CLASS DESCRIPTIONS—WEEK 5, MATHCAMP 2021

Contents

These blurbs don't have theme or class action information, but if you have questions, and you can't make a guess based on class format/blurb, feel free to ask the teacher or your AA!

9:10 Classes

Galois theory crash course (Mark, M$\boxed{\text{TW}\Theta\text{F}}$)
In 1832, the twenty-year-old mathematician and radical (in the political sense) Galois died tragically, as the result of a wound he sustained in a duel. The night before Galois was shot, he hurriedly scribbled a letter to a friend, sketching out mathematical ideas that he correctly suspected he might not live to work out more carefully. Among Galois' ideas (accounts differ as to just which of them were actually in that famous letter) are those that led to what is now called Galois theory, a deep connection between field extensions on the one hand and groups of automorphisms on the other (even though what we now consider the general definitions of "group" and "field" were not given until fifty years or so later). If this class happens, I expect to be rather hurriedly (but not tragically) scribbling as we try to cover as much of this material as reasonably possible. If all goes well, we might conceivably be able to get through an outline of the proof that it is impossible to solve general polynomial equations by radicals once the degree of the polynomial is greater than 4. (This depends on the simplicity of the alternating group, which we won't have time to show in this class.) Even if we don't get that far, the so-called Galois correspondence (which we should be able to get to, and probably prove) is well worth seeing!

Class format: Interactive lecture (over Zoom). I'll be using a document camera like a "blackboard" (and scanning the notes afterward), looking out at your faces even when you can't see mine (when I'm not actually writing, you will see mine), and asking questions to help us go through the material together.

Prerequisites: Group theory; linear algebra; some familiarity with fields and with polynomial rings.

Chilies	Homework
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Finite fields: the power of Frobenius (Eric, M$\boxed{\text{TW}}\Theta$F)
Many many aspects of the algebra of finite fields boil down to understanding a single map: the Frobenius homomorphism $x \mapsto x^p$! In this class we'll pick up where Viv's week 3 class left off and dive into the algebraic side of finite fields. We'll see how the Frobenius homomorphism and its powers are a tool for distinguishing between different finite fields, we'll see how they fit together, and we'll understand (in the 2 day version) what the algebraic closure of $\mathbb{F}_p$ is and the structure of its subfields.

Class format: Mixture of lecture and in-class worksheets.

Prerequisites: Viv's week 3 finite fields class or similar background.

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Fractal projections and a number theory question (Alan, M$\boxed{\text{TW}}$ΘF)
Let $K_0 \subset \mathbb{R}^2$ be the unit square. Divide K_0 into 16 squares of equal size, and let $K_1 \subset K_0$ be the union of the four corner squares. Repeat the same procedure on each of the four squares of K_1 to get K_2 (a union of sixteen squares), and so on. We define the four corner Cantor set to be the limit set $K = \bigcap_{n=0}^{\infty} K_n$.

In this class, we will discuss some interesting properties of the projections of the four corner Cantor set, including connections to the following number theory fact: If m and n are odd integers, then m/n can be written as the ratio of two numbers of the form $\sum_{j=0}^{\ell} \epsilon_j 4^j$, where $\epsilon_j \in \{-1, 0, 1\}$. (Incidentally, this number theory fact is proved in a paper called "An awful problem about integers in base four.")

Class format: Lecture

Prerequisites: None

Chilies	Homework
🌶️🌶️	HW Optional

Party parrot workshop (Zoe, M$\boxed{\text{TW}}$ΘF)
So many dancing parrots. The party begins!

Mathematical content will be provided.

Class format: IBL

Prerequisites: none

Chilies	Homework
🌶️	HW Recommended

Axiomatic music theory (J-Lo, MTW$\boxed{\Theta\text{F}}$)
What makes some combinations of notes more pleasant to listen to than others? Why does the chromatic scale have 12 notes? At first glance, music might seem like a collection of completely arbitrary facts that just coincidentally combine in ways that happen to sound nice.

