CLASS DESCRIPTIONS—WEEK 2, MATHCAMP 2022

Contents

9:10 Classes

Algorithms for large primes (🌶🌶, *Zach Abel*, TWΘFS)
Much of modern internet security relies on a counterintuitive principle: **testing** whether large numbers are prime is fast, but **factoring** those same numbers is believed to be infeasible, even with state-of-the-art supercomputers and factoring algorithms.

For example, consider this 617-digit number n:

3049393803906409820462572243298853574672149664378108215389188696453420214699722967584199470131652491
3849210517415875076785196312119495759970859252434309129302173156352106846709170430429056753647687903
1227528692058927690483709214285585719241101990073778161131981122159963106459662254167802232291640108
9348914343202481190896533900420837116144945653222123954830825359910625724337519235659570699858976093
3034168762845787208048115384026599867498109469257288083679805389339036591501281524285494832182868787
4342301743019419306688013850612219622243010119848476991152725406666046444056748106004723607644097968
61925466465327459.

This number n is **not** prime and $n + 8$ **is** prime, and a typical laptop can **verify** both of these facts in fractions of a second. By contrast, the technology to **factor** n (and numbers like it) into primes does not yet exist, and most encrypted communications (in particular, most internet traffic) depends on this fact! The example n above is copied directly from the public certificate that protects `https://www.amazon.com`, but this security could be breached by anyone who can factor n into primes, so Amazon and all of its users rely on this not being feasible.

To factor a large number and/or test whether it is prime, the naïve "trial division" algorithm considers all potential factors individually: "is it divisible by 2? 3? 4? 5? etc." But for numbers with hundreds of digits, this is way too slow, since the universe will literally suffer heat death before this algorithm makes noticeable progress.

So how is it possible to conclude that a large number (like n) is composite *without* factoring it? How can we be sure that a large number (like $n+8$) is prime *without* testing all of its possible prime factors? We'll explore clever algorithms that enable efficient tests like these, and the elegant underlying number theory. Topics may include: primality certificates; probable vs provable primes; the Great Internet Mersenne Prime Search; generating large primes; the AKS primality test.

Homework: Optional

Class format: Lecture

Prerequisites: Modular arithmetic: should understand modular inverses and Fermat's Little Theorem. I plan **not** to assume or use any knowledge of abstract algebra.

Extremal graph theory (🌶🌶🌶🌶, Yuval, TWΘFS)

A basic fact in graph theory is that every tree with n vertices has exactly $n-1$ edges. Said differently, if an n-vertex graph has no cycles, then it has at most $n-1$ edges.

What if, rather than excluding *all* cycles, we only exclude the triangle C_3? Then suddenly we can put in a lot more edges: the complete bipartite graph $K_{\lfloor n/2\rfloor,\lceil n/2\rceil}$ has $\lfloor n^2/4\rfloor$ edges and no C_3. This really is a *lot* more edges—rather than growing linearly in n as before, it now grows quadratically.

Can we put in any more edges? It turns out we can't: $\lfloor n^2/4\rfloor$ is the most edges a C_3-free n-vertex graph can have. We'll prove this on the first day of class.

What if, instead of excluding C_3, we exclude the five-cycle C_5? It turns out that again, the most edges we can have is $\lfloor n^2/4\rfloor$. This result is way too hard to prove in a Mathcamp class, though we will prove a slightly weaker version of it. The exact same thing is true if instead we exclude C_7, C_9, or any other odd cycle.

What if, instead, we exclude C_4? Somewhat shockingly, the answer is suddenly a lot smaller: the maximum number of edges in a C_4-free n-vertex graph is around $n^{3/2}$, which grows much more slowly than the quadratic behavior we saw before. We'll prove this on the second day of class. Similar techniques allow one to show that if we exclude C_6, the answer grows like $n^{4/3}$.

So now we know the answer if we exclude C_3, C_4, C_5, C_6, or C_7. What happens if we exclude C_8? ***No one has any idea.***

This class will be an introduction to the wild wild world of extremal graph theory, where the problems are simple, the techniques are beautiful, the results are deep and powerful, and there are a million natural questions that seem completely impossible to answer.

