CLASS DESCRIPTIONS—WEEK 3, MATHCAMP 2023

Contents

9:10 Classes

Consistency of arithmetic by killing hydras (🌶🌶🌶🌶, Della, TWΘFS)
Steve proved last week—and Gödel proved in 1931—that if the basic axioms of number theory (formalized as Peano Arithmetic) are consistent, then they can't prove that they're consistent. I'm going to ignore this inconvenient fact, and prove that PA is consistent from as simple axioms as I can (no set theory!).

To do this, we'll turn proofs into trees and play something like the hydra game on them. Once the hydra is defeated, we'll have an extra simple proof that definitely isn't a proof of a contradiction. Because of Gödel, we'll need a pinch of axioms beyond PA: as with hydras, defeating our proofs will require ordinal numbers.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: You should be familiar with Peano Arithmetic, Gödel's second incompleteness theorem, and ordinal numbers up to ε_0. If you took Steve's week 2 class and remember Susan's colloquium about killing hydras, you're prepared.

Functions of a complex variable (Week 1 of 2) (🌶🌶🌶, Mark, TWΘFS)
Spectacular (and unexpected) things happen in calculus when you allow the variable (now to be called $z = x + iy$ instead of x) to take on complex values. For example, functions that are "differentiable" in a region of the complex plane now automatically have power series expansions. If you know what the values of such a function are everywhere along a closed curve, then you can deduce its value anywhere inside the curve! Not only is this quite beautiful math, it also has important applications, both in- and outside math. For example, functions of a complex variable were used by Dirichlet to prove his famous theorem about primes in arithmetic progressions, which states that if a and b are positive integers with $\gcd(a, b) = 1$, then the sequence $a, a + b, a + 2b, a + 3b, \ldots$ contains infinitely many primes. This was probably the first major result in analytic number theory, the branch of number theory that uses complex analysis as a fundamental tool and that includes such key questions as the Riemann Hypothesis. Meanwhile, in an entirely different direction, complex variables can also be used to solve applied problems involving heat conduction, electrostatic potential, and fluid flow. Dirichlet's theorem is certainly beyond the scope of this class and heat conduction probably is too, but we'll prove an important theorem due to Liouville that 1) leads to a proof of the so-called "Fundamental Theorem of Algebra", which states that any nonconstant polynomial (with real or even complex coefficients)

has a root in the complex numbers and 2) is vital for the study of "elliptic functions", which have two independent complex periods, and which may be the topic of a week 5 class. Meanwhile, we should also see how to compute some impossible-looking improper integrals by leaving the real axis that we're supposed to integrate over and venturing boldly forth into the complex plane! This class runs for two weeks, but it should be worth it. (If you can take only the first week, you'll still get to see a good bit of interesting material, including one or two of the things mentioned above.)

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Multivariable calculus (the week 1 crash course will be enough). Familiarity with Green's Theorem won't hurt, but the theorem will be stated, so it's fine if you didn't take the one-day class about it in week 2.

Music: the number theory of sound (🌶🌶, *J-Lo*, TWΘFS)

A "major scale" is obtained by dividing an "octave" into seven steps, where the third and seventh of these are "half" and the rest are "whole," for a total of twelve half steps. To form a "major triad," a particularly pleasant combination of notes, you can combine a "third" (four half steps) and a "fifth" (seven half steps).

Regardless of your music theory background, the description above may seem like a jumble of numbers with more idiosyncracies than the imperial system of units. Even if we ignore the naming conventions, why divide the octave into 12 equal pieces? Why do $\frac{4}{12}$ and $\frac{7}{12}$ of an octave sound so natural, while $\frac{6}{12}$ of an octave sounds so unsettling? What even *is* an octave?