But it turns out that many of these complicated musical constructions can be derived as corollaries from just a couple of basic conditions that we might want our music to satisfy! You will reach these conclusions for yourselves by working through a set of problems in groups. The rich, complex intricacy? That's actually number theory under cover!

Class format: Almost entirely problem-solving in breakout rooms (a bit of lecture at the start and end of each day)

Prerequisites: None, but the class will be much easier to follow if you have some musical background (being able to play an instrument or read sheet music should be sufficient)

Chilies	Homework
🌶️🌶️	HW Optional

Compactness in combinatorics of coloring (Aaron, MTW$\boxed{\Theta\mathrm{F}}$)
We take a look at some of the things you can do if you throw the Compactness Theorem at combinatorics problems that involve coloring graphs and hypergraphs. We'll see how this can let us understand colorings of infinite graphs using finite graphs, and more surprisingly, understand colorings of finite graphs using infinite graphs.

Class format: Lecture on virtual whiteboard with probably a bit of optional homework

Prerequisites: Model Theory, some graph theory

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Ultra-fantastic ultra filters (Zoe, MTW$\boxed{\Theta\mathrm{F}}$)
We want to figure out what it means for a subset of the Reals to be big! We will discuss what are the ideal properties for a 'Big' set to have. The surprising aspect is that when we have a good list of these properties it is actually hard to show that there exists an object with these properties! These are surprisingly useful objects in set theory and combinatorics.

Class format: Lecture

Prerequisites: none

Chilies	Homework
🌶️–🌶️🌶️🌶️	HW Recommended

10:10 Classes

Arithmetic progressions and primes and parrots (Viv, $\boxed{\mathrm{MTW}}\Theta\mathrm{F}$)
Did you know that there are infinitely many primes?

This fact has been proven many times in many different ways, but in this class we're actually going to ask a different question.

Did you know that there are infinitely many primes congruent to $a \pmod q$, as long as a and q are relatively prime?

Dirichlet showed this in the 1900s using some of my favorite ideas in all of mathematics. The ideas that he used include group theory, complex analysis, and his class number formula (see week 2). The ideas that we will use. . . are a subset of those, but a pretty big subset.

Class format: Lecture

Prerequisites: Familiarity with complex numbers. Some knowledge of groups and basic complex analysis (such as the word "pole") is helpful.

Chilies	Homework
🌶️🌶️🌶️	HW Recommended

Supervised machine learning: the essentials (Jorge, $\boxed{\mathrm{MTW}}\Theta\mathrm{F}$)
Machine learning (ML) is a type of artificial intelligence (AI) that allows software applications to become more accurate at predicting outcomes without being explicitly programmed to do so. One subfield of ML is known as *supervised learning*, in which we use historical data (e.g. past measurements) as inputs in order to obtain these predictions.

In order for ML to pull off the futuristic cool stuff one day (such as self-driving cars), we'll need to start small. An example: I have a polynomial curve of unknown degree (but let's say the degree is less than 10), and I give you the coordinates of 10 points on the curve. We want to find a polynomial formula that accurately describes these points and any future points on my curve that I give you. How do we go about this?

The tricky part about it is that solving the problem exactly (via the so-called Lagrange interpolation formula) won't necessarily give you a better solution. In fact, it is likely to give you something terrible that won't be even close to how the actual curve looks like, due to a phenomenon called *overfitting*.

In order to solve the problem correctly, the class will go over the essentials of supervised learning: doing a linear regression in order to find suitable formulas and using *regularization* to avoid overfitting. What do all these buzzwords mean? Come to class and find out!