Homework: Recommended

Class format: Lecture

Prerequisites: Graph theory

On beyond i (🌶🌶, Steve, TWΘFS)

There is a nice progression of number systems, $\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R} \to \mathbb{C}$: we start with the natural numbers, and at each stage we fix some problem. So, for example, we go from $\mathbb{Q}$ to $\mathbb{R}$ to "fill in the holes," and we go from $\mathbb{R}$ to $\mathbb{C}$ so that equations like $x^2+1=0$ will have solutions. Once we get to $\mathbb{C}$, though, we seem to be done: there are no holes as in the case of $\mathbb{Q}$, and the fundamental theorem of algebra tells us that every polynomial which is not constant already has a root over $\mathbb{C}$. So there's no need to keep going.

So let's keep going! Having only *one* square root (up to $\pm$) is boring. We want more! There are number systems past the complex numbers—strange things like the quaternions, octonions, and sedenions—which satisfy this perfectly normal craving. In this class, we'll begin by playing around with these systems, and then turn to the underlying bit of abstract mathematics which lets us build these and many others. Oh, and we'll also look at reasons why someone might be interested in these systems *other* than curiosity and a love of the bizarre.

Homework: Recommended

Class format: Lecture

Prerequisites: Complex numbers (but not complex analysis!), knowing the definition of a ring is helpful but not necessary

Ring theory (🌶🌶, Kayla, TWΘFS)

If you like it, you should ideally put a ring on it! When we first learn about number systems, we learn the basic operations: addition, subtraction, multiplication, division. If we lose the context of strictly looking at integers, real or complex numbers, for which sets can we still do these operations? Rings are algebraic structures in which addition and multiplication exist and act as we'd expect. This abstract way of thinking about algebraic structures is the backbone of many other interesting topics (and classes to come at camp!) such as commutative algebra, algebraic geometry, representation theory, field and Galois theory. In this class, we will see a quick introduction to the beautiful world of ring theory.

Homework: Recommended

Class format: Lecture

Prerequisites: None

Required for: Commutative algebra and algebraic geometry (W3); A curious connection between p-adic distances and triangulations of a square (W4); Finite fields (W4); Introduction to Galois theory (W4)

The residue theorem (🌶🌶🌶, Kevin, TWΘFS)

Complex analysis studies functions whose input and output are both complex numbers $z = a+bi$ rather than real numbers. Many of the same concepts that come up in calculus extend to the complex setting, but miraculous things start to happen! For example, the residue theorem says that integrals around a closed curve in the complex plane can be evaluated simply by studying the function's behavior near the points inside the curve where it's undefined. This result is tremendously important not only in complex analysis, but also in other fields of math from combinatorics to number theory. It even helps us to evaluate real integrals! In this class, we'll start with the definition of complex differentiation and build our way up to this remarkable theorem and several related results and applications.

Homework: Recommended

Class format: Lecture. The (recommended but not required) homework will guide you through some proofs that we won't cover in detail during lecture: you'll be able to follow the lectures without doing the homework, but the homework will be necessary if you want to prove everything we cover.

Prerequisites: Single-variable calculus (derivatives, integrals, and power series); the multivariable calculus that we need will be covered in the class/homework.

10:10 Classes

Bonus group theory part 2 (🌶🌶🌶, Ben, TWΘFS)

In Susan's group theory class[1], you learned about Lagrange's theorem, which says that if H is a subgroup of a finite group G, then the size of H divides the size of G.

We might wonder about the converse of this—suppose we have a group G of order, say, $12 = 2^2 \cdot 3$. Does G necessarily have subgroups of orders $1, 2, 3, 4, 6$, and 12? If it does, how many of these subgroups can it have?

In this class, we'll discuss the Sylow theorems, which (among other things) tell us that our G has to have subgroups of order $2, 3$, and 4. (If you're wondering about 6, it turns out there is a group of order 12 with no subgroup of order 6.)