In this class, we begin with the basic (but vague) question underlying all of the above: *which notes "work" together?* For most of the class you will explore the question yourselves: you will run auditory and graphical experiments, come up with hypotheses, and discuss theoretical models to understand the situation. Along the way you will find that this deceptively simple question has a very rich and complex structure (SPOILERS[1]). I will share some mathematical tools that can be used to better understand this structure (continued fractions and Diophantine approximation, to name a few), but the goal is for you to develop a theory describing these observations yourselves. As a result, this course will be somewhat open-ended, and its direction may change depending on where our exploration leads.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

Problem solving: olympiad inequalities (🌶🌶, Ian, TWΘFS)

One day, your teacher decides to change the way to calculate the average of your exam scores. If you have n scores, instead of adding the scores and dividing by n, the teacher decides to multiply all your scores and take the n^{th} root. Will you form a mob and complain to the teacher? Do you have grounds for such action? In this class, we will be discussing when one thing is always greater than (or equal to) another: inequalities!

In this five day course, we will be discussing rearrangement inequalities, AM-GM, Cauchy, Jensen, and a mix of useful inequalities for Math Olympiad. This class is more for campers who have not seen Olympiad inequalities before.

Homework: Recommended

Class format: Lecture + IBL

Prerequisites: Basic inequality (e.g. what happens to the inequality when we multiply -1?)

[1] it's number theory

Solving equations with origami (🌶🌶, Eric, TWΘFS)
Put a piece of paper in front of you. Mark two points on it. Pretend that your paper is a piece of the complex plane, and that your two marked points are 0 and 1. Which other points in the plane can you construct by folding your paper and marking where your existing points fold to? If you allow arbitrary folds you can hit anything, but what if we restrict ourselves to folds we can "line up" using our existing points and lines?

We'll be able to answer this question precisely by developing a system of axioms for one-fold origami and analyzing their algebraic potential. Along our journey to understanding the limits of the algebraic power of origami we'll travel the world (Japan to India to Austria to Italy and back), gently encounter some flavours of math from abstract algebra and algebraic geometry, and employ a truly wonderful piece of 19^{th} century mathematics to solve equations by shining lasers on turtles. If that's not enough for you, one of the historical characters we'll encounter has possibly the greatest name in mathematics: Margherita Piazzola Beloch (arguably the first person to really understand the algebraic power of origami).

Homework: Recommended

Class format: This will be a very active class! There will be some lectures, we'll do lots of paperfolding activities in class (I will supply the paper), and you'll spend a fair bit of time working through constructions on your own or in small groups.

Prerequisites: Comfort with the idea of dimension of a vector space. You do not need any prior knowledge of origami to follow this course.

10:10 Classes

A very chill intro to measure theory + dimension (🌶, Charlotte, TWΘFS)
Suppose I gave you a set in $\mathbb{R}^2$, and asked you how large it is. If the shape were simple enough, you'd probably compute its area. But what if the shape were not so simple? What if I asked you to provide some other sense of size? In this class we will take an exploratory approach to different ways that we can measure the size of sets in $\mathbb{R}$ and $\mathbb{R}^2$, with a particular focus on measure theory and dimension.

You can think of a measure in

$$\mathbb{R}$$

and

$$\mathbb{R}^2$$

as a generalization of length and area, respectively. We'll discuss what properties we want our definition of measure to satisfy, try out a couple of definitions, and see what kinds of sets our definition of measure works well with. Most measure theory classes involve a lot of detailed proofs and a lot of epsilons, which can make the ideas seem much more complicated than they actually are. We'll focus less on rigour and more on the ideas of measure theory.

We'll also spend some time looking at different types of fractals sets, which loosely, are sets that no matter how far you zoom in on, exhibit fine structure and possibly, self-similarity (i.e., a fractal may be a union of multiple smaller copies of itself!). There's a notion of size, called dimension, which attempts to quantify the fine structure of a fractal. We'll look at multiple ways that we can define a set's dimension, and discuss how it agrees with the intuitive sense of dimension that we already have.

These explorations will lead us to some intriguing or even counterintuitive examples, like a set with largest possible dimension but smallest possible measure, and a function that is basically flat everywhere, yet increases from zero to one.

