariable calculus crash course** (🌶🌶🌶, Mark, week 1)
- **Functions of a complex variable** (🌶🌶🌶, Mark, weeks 2 & 3)
- **Introduction to analysis** (🌶🌶, Alan & Charlotte, week 2)
- **The calculus of variations** (🌶🌶🌶🌶, Ben, week 3)
- **The derivative as a linear transformation** (🌶🌶, Alan, week 4. Also in Smörgåsbord)
- **The inverse and implicit function theorems** (🌶🌶🌶, Alan, week 4. Also in Smörgåsbord)
- **Nowhere differentiable but continuous functions are everywhere!** (🌶🌶🌶, Charlotte, week 4)
- **The fundamental theorem of algebra and its many proofs** (🌶🌶🌶🌶, Jorge, week 4. Also in Smörgåsbord)
- **PDEs part 1: Laplace's equation** (🌶🌶🌶🌶, Assaf, week 4)

4. Computer science & algorithms

The study of telling computers how to solve problems began with Euclid over 2000 years ago. Much later, when computers appeared, people began to finally see the point. There's a ton of math that goes into making your computing devices work, and understanding how to make them faster, safer, and smarter. These classes explore many of the different ways people use math to understand how to do things with computers.

Classes in this theme:

- **Sparsest cut** (🌶🌶🌶, Alan, week 1. Also in Discrete analysis, Graph theory)
- **Introduction to quantum computing** (🌶🌶, Jorge, week 1. Also in Math in real life)
- **Cryptography and how to break it** (🌶🌶🌶, Linus, week 1. Also in Number theory)
- **Algorithms on your phone** (🌶🌶, *Agustin Garcia*, week 2. Also in Smörgåsbord, Math in real life)
- **Sit down and (don't) solve SAT?** (🌶, Zoe, week 2)
- **Factoring large prime numbers** (🌶🌶, Linus, week 3. Also in Number theory)
- **Learning online learning online** (🌶🌶, *Eric Neyman*, week 4. Also in Math in real life)

5. Discrete analysis

These classes are all about taking some ideas and perspectives from analysis (limits! inequalities!), but applying them to problems in *discrete* settings: you're more likely to think of things in terms of steps, rather than the smooth, gradual changes you might find in "regular" analysis. In this theme, you'll use techniques inspired by analysis to study questions in combinatorics, geometry, algebra, and more!

Classes in this theme:

- **Sparsest cut** (🌶🌶🌶, Alan, week 1. Also in CS & algorithms, Graph theory)
- **How to count primes** (🌶🌶, Viv, week 1. Also in Number theory)
- **Incidence combinatorics** (🌶🌶🌶🌶, Aaron, week 1. Also in Graph theory)
- **Continued fractions** (🌶🌶, Ben, week 1)
- **Kakeya sets over finite fields** (🌶🌶, Charlotte, week 1)
- **A pair of fractal curves** (🌶🌶, Ben, week 2. Also in Smörgåsbord)
- **Euclidean geometry beyond Euclid** (🌶🌶, *Yuval Wigderson*, week 2. Also in Smörgåsbord, Rigid shapes)
- **The probabilistic method** (🌶🌶, Mia, week 4. Also in Smörgåsbord, Graph theory)
- **Evolution of random graphs** (🌶🌶🌶, Misha, week 4. Also in Graph theory)
- **Finite Fourier analysis** (🌶🌶🌶, *Mike Orrison*, week 4)

6. Games

These classes are all about studying *strategies*. What does it take to win a game? Can you tell in advance whether you're guaranteed to win or lose? Can strategies change over time? And what happens if you're playing multiple games at once; is there a way of combining strategies together?

Classes in this theme:

- **Better sheep through modeling** (🌶, J-Lo, week 1. Also in Smörgåsbord, Math in real life)
- **The pirate game** (🌶, Ben, week 2. Also in Smörgåsbord)
- **Surreal numbers** (🌶🌶, Aaron, week 3. Also in Algebraic structures)

7. Graph theory

A graph is a collection of objects ("vertices") and connections between them ("edges"). There's a ton of interesting questions to study about graphs, and you can sample a few of them through these classes!

Classes in this theme:

- **Sparsest cut** (🌶🌶🌶, Alan, week 1. Also in CS & algorithms, Discrete analysis)
- **Incidence combinatorics** (🌶🌶🌶🌶, Aaron, week 1. Also in Discrete analysis)
- **Introduction to graph theory** (🌶🌶, Marisa, week 2)
- **Graph colorings** (🌶🌶, Mia, week 3)
- **The probabilistic method** (🌶🌶, Mia, week 4. Also in Smörgåsbord, Discrete analysis)
- **Evolution of random graphs** (🌶🌶🌶, Misha, week 4. Also in Discrete analysis)

8. Math in real life

One of the most important characteristics of mathematics is that it is *useful*; these classes will give you a taste for the many ways that math can explain and/or impact the world we live in.