These theorems are also useful for some classification problems that we'll investigate, for instance:

- How many groups of order $15 = 3 \cdot 5$ are there?
- Why is this different than the number of groups of order $21 = 3 \cdot 7$?
- And why do I keep writing out prime factorizations?

Homework: Recommended

Class format: Lecture

Prerequisites: Group theory—Susan's class covers all of the needed background (groups, subgroups, Lagrange's theorem, normal subgroups)

[1] Or in whatever class you first met groups in, I'm willing to bet.

Equidistribution (🌶️🌶️, Viv, TWΘFS)
What does the sequence

$$0.1415..., 0.2831..., 0.4247..., 0.5663..., 0.7079..., 0.8495..., 0.9911..., 0.1327..., ...$$

have that the sequence

$$0.6666..., 0.3333..., 0, 0.6666..., 0.3333..., 0, ...$$

doesn't?

Well, a lot more terms, for one! The first sequence above is the portion after the decimal point of multiples of π, and the second is the same thing for multiples of $\frac{2}{3}$. If we kept going, we'd find that the decimal portions of multiples of π defy simple categorization: as a set, they don't stay in specific sections of the interval $[0, 1)$ (or on specific points like $0, 0.3333...$ and $0.6666....$), but rather move all over the place, a lot. In fact, this sequence seems to love every part of the interval $[0, 1)$ equally! This property is known as *equidistribution.* In this class, we'll define and build up intuition for the concept of equidistribution of sequences mod 1. We'll prove a beautifully simple way of checking that a sequence is equidistributed, and then we'll use it to show that the sequence $(n\alpha \bmod 1)_{n\geq 1}$ is equidistributed mod 1 if and only if α is irrational.

Homework: Recommended

Class format: Lecture

Prerequisites: Single-variable calculus (especially limits, sequences, and series), complex numbers (i.e. comfort with i and $e^{2\pi i\theta}$ being the unit circle, but no more).

Fractal geometry (🌶️🌶️, Steve, TWΘFS)
The usual three dimensions are fun and all, but they get kind of boring after a while. One way to liven things up is to add more dimensions; billion-dimensional shapes are probably super cool! But you know what I like even more than big numbers? *Wrong numbers.* I want a two-and-a-half-dimensional shape. Or a π-dimensional shape. Or a shape with a decent number of dimensions, but for terrible reasons.

It turns out that we can make this happen! The answer is *fractals*, a particularly weird and beautiful kind fo shape. Fractals crop up throughout mathematics in all sorts of weird ways, and have lots of fascinating properties *besides* just being dimensionally weird. This class will be about what dimensions are, why fractals have silly numbers of them, and how awesome that is.

Homework: Optional

Class format: Lecture

Prerequisites: None

Lehmer factor stencils (🌶️, Aaron and Eric, TWΘFS)
In the 1920s, one of the fastest known ways to factor large numbers was with Lehmer Factor Stencils. If you wanted to factor, say, 1229209, you could go to one of a few libraries, borrow a massive set of paper stencils, and then start doing calculations with an adding machine. After a while, your calculations might pop out the numbers $-21, -5, 11, -2, 103, 3$. You lay stencils labelled $-21, -5, 11, -2, 103, 3$ on top of each other, and amid the grid of numerically-labelled holes, light shines through exactly one, labelled 827. You check if 1229209 is divisible by 827, it is not divisible, and you conclude that 1229209 is prime.

Obviously this is not the fastest way to factor large numbers anymore, but in this class, we'll go back in time, grab our stencils, and factor away. Along the way, we'll learn how to create our own set of stencils, and how holes in paper can know so much about factoring numbers.

Homework: Optional

Class format: IBL—we will do proof exercises to understand why the stencils work, derive an algorithm to use them efficiently, and factor numbers using the stencils (and probably a 4-function calculator, it's not the 20s anymore).

Prerequisites: Modular arithmetic

The Hales–Jewett theorem (🌶️🌶️🌶️🌶️, Misha, TWΘFS)
The Hales–Jewett theorem is a classic result in Ramsey theory that, informally, says that "high-dimensional tic-tac-toe can never end in a draw." It is known for (1) many applications to other problems, and (2) eeeeenormous upper bounds. We will see two proofs of this theorem, and also visit exciting locales such as hypergraphs, arithmetic progressions, and point constellations.