You may have noticed that Tanya's also teaching a class about measure theory this week: you should take my class if you'd prefer a slower-paced, worksheet-based, geometric and example focused intro to measure theory.

Homework: Optional

Class format: Mostly group work and worksheet based.

Prerequisites: Familiar with the idea that $\mathbb{Q}$ is countable while $\mathbb{R}$ is not, and an intuitive idea of what limits are. Familiarity with derivatives and integrals could be useful, but not required.

Graph colorings (🌶🌶, Mia, TWΘFS)
A new island has been discovered in the Arctic Ocean! While the geographers are arguing over how to divide the island, the cartographers begin to wonder about the map: how many colors are needed to color the countries so that any two countries that share a border get different colors? The Four Color Theorem says just four. We won't prove this—it took over 100 years and a computer program that checked 1,936 different cases to prove this theorem—but we will use this question as a springboard to others.

Suppose the countries decide that they have *non-negotiable* color preferences. For instance, the country Zudral demands to be cyan or magenta. And the country Scaecia refuses to be anything but light blue, sky blue, or cornflower blue. Given that each country now has a list of allowable colors, how does that change the cartographer's ability to color the map?

Or what if we are allowed countries to be shaded in with several colors? In this case, Zudral could be indecisive and be one half cyan and one half magenta. Or what if we changed up the objective entirely and instead of focusing on the total number of colors used, we tried to minimize the number of colors "seen" on the neighboring countries?

Secretly, the questions above can be changed into questions about graph colorings; specifically, list coloring, fractional coloring, and "local coloring" respectively. With each new coloring, there arises a new chromatic number and we return to our central questions:

(1) What are the bounds for this chromatic number?
(2) Can we construct a family of graphs that forces this chromatic number arbitrarily far from its bounds?

Note: Although maps are an excellent motivating example, we will be focusing on general graphs, not just planar graphs!

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Basic definitions from graph theory

Guess Who? (Week 1 of 2) (🌶 → 🌶🌶, Tim!, TWΘFS)

The children's board game *Guess Who?* is the hardest game that you thought was easy. In case you haven't played it, the rules are simple: you ask yes/no questions to try to determine your opponent's mystery character from among 24 possibilities, and whoever guesses correctly with fewer questions wins.

You may think (like many mathy folks I have talked to) that you know the optimal strategy. In fact, YouTube star (and former NASA engineer) Mark Rober made a whole video about this "BEST strategy." But I realized watching the video that this strategy isn't really the "BEST." Actually, it can be totally destroyed.

I wrote a little program to compute the real optimal strategy, and the answer is bizarre. The solution exhibited a pattern so unexpected that it left me asking "Where did this come from?" even *after* I had written out a proof.[2]

[2]Note: "Where did this come from?" is a different question than "What is the proof?" I had a proof! But being able to prove that something has weird behavior is different question than what is *causing* that behavior.

At this point, I became pretty obsessed with this game. I thought about this game on and off for years trying to understand why this simple children's game exhibited such a surprising pattern. And I finally got to a pretty satisfying answer.

The answer goes through several areas of math; for each topic, we'll see the fundamentals of the topic, then apply what we learned to the *Guess Who?* problem. Over the course of **two weeks**, we'll see decision trees, **information theory** and entropy, **matrix games**, relaxations, **linear programming** and convex optimization, **continuous random variables** and cumulative distribution functions, and **network flows**. By the end of the class, you'll have an introduction to each of these topics, as well as an answer to the *Guess Who?* mystery (and some open questions you can think about).

The class is 🌶 to 🌶🌶🌶🌶 over the two weeks as we move through the topics. It's a sampling platter of chili levels — you may get to see some material at a slightly higher chili level than you are used to, and other material at a slightly lower chili level; I hope you will enjoy the variety. Overall, the first week will start at 🌶 but will then be mostly 🌶🌶, while the second week will be mostly 🌶🌶🌶 with a bit of 🌶🌶🌶🌶.