Class format: Lecture

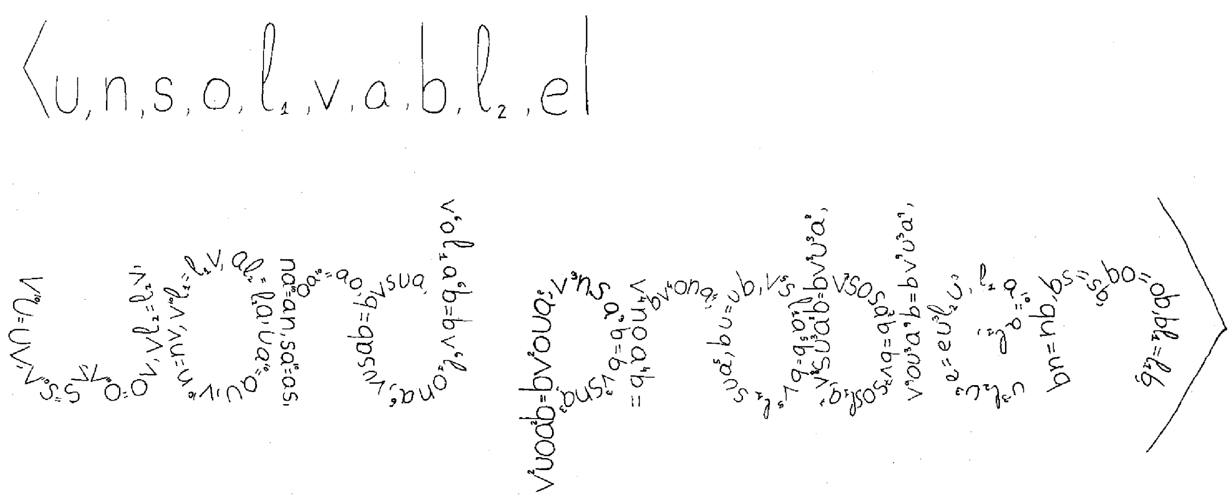
Prerequisites: Linear algebra (vector spaces, bases, linear combinations, matrix equations) and some probability (knowing about Gaussian distributions and conditional probability)

Chilies	Homework
	No HW

The word problem for groups (Assaf, MTW ΘF)

A group can be thought of as a collection of reversible operations, with some rules about how they relate to each other. Such a way of thinking is called a *group presentation*, and as an example, we have that the cyclic group of order 3 can be written as $\langle x | x^3 = e \rangle$, where x is a generator, and $x^3 = e$ is a relation. Given a group presentation with a finite set of generators and relations, does there always exist an algorithm to tell if a word in the generators is the identity? This is called the Word Problem, and it was posed by Max Dehn in 1911. The surprising answer, shown by Pyotr Novikov in 1955, is no - there does not exist such an algorithm. In this class, we will prove this result by studying how we can embed "uncodable" sets in groups. This will give us a candidate for a "bad" group for which a solution to the word problem would contradict the "uncodablity" of the set.

Additionally, this class comes with a really sweet t-shirt design that we should totally reprint this year, and which contains a group presentation that has unsolvable word problem (we will not prove that this specific presentation does not have solvable word problem in this class):



Class format: Lectures with some homework

Prerequisites: group theory

Chilies	Homework
🌶️🌶️–🌶️🌶️🌶️	HW Recommended

Vitali's curse (Gabrielle, MTW ΘF)
I failed my real analysis qualifying exam in fall 2019 because I did not read the problem and answered a far easier question than what I had been asked. (Pro tip: Don't do that.) And because I didn't have the Vitali Covering Lemma in my heart. As punishment, I have been doomed to roam the earth, telling people about the Vitali Covering Lemma. We'll learn a little bit of measure theory, answer the question I thought I was supposed to answer, prove the Vitali Covering Lemma, and finally vindicate Past Gabrielle by solving the problem I was supposed to solve. Time permitting, we'll see less redemption-based applications.

Class format: Lecture

Prerequisites: Failure. Also helpful to have comfort with epsilon-delta definitions of continuity and differentiability.

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Conjugation in the symmetric group (Emily, MTW ΘF)
Two elements x, y of a group G are said to be conjugate if there exists a $g \in G$ such that $gxg^{-1} = y$. If we consider the set of all elements that are conjugate to x in G, this is called the conjugacy class of x. Now if we let G be the symmetric group S_n, the conjugacy classes partition S_n is a rather nice way - by cycle type of the permutations. In this class, we will prove this beautiful fact, as well as find a formula for the size of each conjugacy class.