If you think inequalities like

$$r\text{-Fun}(t) \le \underbrace{r\text{-HJ}^{r\text{-HJ}^{r\text{-HJ}^{\cdot^{\cdot^{\cdot^{r\text{-HJ}(2)}}}}}(2)}(2)}(2)}_{rt\text{ levels}} \le \underbrace{r^{4^{r^{4^{\cdot^{\cdot^{\cdot^{r^{4^{r}}}}}}}}}}_{2rt-1\text{ levels}}$$

are fun, think about taking this class!

Homework: Recommended

Class format: Lecture

Prerequisites: None

The Ra(n)do(m) graph (🌶️🌶️, Travis, TWΘFS)
Take a collection of vertices and draw an edge between each of them with probability 1/2. If the collection of vertices is the natural numbers, it turns out that there's only one graph that results from this process. It's called the Rado graph, and this is only the first of its super-cool properties. We'll talk about this and as many other Rad(o) facts as we can.

Homework: Optional

Class format: Lecture

Prerequisites: You should know what a graph is. You should also know why, if I flip three coins, the probability that they all end up heads is $\frac{1}{8}$. If you know those things, you're good.

11:10 Classes

Counter? I hardly know 'er! (🌶️, Narmada and Travis, TWΘFS)
Turns out there's more to counting than using your fingers. In this class, we'll introduce some of the techniques used to sneakily count things that don't want to be counted. (Topics will include basic counting techniques, bijective proofs, formula discovery, and recurrences. If you've seen this before, this class may not be for you. If you haven't, it'll be oodles and oodles of fun. (First counting lesson: that's two oodles (AKA one pair of oodles (AKA one poodle)).))

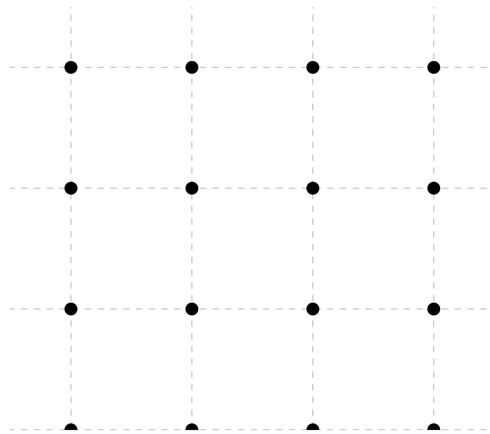
Homework: Recommended

Class format: Mostly group work with some lecture

Prerequisites: None

Erdős' distinct distance problem in the plane (🌶️🌶️🌶️, *Neeraja Kulkarni*, TWΘFS)
If P is a set of N distinct points in the plane, the set of distances between points in P is called the distance set $\Delta(P)$. The size of the distance set is at most $\binom{N}{2}$ and we can find examples where this upper bound is realized (for instance, choose the endpoints of a scalene triangle). A much more difficult

question is to ask how small the distance set can be. For example, $P = \{(1,0),(2,0),\ldots,(N,0)\}$ gives $|\Delta(P)| = N$. Paul Erdős discovered a better example by taking his points in a square lattice, that is, taking all points with integer coordinates between 0 and $\sqrt{N}$:



For this set, $|\Delta(P)|$ works out to be about $N/\sqrt{\log N}$. Based on the lattice example, Paul Erdős conjectured in 1946 that $|\Delta(P)| \geq N/\sqrt{\log N}$. This conjecture was proved by Guth and Katz in 2015 (or rather almost proved, as they showed a lower bound of $N/\log N$). In this course we will look at their proof, which uses topological tools such as the polynomial ham sandwich theorem, algebraic geometry tools and clever incidence geometry arguments.

Homework: Recommended

Class format: Lecture

Prerequisites: Comfort with vector geometry. Familiarity with critical points of a function would be very helpful.

My two favourite type of sets: Cantor sets and Kakeya sets (🌶🌶🌶, Charlotte, TWΘFS)
KAKEYA SETS are sets in the plane that contain a unit line segment in every single direction. Seems like they'd be large, eh?