The class will go through a satisfying story arc in just its first week, so it's possible to take the first week without the second.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None

How to build a donut (🌶🌶🌶, Kayla, TWΘFS)
Whether you prefer Krispy Kreme, Dunkin, or something fancy from Portland, donuts are a well-loved treat. People who especially love donuts call themselves topologists. In this class, we will be baking as topologists! In particular, we will be doing things a bit out of order starting with the icing first. Using a chocolate icing function, we will characterize special properties of donuts topologically to construct the donut. Once we devour this example, we will formulate some basic topological notions and develop more general techniques to show that defining special icing functions from a topological space to $\mathbb{R}$ can completely determine any topological pastry up to continuous deformation.

Homework: Optional

Class format: Interactive lecture.

Prerequisites: Calculus, up to knowing what a derivative is.

How to count rings (🌶🌶🌶, Kevin, TWΘFS)
How many ways can we make the abelian group $\mathbb{Z}\oplus\mathbb{Z}$ into a ring? What about $\mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}$? $\mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}$? It turns out that we can answer these questions in a simple and satisfying way (but only if there aren't too many $\mathbb{Z}$'s!).

This class will be an introduction to the parametrization of rings of low degree (2, 3, and 4). Along the way, I'll provide an introduction to some basic ideas from algebraic number theory and arithmetic statistics, where the study of low degree rings has some cool applications. For those who enjoyed my class on Bhargava's cube, I'll be talking about the algebraic/number-theoretic side of the story, which relates binary quadratic forms and $2 \times 2 \times 2$ cubes to ideals in quadratic rings.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Basic ring theory and abstract linear algebra (e.g. abstract vector spaces and linear operators, bases, trace/determinant)

11:10 Classes

All aboard the Möbius (🌶🌶🌶🌶, Narmada, TWΘFS)
BüS SCHEDULE

McMobil from **Möbius function** to **Prime Number Theorem**
Scheduled in week 3
What's it like on board?
Lots of Greek letters · The *primest* numbers · Funky finite sums
Stops

- Arithmetical functions
- Möbius inversion
- Sketchy descriptions of the distribution of primes by a lapsed number theorist

Homework: Optional

Class format: Lecture

Prerequisites: basic modular arithmetic

Calculus of variations (🌶🌶🌶, Ben & Steve, TWΘFS)
The calculus of variations, as a technique, is only a little younger than the calculus as a whole; Newton is the first person known to have used it, in the 1680s. Broadly speaking, where the "normal" calculus finds *points* that optimize *functions*, the calculus of variations finds *functions* that optimize functions-of-functions, or *functionals*.

In this class, we'll aim to study some of the general theory of the subject: this will help draw out some of the similarities with the usual differential calculus. We will also aim to study some examples—both simple and complex—to see the power of the methods, to see where they leave a lot of work to be done, and to explore some of the historical context of the subject.

Homework: Recommended

Class format: Lecture

Prerequisites: Multivariable calculus (familiarity with the multivariable chain rule, in particular)

Generating functions, Catalan numbers, and partitions (🌶🌶, Mark, TWΘFS)
Generating functions provide a powerful technique, used by Euler and many later mathematicians, to analyze sequences of numbers; often, they also provide the pleasure of working with infinite series without having to worry about convergence.

The sequence of *Catalan numbers*, which starts off $1, 2, 5, 14, 42, \ldots$, comes up in the solution of many counting problems, involving, among other things, voting, lattice paths, and polygon dissection. We'll use a generating function to come up with an explicit formula for the Catalan numbers.

A *partition* of a positive integer n is a way to write n as a sum of one or more positive integers, say in nonincreasing order; for example, the seven partitions of 5 are

$$5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, \text{ and } 1+1+1+1+1\,.$$

The number of such partitions is given by the partition function $p(n)$; for example, $p(5) = 7$. Although an "explicit" formula for $p(n)$ is known and we may even look at it (in horror?), it's quite complicated. In our class, time permitting, we'll combine generating functions and a famous combinatorial argument due to Franklin to find a beautiful recurrence relation for the (rapidly growing) partition function. This formula was used by MacMahon to make a table of values for $p(n)$ through $p(200) = 3972999029388$, back when "computer" still meant "human being who does computations".