But we won't stop there! What happens if we restrict the g's used above to be elements of a subgroup of G, say A_n? Will the conjugacy classes in A_n also be completely determined by cycle type? Come to this class to find out!

Class format: Interactive lecture

Prerequisites: Group theory—should be comfortable with the symmetric group and cycle notation.

Chilies	Homework
🌶️🌶️🌶️	No HW

Perfection (Mia, MTW ΘF)
Commutative algebraists have their excellent rings and algebraic geometers have their wonderful compactifications, but no one achieves perfection like graph theorists. In this class, we will prove none other than the Perfect Graph Theorem which, in addition to having an excellent[1] name, has an exceedingly clever proof.

So, what is perfection? In Graph Colorings, we proved that $\omega(G) \leq \chi(G)$ and then asked, can we push those two invariants arbitrarily far apart. An alternative question one might ask is, what graphs achieve equality? Or even better, which graphs achieve equality and have that their subgraphs achieve equality too? The answer, perfect graphs! And what's more, the Perfect Graph Theorem gives us an elegant characterization of these graphs.

Note: Graph Colorings is not a prerequisite.

Class format: Interactive Lecture

[1] Excellent under the English definition, not the algebraic one.

Prerequisites: Graph Theory

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Taxicab geometry (Samantha, MTW$\boxed{\Theta\text{F}}$)
Taxicab geometry is the geometry resulting from taking the Euclidean plane but defining the distance between points (a, b) and (c, d) to be $|a-c|+|b-d|$. It turns out that this geometry has very different properties from Euclidean (and non-Euclidean) geometry! For example, circles are now square-shaped! In this class, you'll explore some of the basic properties of taxicab geometry; this class will be structured very similarly to my week 4 non-Euclidean course, but with more proofs!

Class format: IBL

Prerequisites: None

Chilies	Homework
🌶️	No HW

The quantum factorization algorithm (Jorge, MTW$\boxed{\Theta\text{F}}$)
What makes quantum computing special, apart from just providing us with an overly complicated way of transmitting information? I'm so glad you asked...

Just before the turn of the century, Peter Shor came up with an algorithm that can use quantum computers to factor numbers in $O(b^3)$ time, where b is the number of bits of the number being factored. In comparison, not even the state-of-the-art algorithms on classical computers are able to factor arbitrary numbers in polynomial time. Because of this, and given that the most common cryptographic algorithm (RSA) depends on the intractability of factoring large pseudo-primes by brute force, there is a large interest in quantum computers from a security standpoint.

Shor's paper is very interesting in its own right, and its study can be split into two. First, we study the "classical" part of the algorithm, which shows how we could factor numbers quickly if we were able to find the order of numbers with respect to the right modulus. The quantum part of the algorithm is precisely how to find these orders, and introduces the very interesting quantum Fourier transform.

Class format: Lecture

Prerequisites: "Intro to quantum computing" week 1 class. Linear algebra: bases, dimension, and the matrix representation of an operator. Also, some familiarity with elementary number theory: divisors, primes, modular arithmetic.

Chilies	Homework
🌶️🌶️🌶️🌶️	No HW

12:10 CLASSES

A combinatorial proof of "the" quintic formula (J-Lo, $\boxed{\text{MT}}$WΘF)
If we are only allowed to use $+$, $-$, $\times$, $\div$, and $\sqrt[n]{\ }$ for any n, it is impossible for us to write down the roots of a general quintic equation. However, if we allow ourselves to use other functions, then solutions do exist! One of these solutions uses the extra function

$$F(x)=\sum_{k=0}^{\infty}\binom{5k}{k}\frac{x^{4k+1}}{4k+1}.$$

We will find a combinatorial proof of this version of the quintic formula using a generalization of the Catalan numbers.