CANTOR SETS are sets that are constructed iteratively. The standard Cantor set is constructed by starting with the unit interval, dividing it into three subintervals, and throwing away the middle one. Then we divide our remaining two intervals into three parts, and again throw away the middle ones. We do this forever.

These two types of sets are very interesting in their own right, but in this class, we will discuss a very cool connection between the two. In particular, we will use a Cantor set to construct a Kakeya set with zero(!) area.

Homework: Recommended

Class format: Lecture

Prerequisites: Know what an open set is and what a limit is. Have experience working with proofs involving epsilons.

Teichmüller theory of the torus (🌶🌶, Arya and Assaf, TWΘFS)
Take a paper square, and glue opposite sides. If done correctly (i.e., in $\mathbb{R}^4$), you will get a torus which is flat—just like the paper you used to create it. In this class, we will study the geometry of this type of construction. We will look at the "space of all flat tori" (Teichmüller space) and study it using Lattices (in $\mathbb{R}^2$), Loops (on the torus), and Linear algebra. Along the way, we'll meet some beautiful

critters like the curve graph, the Farey tessalation of the circle, and Möbius transformations in the upper-half plane.

Be warned—this class *will* involve some divison by zero, under staff supervision.

Homework: Recommended

Class format: Lecture

Prerequisites: Linear algebra (2-dimensional vector addition, multiplication, and 2×2 matrices), complex numbers will be helpful

The continuum hypothesis (week 1) (🌶🌶🌶🌶, Susan, TWΘFS)
How do you prove that a statement is unprovable? Well, that sort of depends on *why* the statement is unprovable. If it's unprovable because it's false, you can prove its negation—done! But what if it's neither true nor false? There's a huge class of mathematical statements that are actually independent of our standard collection of mathematical axioms (the Zermelo-Fraenkl axioms with choice, or ZFC for short). One excellent example of an independent statement is the continuum hypothesis.

The continuum hypothesis is a famous conjecture about the nature of infinity. A lot of the early exploration of infinite sets was done by Georg Cantor in the late 1800s. Cantor discovered the somewhat surprising fact that there are different sizes of infinity. Some familiar infinite sets turn out to be the same size, like the naturals and the rationals (which in and of itself is a bit surprising if you're used to thinking of the rationals as "bigger," but hey, that's infinity for you). In 1874, Cantor published a proof that the real numbers were a strictly larger size of infinity than the natural numbers.

The obvious followup question is: are there any infinities in between? The continuum hypothesis is the statement that, no, the size of the continuum (the real numbers) is the very next size of infinity. However, this question remained open for nearly ninety years, until 1963, when Paul Cohen proved that the continuum hypothesis is independent of ZFC. His technique was essentially to build two miniature set theoretic universes—one in which the continuum hypothesis was true, and one in which it was false.

In this class, we'll take a fast march through the proof of the independence of the continuum hypothesis. There are no prerequisites beyond a basic familiarity with cardinality, but be prepared to move fast!

Homework: Recommended

Class format: This will be a standard lecture class. Homework problems are not required, but you should be prepared to go over your notes and ask me questions in between classes.

Prerequisites: None

1:10 Classes

Computer-aided design (🌶, *Elizabeth Chang-Davidson*, TWΘFS)
Computers are awesome! They can do so many cool things! In particular, if you can imagine some shape or machine, you can make a computer draw it in 3D. Once the computer knows what it is, then it can show you what it would like from any angle, and you can tweak it without having to redraw the whole thing. You can also turn it colors and zoom in on small details. Basically, anything you can do in your head, you can show to other people, with the computer.

In addition, once you have told the computer about it, the computer can print out pictures or files that let machinists or machines make the part in real life. Computer aided design is useful for all kinds of things, from making robots to race cars to mathematical art.