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Summation notation; geometric series. Some experience with more general power series may help, but is not really needed. A bit of calculus may come in handy, but you should be able to get by without.

Polytopes (Week 2 of 2) (🌶🌶🌶, Susan, TWΘFS)
This week in Polytopes, we'll clean up any loose ends from our proof of the equivalence of our two definitions of polytopes and move on to consider a cool class of examples: cyclic polytopes!

If you choose n arbitrary points from the curve parametrized by $(t, t^2, t^3, \dots, t^d)$ and take the convex hull, you obtain a polytope that has precisely those n points as vertices. That is to say, no point falls within the convex hull of the others. And this is true no matter how many points you choose! Also, these polytopes have way more edges than our three-dimensional intuition would suggest. In four dimensions and above, the line segment between any pair of vertices is an edge of the polytope!

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Polytopes Week 1

The sum-product conjecture (🌶🌶, Neeraja, TWΘFS)
Let A be any set of N integers. The *sum set of* A is the set

$$A + A := \{a + b : a, b \in A\}$$

and the *product set of* A is the set

$$A \cdot A := \{a \cdot b : a, b \in A\}.$$

If A is given by N randomly chosen integers, we would expect that both the sum set and the product set have size around N^2. However, each of the two can be made much smaller by choosing A carefully; for example, choosing A to consist of elements of an arithmetic progression, i.e. $A = \{a+d, a+2d, a+3d, \dots, a+Nd\}$ makes $|A + A| = 2N - 3$. The sum-product conjecture suggests that no set A has both a small sum set and a small product set. Roughly speaking, the conjecture says that if N is large enough, then for any set A,

$$\max\{|A + A|, |A \cdot A|\} \approx N^2.$$

This unsolved conjecture has a number of applications in many different fields, including number theory, incidence geometry, and computer science. In this class we will survey some of the partial progress made towards proving the conjecture.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: A little experience with epsilon-delta proofs may be helpful, but is not required

1:10 Classes

Coxeter groups (🌶, Kayla, TWΘFS)
Coxeter groups are a special class of groups that capture the symmetries of objects. For example, how many symmetries are there of a cube? Icosahedron? Coxeter groups give a combinatorial way for us to think about these types of problems. We will look at examples of of Coxeter groups and state a classification theorem of finite Coxeter groups that is one of my all time favorite theorems!

Food for thought: what is the sequence $1, \infty, 3, 5, 3, 4, 4, 4, 3, 3, 3, 3, \dots$?

Homework: Optional

Class format: Interactive lecture

Prerequisites: Group theory encouraged!

Latin squares (🌶🌶, *Zoe Wellner*, TW ΘFS)
In addition to sudokus being a type of Latin square, Latin squares are quite the mysterious object. They have various connections to projective geometry and there are some questions about their properties that have been unknown for over 60 years! In this class, we will learn about Latin squares and why some of the questions about them are hard.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

Linear algebra through knots (🌶🌶🌶, Raj, TW ΘFS)
In this course, we will learn how to visualize maps between vector spaces using braids. The point is that long products of maps can be manipulated very easily by untwisting and unturning the corresponding braids.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Linear algebra.

Logic puzzles (🌶, Misha, TW ΘFS)
Here are a few logic puzzles, some more well-known than others:

(1) You are shown two closed boxes, each containing either cookies or carrots. (There is no guarantee that one contains cookies and the other contains carrots.) They are labeled as follows:
  Box 1: At least one of these boxes contains cookies.
  Box 2: The other box contains carrots.
  You are told that either both labels are true, or both are false. If you want cookies, which box should you open?