Class format: Interactive lecture

Prerequisites: None

Chilies	Homework
🌶️🌶️🌶️	HW Recommended

Ben's favorite game theory result (Ben, $\boxed{\text{MT}}$WΘF)

In my Week 2 class "The Pirate Game," I spent several minutes bad-mouthing the Prisoners' Dilemma, saying that I just found it utterly boring and completely, profoundly devoid of anything resembling interest.

That might have been a bit harsh, and I'm here now to also admit that it was a bit unfair. I do know of one theorem that makes the Prisoners' Dilemma a little more interesting—well, a LOT more interesting, truth be told. This is one of the so-called "Folk Theorems" of game theory.

Usually, when we analyze sequential games, we start at the "end" of the game and work backwards. But what if we don't know when the game is going to end? In practical "games" that occur in "real life," this is often the case because most of us cannot predict the future.[2] It turns out that analyzing such games is much more difficult, and therefore much more interesting.

In this class, we'll talk more about credible threats, about how to even start thinking about "infinite games" (i.e. games where there's no fixed end point), and a kind of strategy called a "grim trigger." Time permitting, we'll also talk about why all the game theorists HATE Ben's Favorite Game Theory Result. If even more time permits, we might even talk about mercy, forgiveness, and grace, but I really can't make any promises on that front.

Class format: Mostly interactive lecture

Prerequisites: Familiarity with geometric series

Chilies	Homework
🌶️	HW Optional

Sperner's lemma and Brouwer's fixed point theorem (Aaron, $\boxed{\text{MT}}$WΘF)

Take your favorite triangle, and paint its corners red, green and blue. Now subdivide it into a lot of tiny triangles, and paint all the corners of all of those with the three colors, so that each color on the edge between red and blue is red or blue, and each color on the edge between green and blue is green or blue, and each color on the edge between red and green is red or green.

Sperner's Lemma says there is at least one tiny triangle with a corner of each color.

Take your favorite continuous map from the disk to itself. Brouwer's Fixed Point Theorem says it has a fixed point.

In this IBL class, we will prove this very discrete lemma, and use it to prove this very continuous theorem.

Class format: IBL: We will solve problems in breakout rooms.

Prerequisites: Sperner's Lemma will require just a bit of graph theory, Brouwer's FPT will require the idea of Cauchy sequences.

Chilies	Homework
🌶️🌶️	HW Optional

[2]If you can predict the future, please contact me exactly one year ago to let me know.

There are less than 10^{39} Sudoku puzzles (Linus, $\boxed{\text{M}}$TWΘF)
We'll introduce "entropy," which measures how much information you learn when you reveal the value of a random variable. We'll use it to upper bound $\binom{N}{k}$ as well as the number of 9-by-9 Sudoku puzzles.

Class format: Lecture

Prerequisites: linearity of expectation

Chilies	Homework
🌶️🌶️🌶️🌶️	No HW

A property of a^n (Mia, M$\boxed{\text{T}}$WΘF)
Suppose I asked you to give me a power of two starting with '6'; you'd probably quickly reply '64'. And if I asked you to give me a power of two starting with '41', well, that get's a little trickier, but perhaps you know that $2^{22} = 4194304$. However, if I asked you to find me a power of two starting with '616263646566', that's a whole other story. Rather remarkably though, such a power exists. In fact, for any sequence of digits S (which is not a power of 10), there exists a power of two that starts with those digits! The delightful proof comes from Ingenuity in Mathematics and will be the starting point for the class.

Class format: Interactive lecture

Prerequisites: None

Chilies	Homework
🌶️	No HW

Arithmetic progressions and primes and parrots (Charlotte, MT$\boxed{\text{WΘF}}$)
The Green–Tao Theorem is a celebrated result in mathematics: the primes contain arbitrarily large arithmetic progressions. That is, for any $k \in \mathbb{N}$, the set of primes contains some sequence of points in the form a, $a + d$, $a + 2d, \ldots, a + kd$.