Homework: Recommended

Class format: A lot of time to work on your own projects

Prerequisites: None

Eigenstuff! (🌶️🌶️🌶️, Mark, [T]WΘFS)
If after a sunny day, the next day has an 80% probability of being sunny and a 20% probability of being rainy, while after a rainy day, the next day has a 60% probability of being sunny and a 40% probability of being rainy, and if today is sunny, how can you (without taking 365 increasingly painful steps of computation) find the probability that it will be sunny exactly one year from now?

If you are given the equation $8x^2 + 6xy + y^2 = 19$, how can you quickly tell whether this represents an ellipse, a hyperbola, or a parabola, and how can you then (without technology) get an accurate sketch of the curve?

These are two of many problems that can be solved rather efficiently using "eigenstuff" – more formally, eigenvalues and eigenvectors of square matrices. In this class we'll define what those are and quickly look at a few examples of the cool things that can be done with them. They will also come up in the representation theory class in weeks 3 and 4; however, they won't come up at the very beginning of that class, and if you don't make it to the "eigenstuff" class but you want to take representation theory, I'm willing to try to get you caught up on them early in week 3.

Homework: Optional

Class format: Lecture

Prerequisites: Linear algebra (specifically, linear transformations and their matrices, the idea of a basis, and matrix multiplication)

Grammatical group generation (🌶️, Eric, TWΘ[FS])
Do you like silly word games? Normal subgroups and presentations of groups got you down? Come to this extremely light-hearted romp through the world of grammatically generated groups! In this class, based on a real actual published math paper, we will use group theory to understand how many homophones and anagrams the English language has. If you think this sounds silly, that's because it is silly. But we'll do it anyways, and see some cool group theory along the way! Be prepared for terrible jokes and words you will never see used in any other context.

Homework: Optional

Class format: 50/50 mixture of interactive lecture and small group/solo work

Prerequisites: Group theory: familiarity with what normal subgroups and quotient groups are.

Hyperplane arrangements (🌶️🌶️, Emily, [TWΘFS])
They sound fancy, but hyperplane arrangements are pretty simple to define. In $\mathbb{R}^2$, they are collections of lines; in $\mathbb{R}^3$, they are collections of planes (and we can keep going into higher dimensions!). For example, cutting a pizza into slices produces a hyperplane arrangement, where the cuts are the hyperplanes. We will discuss how to classify the different pieces of hyperplane arrangements, and how to do operations on them. Another thing that we will explore is how to count the number of slices that hyperplanes cut $\mathbb{R}^n$ into. This is obviously very easy in the case of a pizza, but in general it is not always so nice (especially when we are constructing arrangements that we cannot easily visualize). Some tools that we will use are posets, the Möbius function, and characteristic polynomials.

Homework: Recommended

Class format: Lecture with some group work

Prerequisites: None

Information theory (🌶️🌶️, Linus, [TWΘ]FS)
Exactly how much does learning today's weather tell you about tomorrow's? Approximately how many possible Tweets are there, if we restrict to reasonable English Tweets only? These questions can be answered using *entropy*, a notion of the amount of information contained in a random variable.

In this class, we'll introduce entropy and use it to give slick proofs of a few theorems in discrete math, such as upper bounding the number of $n^2 \times n^2$ Sudoku puzzles.

Homework: Recommended

Class format: Lecture

Prerequisites: None

Maximally colorful mathematics (, Zoe, TWΘFS)
Brouwer's fixed point theorem gives us such fun facts as the existence of a cake cutting such that everyone having cake is happy with their slice, and many more! This theorem has several equivalent statements and switching statements makes for easier proofs in different settings. In particular, some of these statements allow for the ease of "colorful generalizations!" With more and more colorful generalizations we get to see more and more connections to different types of mathematics including combinatorics and discrete geometry.

With fabulously far reaching consequences, come work through these elegant techniques towards approaching problems with more than enough styles of proofs to keep us happy for a week.

Homework: Recommended

Class format: IBL

Prerequisites: None

The category of sets (, Nic, TWΘFS)
If you've heard of category theory before, there's a decent chance you heard that it has a reputation for being horribly abstract and impossible to understand, and that you need to know a lot of math before you start learning it. This class is my attempt to convince you that almost none of that is true.