(2) Every mentor at Mathcmap either always tells the truth, or always lies. You run into Tanya and Travis at dinner, and decide to find out whether they tell the truth or lie.
  "Are you a truthteller?" you ask Tanya, who responds, "Banana." This is a word in food tongue that either means "Yes" or "No", but you have forgotten which one.
  "Does banana mean 'Yes'?" you ask Travis. Travis replies, "It does. You shouldn't believe Tanya, though; she always lies."
  What can you conclude about Tanya and Travis?

(3) X and Y are two integers bigger than 1; their sum is 100 or less. Sam is told their sum $X+Y$, and Priya is told their product $X \cdot Y$. Both Sam and Priya always tell the truth, and know all the information in this paragraph.
  Sam and Priya then hold the following conversation:
  Sam: I know that Priya does not know X and Y.
  Priya: Now I know X and Y.
  Sam: Now I also know X and Y.
  What are X and Y?

In this class, we will explore logic puzzles like these, solve many of them, talk about how we can solve them more easily, and see some ways in which they're connected to other kinds of math.

Homework: Recommended

Class format: All over the place; there will be some lecture, some independent work on logic puzzles, and some discussion.

Prerequisites: None.

Neural codes (🌶🌶🌶, *Zoe Wellner*, TWΘFS)
The stimuli of a given neuron can be modeled by a convex set and combinatorial objects known as neural codes can extract information about the space covered by these regions. Although these were initially of interest to determine how the brain stores information, topologists and geometers have expanded on the questions we ask about these objects. We will be looking at how codes behave in different dimensions and get used to working with the spaces that arise from codes and other related objects.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

Non-standard analysis (🌶🌶, Krishan, TWΘFS)
Intuitively, a function on the reals is continuous if it sends points which are close together to other points which are close together. This idea is relatively simple, but unfortunately, the definition of continuity from real analysis is more complicated. In real analysis, we introduce epsilons and deltas as bookkeeping devices to make the intuition precise, but what if I told you that we could do without them?

In non-standard analysis we have access to actual infinitesimal numbers. So we can say that a function on the reals is continuous when it sends points that are infinitesimally close to other points that are infinitesimally close. Similarly, we have access to infinite numbers, so we can say that a sequence converges to a limit, L, if the "infinite" terms in the sequence are infinitesimally close to L.

This class will mention some ideas from logic, but logic will not be the main focus, and no prior experience with logic is required to follow the class.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Familiarity with epsilon-delta proofs

Predicting the future (🌶🌶, *Rice Neyman*, TWΘFS)
You're deciding where to hold Mathcamp in 2026 and want to make sure that the campus is safe for campers. And so you decide you need to know how many bears there will be on the Champlain College campus in 2026. But you're no bear expert, so you decide to consult the local ursinologist (bear expert) for a forecast. You're happy to spend some money to make sure that the forecast is good, so you decide that you will pay the ursinologist according to the accuracy of their prediction. (They make a prediction about the number of bears, you wait until 2026, check how many bears there are, and the more accurate the prediction, the more you pay them.) It turns out that unless you're careful with your payment scheme, you could incentivize the ursinologist to lie to you! How should you set up the payment scheme so that a self-interested ursinologist will tell you the truth?

But then you decide that it's really important to know how many bears there will be, so you decide to ask not one but *three* ursinologists for a forecast. And they all give you different answers: one says that there will be 5 bears on campus, another says 10 bears, and a third predicts a whole 100 bears! What's the best way to aggregate these predictions into a single number? It turns out that the answer depends on how you decided to pay the ursinologists!

On Day 1 we will talk about *forecast elicitation*: how to incentivize forecasters to tell you their true beliefs. On Day 2 we will talk about *forecast aggregation*: combining multiple forecasts into one.

And on Day 3 we will talk about *prediction markets*, which do both of these things at the same time. Come learn about the secrets of predicting the future!

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

The Borsuk–Ulam Theorem (🌶, Arya, TWΘFS)

In this class, we shall discuss the Borsuk–Ulam theorem, which states that at any point of time, there are two diametrically points on Earth that have the same temperature and air pressure. One can proceed to make deep statements about the topology of spheres, or one can proceed to make combinatorial statements about triangulations and colourings.