In this class we'll look at something much easier to prove, that the primes get arbitrarily close to arbitrarily long arithmetic progressions. Proving this should help shed some light on why the Green–Tao Theorem is true – and we'll see that it has a lot to do with the "size" of the set of prime numbers.

Consequently, we'll spend some time looking at a variety of ways to define the "size" of a subset of the natural numbers, and consider whether or not sets that are large or small in these notions of size should or could contain arbitrarily large arithmetic progressions.

Class format: Mostly interactive lecture with some time for problem solving

Prerequisites: You should be comfortable with limits and infinite series. It would be helpful if you've already seen sups, infs, and limsups before, but I'll introduce these quickly.

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Draw every curve at once (Ben, MT$\boxed{\text{WΘF}}$)
Isn't it so tiresome to have to draw different things at different times? When I want to write an "A,"

[3] For sufficiently inconvenient values of convenient.
[4] For sufficiently impossible values of possible.
[5] For sufficiently difficult values of easy.
[6] For sufficiently… weird values of curve, of course

I have to use an entirely different process from when I write a "B," and wouldn't it be a lot more convenient if I could just draw everything at once? Every single curve imaginable, all packed into one shape?

Not only would this be marvelously convenient[3], it is also possible[4] and easy[5]! You, too, can draw a universal curve[6] and in this class, we'll see how!

Class format: Interactive Lecture

Prerequisites: Knowing what metric spaces are

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Martin's axiom (Susan, MT $\boxed{\text{W}\Theta\text{F}}$)

You probably know that there are different sizes of infinity. The real numbers are provably larger than the natural numbers. In fact, the power set of any set A is larger than A. In fact in fact, the power set of $\mathbb{N}$ turns out to be precisely the size of $\mathbb{R}$. This begs the question: are there sizes of infinity between $|\mathbb{N}|$ and $|\mathbb{R}|$?

In this class, we will not answer that question. We will blow right past that question, assume that we're in a universe with intermediate sizes of infinity, and ask ourselves: how do those intermediate sizes of infinity behave? Using an extra-set-theoretic axiom called Martin's Axiom, we can show that intermediate sizes of infinity behave in several crucial ways like the natural numbers.

If you've ever looked at an induction proof and thought, "Man, this is cool and all, but I really wish it involved more posets," this could be the class for you!

Class format: lecture

Prerequisites: none

Chilies	Homework
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Mechanical computers (Tim! Black, MT $\boxed{\text{W}\Theta\text{F}}$)

A few years ago, I came across a YouTube video about a game called Dr. Nim (https://youtu.be/9KABcmczPdg). It's a little computer that can play a simple game against a human with perfect strategy. It's a fairly simple game — one-pile nim — but the amazing part is that there are no electronics. Just using mechanical pieces and gravity, the "computer" chooses how many marbles to drop, and drops them all on its own. It's not the first example of a mechanical computer — people started dreaming up computers long before modern electronics. But is Dr. Nim really a computer at all? It can really only do one thing.

A new game came out in 2018 called Turing Tumble (https://www.turingtumble.com/). It's a marble contraption similar to Dr. Nim, but has components that can be plugged in and arranged on a big peg board. It can do a lot of things; for example, it can do addition and multiplication, and it can even simulate Dr. Nim! Does this count as a computer? It certainly can't output to an electronic display. What does it mean to be a computer?

In this class we'll:

- Look at early attempts at inventing computers (back when mechanical contraptions were the only option).
- Play with Turing Tumble (you can use an online simulator) and see what we can get it to do.
- Discuss what it means to be a computer, and define *Turing Completeness* as one answer to this question.
- See other examples of Turing Complete Systems, and try to prove that Turing Tumble is Turing Complete.

Class format: Lecture/Discussion

Prerequisites: None

Chilies	Homework
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1:10 Classes

Completeness of the real numbers (Alan & Charlotte, $\boxed{\text{M}}$TWΘF)
Our Week 2 Introduction to Analysis class was *full of holes* (:partyparrot:) because there were many topics we did not have time to cover (:actualsadparrot:). This class is an attempt to *fill in some holes* (:partyparrot:) by discussing the completeness of the real numbers. Recall that the rational numbers are full of holes! The way to fill in the holes is by "completing" them, thus obtaining the real numbers. This can be done in various ways, including via Cauchy sequences, monotone sequences, least upper bounds, and more. In this class, we will discuss these various ways to think about completeness of the reals.