While it's kinda true that category theory is abstract, that's only because it's so widely applicable; the ideas show up in almost every corner of modern math! In this class, we'll explore category theory in a simple, familiar setting: that of sets and functions between them. It turns out that if we build up the core concepts of set theory by focusing on *functions* rather than *elements*, then these definitions—emptiness, products, unions, intersections, power sets, and more—will have generalizations to a wide variety of other mathematical objects. (Plus, it's also just a fun way to think about set theory.) Once you see how it works for sets, my hope is that category theory will feel much more approachable.

Homework: Required

Class format: Lecture

Prerequisites: Some facility with basic set theory ideas like union, intersection, product, and so on; no prior exposure to category theory is expected. Group theory and linear algebra will pop up in a couple optional, totally skippable exercises.

The probabilistic method (, Yuval, TWΘFS)
A set A of integers is called *sum-free* if there do not exist $x, y, z \in A$ satisfying $x + y = z$. Erdős proved the following amazing theorem: given any set S of integers, there is a sum-free subset $A \subseteq S$ with $|A| \geq |S|/3$. In other words, given *any* set of integers, you can pick out one-third of them so that no two numbers you've picked out sum to a third.

This is a theorem about numbers, so it ought to have a number-theoretic proof. But the only known proof uses almost nothing about numbers. Instead, the only way we know how to prove this theorem is by using *randomness*.

This is the heart of the probabilistic method, which is one of the most powerful techniques in modern combinatorics. Rather than proving something "directly," you impose some kind of randomness, and then show that your desired result holds with positive probability. In this class, we'll see a few examples of this idea in action, including a quick proof of the result above.

Homework: Optional

Class format: Lecture

Prerequisites: It would be helpful to know basic probability concepts like random variables and expectation; the first day of Martingales is more than enough.

Colloquia

Project selection (Staff, Tuesday)
This is not a colloquium.

Many Mathcampers enjoy working on some kind of long-term project throughout camp: on their own, or in groups, and possibly with guidance from a staff member. These projects range from reading math papers to folding origami to doing original research to baking. They can take lots of time every day or just some planning once or twice a week. If this sounds appealing to you, and you have a project you'd like to work on, just talk to any of the Mathcamp staff about it! We'd be happy to help out. If this sounds appealing to you, but you don't have a project in mind yet, then come to this event: the project selection fair! Staff have many of their own project ideas for you to sign up for.

Exploring extreme x in e^x (Assaf, Wednesday)
This expository expo expounds experiments explicitly expanding exponents. The expanded expression expels explosive exploration of $\exp(x)$ and explains its expansive exploits. Explicitly, experience how exponentiation exports expanses to groups and exposes exploitable expressways to solving ODE and PDEs. Before expiring, expect explicit explanations of extreme examples of $\exp(x)$.

In English: Using calculus, we can write $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. But what if we took that to be the **definition** of e^x? If that's the case, then we can define e^x for some very weird x's ranging from complex numbers to strange algebra systems to matrices to derivatives... whatever that means. In this colloquium, we will find out what this means!

Map coloring tourism (Misha, Thursday)
When it comes to coloring maps, a so-called "four color theorem" may have been proven by computer, but the situation changes when countries impose their own political[2] constraints on the process.

In this colloquium, we will survey the recent history of three famous[3] countries in the Nonspecific Ocean in search of answers. We will go on a tour of the canals and cantons of Circlevania; we will color between the valleys and chasms of Carstenland; we will visit the 68 baronies, 20 counties, and 4 duchies of The Mirzakhanate. We will get four different answers to the question "How many colors do we *really* need to color[4] a map?"

Fruit math memes (Eric, Friday)
You may be familiar with the fruit math meme: an image of some equations where the variables are fruits, accompanied by the claim that "95% of people can't solve this" or some similar figure. Usually these exist to troll people. Occasionally though, a fruit math meme can teach us about the frontiers of number theory! We'll conquer some fruit math memes by learning about elliptic curves, and along the way we'll encounter a Hilbert problem, some Millennium problems, and more.

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[2] Mathematical.

[3] Fictional.

[4] List color.