Classes in this theme:

- **Better sleep through modeling** (🌶, *Olivia Walch*, week 1. Also in Smörgåsbord)
- **Better sheep through modeling** (🌶, J-Lo, week 1. Also in Smörgåsbord, Games)
- **Introduction to quantum computing** (🌶🌶, Jorge, week 1. Also in CS & algorithms)
- **Algorithms on your phone** (🌶🌶, *Agustin Garcia*, week 2. Also in Smörgåsbord, CS & algorithms)
- **The special theory of relativity** (🌶🌶🌶, Jorge, week 2)
- **The Schwarzschild solution** (🌶🌶🌶🌶, *Jon Tannenhauser*, week 3)
- **Lattices that make up the world** (🌶🌶, *Elizabeth Chang-Davidson*, week 3. Also in Rigid shapes, Symmetries)
- **Causal inference: how can we prove that X causes Y?** (🌶, *Mira Bernstein*, week 4)
- **Learning online learning online** (🌶🌶, *Eric Neyman*, week 4. Also in CS & algorithms)

9. Number theory

How well do you know the whole numbers, $1, 2, 3, \ldots$? Do you want to get to know them a whole lot better? These classes will help you to learn about the integers and many of their friends, using whatever tools we can get our hands on.

Classes in this theme:

- **Topics in number theory** (🌶🌶, Misha, week 1. Also in Smörgåsbord)
- **How to count primes** (🌶🌶, Viv, week 1. Also in Discrete analysis)
- **Cryptography and how to break it** (🌶🌶🌶, Linus, week 1. Also in CS & algorithms)
- **Dirichlet's class number formula** (🌶🌶🌶🌶, Viv, week 2)
- **Finite fields and how to find them** (🌶🌶🌶, Viv, week 3)
- **Quadratic forms** (🌶🌶, Gabrielle, week 3)
- **Factoring large prime numbers** (🌶🌶, Linus, week 3. Also in CS & algorithms)
- **What are your numbers worth? or, the part of algebraic number theory we can actually do** (🌶, Eric, week 3)

10. Rigid shapes

In these classes you'll think about shapes: how to slide them around, cut them up, and measure their lengths and angles. The classes you'll see in this theme range from familiar objects in the plane to much wilder shapes, where even just visualizing them is a challenge by itself.

Classes in this theme:

- **Euclidean geometry beyond Euclid** (🌶🌶, *Yuval Wigderson*, week 2. Also in Smörgåsbord, Discrete analysis)
- **Hilbert's 3rd problem** (🌶🌶🌶🌶, *Steve Schweber*, week 2)
- **Problem solving: geometric transformations** (🌶🌶, Misha, week 2. Also in Symmetries)
- **Myth of the 13 Archimedean solids (or, beware the pseudo-rhombicuboctahedron)** (🌶, Lizka, week 3)
- **Lattices that make up the world** (🌶🌶, *Elizabeth Chang-Davidson*, week 3. Also in Math in real life, Symmetries)
- **Non-Euclidean geometries** (🌶, Samantha, week 4)

11. Squishy shapes

In these classes you'll think about shapes: how they curve, bend, and stretch. Don't expect to find very many straight lines here! Some of these courses can be classified as "topology," as they involve identifying which pairs of shapes can be continuously deformed into each other.

Classes in this theme:

- **Insert geometry joke here** (🌶🌶, Zoe, week 1)
- **Topology through Morse theory** (🌶🌶🌶, *Kayla Wright*, week 2)
- **Curvature lies within** (🌶🌶, *Apurva Nakade*, week 3)
- **Using the Cantor set to classify (infinite) surfaces** (🌶🌶🌶, Assaf, week 3)
- **Knot theory** (🌶🌶, Emily, week 4)

12. Symmetries

Flip, rotate, translate — what are all the ways we can take a shape and do something to it so that it ends up looking exactly like it started? These classes study *symmetries* in a wide variety of different contexts.

Classes in this theme:

- **Problem solving: geometric transformations** (🌶🌶, Misha, week 2. Also in Rigid shapes)
- **Lights, camera, group actions!** (🌶🌶🌶, Emily, week 3. Also in Algebraic structures)
- **Lattices that make up the world** (🌶🌶, *Elizabeth Chang-Davidson*, week 3. Also in Math in real life, Rigid shapes)
- **Kleinian groups and fractals** (🌶🌶🌶, *Dan Gulotta*, week 3)
- **The 17 worlds of planar ants** (🌶🌶, *Dror Bar-Natan*, week 4. Also in Smörgåsbord)