Class format: The class is mostly lecture-based. We'll spend some time in breakout rooms discussing problems.

Prerequisites: The Week 2 analysis class, or the Week 4 nowhere differentiable but continuous functions class; more specifically, you should know the epsilon-delta definitions of convergent sequences and Cauchy sequences.

Chilies	Homework
🌶🌶	HW Optional

Counting, involutions, and a theorem of Fermat (Mark, $\boxed{\text{M}}$TWΘF)
Involutions are mathematical objects, especially functions, that are their own inverses. Involutions show up with some regularity in combinatorial proofs; in this class we'll see how to use counting and an involution, but no "number theory" in the usual sense, to prove a famous theorem of Fermat about primes as sums of squares. (Actually, although Fermat stated the theorem, it's uncertain whether he had a proof.) If you haven't seen why every prime $p \equiv 1 \pmod 4$ is the sum of two squares, or if you would like to see a relatively recent (Heath-Brown 1984, Zagier 1990), highly non-standard proof of this fact, consider taking this class.

Class format: Interactive lecture (over Zoom). I'll be using a document camera like a "blackboard" (and scanning the notes afterward), looking out at your faces even when you can't see mine (when I'm not actually writing, you will see mine), and asking questions to help us go through the material together.

Prerequisites: None

Chilies	Homework
🌶️🌶️	No HW

Is math real? (Quinn Perian, $\boxed{\text{M}}$TWΘF)
In the philosophy of math, realism is commonly explained to be a position according to which mathematical objects exist in a sense independent of human thoughts or practices (though this precise definition is itself far from cut and dry). How exactly can we interpret what it means for a mathematical object to exist in this sense? What reasons are there in favor of a realist position? How about against a realist position? What are the alternatives to mathematical realism? In this class, we will try to provide some answers to these questions, and see how different philosophers over time would answer the question "Is math real?"

Class format: The class will be mainly interactive lecture, consisting of lecture along with several opportunities for campers to contribute their own opinion to the discussion of various philosophical positions (on mic or in chat).

Prerequisites: None.

Chilies	Homework
🌶️	No HW

Problem solving: tetrahedra (Misha, $\boxed{\text{M}}$TWΘF)
In the nine years from 1964 to 1972, every IMO competition contained a question with a tetrahedron in it. Since then, no such question has showed up again. In this class, we go back to the halcyon days of yore and solve as many of these problems as we can.

Class format: Extra-interactive lecture, with a slide for every problem that I'll annotate as we solve it.

Prerequisites: None.

Chilies	Homework
🌶️🌶️🌶️🌶️	HW Optional

How to ask questions (Eric, M$\boxed{\text{T}}$WΘF)
In this class you will learn about asking questions and also ask questions, though possibly not in that order. You will have the opportunity to learn practical wisdom on how to ask questions in a mathematical context and how to be intentional about your question asking.

Your homework will be to ask questions, in this class and others.

Class format: Inquiry based learning during lecture about learning through inquiry during lectures.

Prerequisites: None!

Chilies	Homework
🌶️	HW Required

Introduction to auction theory (William Ding, M$\boxed{\text{T}}$WΘF)
There are many ways to sell stuff, and often the price of the stuff that's being sold is not fixed. In this class, we will analyze common auction formats and optimal bidding under a few convenient assumptions. We'll then loosen those assumptions to begin to understand auctions in the real world, specifically the government-run "spectrum auctions" of the '90s and '00s: some were record-breaking

successes, while others, for reasons that we will explore, were so disappointing as to warrant antitrust investigations.