This shall be an introduction to topological combinatorics, relating results in topology (such that continuous functions on the disk have a fixed point) to combinatorial results (the game of Hex cannot end in a draw). Time permitting, we shall talk a bit about algebra and the secret ingredient - projective spaces.

Homework: Optional

Class format: Interactive lecture

Prerequisites: None.

Ultrafilters and voting (🌶🌶, Krishan, TWΘFS)

Imagine you and your friends are trying to decide where to go for dinner. You all have your own personal ranking of the options but somehow you need to combine your individual rankings into a group ranking. If you were hoping that math could help you with this problem then you're out of luck! It turns out that there is no "fair" way to solve this type of problem. This result is known as Arrow's Impossibility Theorem. In this class we will formulate the theorem precisely and will see how this concrete result can be proven using ultrafilters (a type of object most commonly seen in logic).

Homework: Optional

Class format: Interactive lecture

Prerequisites: Familiarity with basic set theory (infinite unions, infinite intersections, etc)

Why do we need measure theory? (🌶🌶🌶, Tanya, TWΘFS)

Suppose you're trying to compute the volume of your aquarium—one way to do so is by multiplying the dimensions of the container. However, what if what you really care about is the number of fish that the aquarium can support. Or, perhaps, instead you're interested in the amount of oxygen in the water. Whilst the number of fish and oxygen levels are not conventional notions of "volume", they are very reasonable measures of size of the aquarium. In this class, we will investigate what it means for a set of real numbers to be measurable and why we need to develop the theory of measures carefully. In particular, we will construct the infamous Vitali set, study the Caratheodory criterion for measurability and compute the volume of the Cantor set.

You may have noticed that Charlotte's also teaching a class about measure theory this week: you should take this class if you already have some familiarity with analysis and are curious to dive deeper into technical details behind foundations of measure theory.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: some basic analysis

Colloquia

Teaching Math to Computers (*Apurva Nakade*, Tuesday)
Will computers ever be able to understand mathematics? Will computers ever be able to prove theorems? How does one even communicate proofs to a computer? What is the meaning of life?

In this talk, I will discuss the project of math formalization using Lean theorem prover. Lean is an open-source programming language which can be used to encode mathematical proofs. There is a vast and rapidly growing library of mathematical proofs in Lean maintained by mathematicians and computer scientists around the world. In the first half of the talk, I will give an overview of the library and show some example proofs in Lean. In the second half, I will discuss type theory (an alternative to set theory) which forms the basis of the Lean programming language.

The talk only requires a basic understanding of mathematical logic and a mild interest in programming. A good sense of humor is recommended but not required.

Antinomy: meditations on Gödel's undecidable sentences (*Ari Nieh*, Wednesday)
The word "paradox" gets thrown around a lot in mathematics, but what does it actually mean? The celebrated logician Quine classified them into three types:

- Veridical: using correct premises and reasoning to reach a true (but surprising) conclusion
- Falsidical: using faulty premises or reasoning to reach a false conclusion
- Antinomy: using "correct" premises and reasoning to nonetheless reach a contradiction

Antinomies are the most disturbing kind of paradox. They indicate something shaky in the foundations upon which your logic is built. In fact, resolving them may require rewriting the fundamental rules of mathematics!

In this talk, we'll explore several entertaining examples of Quine's three types of paradoxes. We'll see that, in some sense, they're secretly all the same type. We'll discuss the influence of paradoxes on the development of logic and set theory in the 20^{th} century. Despite the spooky abstract language, this talk will mostly be fun and accessible, with just a teensy bit of brain-melting.

An introduction to cryptography (*Jess Wernig*, Thursday)

Future of Mathcamp (Staff, Friday)
This is an event we host every year where we ask YOU the campers for feedback on how camp is going. We care about the things you have to say and many times, campers come up with brilliant ideas to improve camp! Please come and help brainstorm how to make Mathcamp better!