Class format: Interactive lecture

Prerequisites: None

Chilies	Homework
🌶️	No HW

Problem solving: linear algebra (Misha, M$\boxed{\text{T}}$WΘF)

Most high school math contests (the IMO included) do not use any topic considered to be too advanced for high school, such as linear algebra. This is a shame, because there have been many beautiful problems about linear algebra in undergraduate contests such as the Putnam Math Competition.

In this class, we will look at linear algebra from a new perspective and use it to solve olympiad problems.

Class format: Extra-interactive lecture, with a slide for every problem that I'll annotate as we solve it.

Prerequisites: None.

Chilies	Homework
🌶️🌶️🌶️	HW Optional

Symmetries of a (hyper)cube (Emily, M$\boxed{\text{T}}$WΘF)

The dihedral group D_4 can be described as the symmetries of a square, which has four rotations and four reflections. We can bump this up to the next dimension and construct a group that is the symmetries of a cube. And bump it up again to a 4th dimension cube. And again and again for every *hypercube*, or n-dimension cube. We will build up the elements of this group when $n = 3$, find the size of this group for any n, and look at a nice way of representing its elements as permutations (but not the kind you are used to!)

Class format: Interactive lecture

Prerequisites: Group theory—know what the symmetric and dihedral groups are

Chilies	Homework
🌶️🌶️	HW Optional

Computability theory and finite injury (Atticus Cull, MT$\boxed{\text{W}}$ΘF)

Computability theory is what you would get if you were to do complexity theory without caring about the efficiency of your algorithms. Instead of bounding ourselves by polynomials, we indulge in the full extent of algorithms. Questions you might consider are, can you come up with a description for every subset of $\mathbb{N}$? Is being able to list the elements of a set the same as being able to tell what's in the set and what isn't? This class will explore fundamental limitations of computation, what it means for sets of naturals to compute each other, and a curious partial order on $\mathcal{P}(\mathbb{N})$, culminating in my hands down favorite proof method: finite injury. It's not as scary as the name suggests - we could be doing infinite injury!

Class format: Lecture

Prerequisites: None

Chilies	Homework
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Problem solving: lecture theory (Misha, MT$\boxed{\text{W}}\Theta$F)
This class will teach you about the dark side of problem-solving: how to make educated guesses, how to use problem statements to your advantage, and how to exploit the one piece of extra information contest writers can't help giving you: that the problem has an answer.

(Due to its nature, this class is primarily focused on US contests like the AMC, AIME, and ARML, where you don't have to prove that your answers are correct.)

Class format: Extra-interactive lecture, with a slide for every problem that I'll annotate as we solve it.

Prerequisites: None.

Chilies	Homework
🌶️	HW Optional

The mathematics of voting (Samantha, MT$\boxed{\text{W}}\Theta$F)
What is the fairest way to vote on something? Perhaps we merely have everyone write down their top choice and the winner is whatever choice received the most votes. Or maybe we allow each voter to rank their preferences, and then we count each preference in a weighted way and allow that to determine the winner. It turns out, neither of these methods is fair, according to economist Ken Arrow's definition of "fair." In this class, we'll consider a few examples of voting systems and their unfairness, then we'll prove Arrow's Impossibility Theorem.

Class format: Lecture

Prerequisites: None

Chilies	Homework
🌶️	No HW

Traffic and the price of anarchy (Assaf, MT$\boxed{\text{W}}\Theta$F)
If we all took the bus, the roads would be empty, and transit would be fast. And yet, for some reason, it always seems like there is gridlock. Why can't we all just co-operate? Why do drivers always have to cut me off when I'm biking to campus? Why is it that closing roads produces better traffic in the rest of the city? How is all of this related to COVID-19 and vaccines?

Everyone wants to be rational, but sometimes our irrational human nature comes out and bites the collective. In this class, we'll explore scenarios where this effect happens. We'll look at Braess' Paradox, the Bus Motivation Problem, and spend some time discussing the formalism of congestion games.

Class format: Lecture

Prerequisites: know what a bus is

Chilies	Homework
🌶️	HW Recommended