# CLASS DESCRIPTIONS—WEEK 2, MATHCAMP 2024

Contents



## 9:10 am classes

**Public key cryptography.** (Athina & Chloe) TWΘFS 50 minutes

Vo uv, aol zjolkbsl ivhyk ohz ybu vba vm yvvt mvy zpnuz, zv Thaojhtw zabkluaz Hspjl huk Ivi dhua av lewhuk pa av pujsbkl hss vm jhtwbz! Ovdlcly aolyl pz h wyvislt: aolf kvu'a dhua zabkluaz myvt vaoly jhtwz av zovd bw av aolpy lcluaz... Lcl myvt MZF dhz zllu ylhkpun aolpy zpnuz!!! Ovd jhu aolf zavw oly???!!!

Come to this class if decoding messages like this sounds fun or if you want to learn about the cryptosystems keeping your digital information safe. Some of the topics we will be seeing include the Diffie–Hellman key exchange, the RSA protocol, and Elliptic Curve Cryptography (and how to attack them!), along with some of the number theory behind these systems.

*Homework:* Recommended.

*Class format:* Interactive lectures

*Prerequisites:* Familiarity with the Euclidean algorithm and basic modular arithmetic will be helpful, but if you want to take this class and are unsure about your background come talk to us!

**Alice and Bob go quantum.** (Narmada) TWΘFS 50 minutes

Imagine that your nemesis gains access to a super-powerful quantum computer. Oh no! Now they can break your classical computer's encryption algorithms! How do you save yourself and the world from doom forever?

...you basically do a lot of linear algebra. This class is an introduction to Quantum Information Theory, which is essentially Linear Algebra Two: Electric Boogaloo. We'll learn how matrices model communication between classical computers, and do some fancy linear algebra to translate this into the quantum world. We'll focus on the combinatorial motivation, which means we'll define what a quantum confusability graph is, and what it means for it to be connected. If there's time, we'll look at how quantum error-correction is vastly different from classical error-correction.

*Homework:* Optional.

*Class format:* lecture with some group work

*Prerequisites:* Linear algebra: a good understanding of linear independence, span, and basis (for vectors in $\mathbb{R}^n$; familiarity with multiplying non-square matrices; know what the inner product is for $\mathbb{R}^n$ and $\mathbb{C}^n$; be comfortable with the words 'orthogonal' and 'projection'.

Graph theory: be comfortable with the terms 'vertex', 'edge', 'independent set', and 'connected component'.

Other: please don't ask me what a quantum computer is.

**Measure and Martin's axiom (week 1 of 2).** (Susan) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

If we want to develop a notion of "length" on the real line, there are some properties we know it ought to satisfy. For instance, the length of $(a, b)$ ought to be $b - a$. And the length of a single-point set ought to be zero.

In this class, we will explore the behavior of sets with size zero. Any countable union of measure-zero sets has measure zero. However, we can take a union of continuum many measure-zero sets and obtain a set that does not have measure zero. (For instance, the union of all single-point sets $\{x\}$ with $0 \leq x \leq 1$ is the unit interval, which has measure 1.)

So what happens if we take an uncountable–but not continuum-sized–collection of measure-zero sets? This question turns out to be independent of the standard ZFC axioms of set theory. We'll explore one answer that arises in a universe where we've added an extra-set-theoretic axiom called Martin's Axiom to the mix.

This class should be fun for campers who want to mess around with really big numbers (like $\aleph_1$) and really small numbers (like $\epsilon > 0$) at the same time.

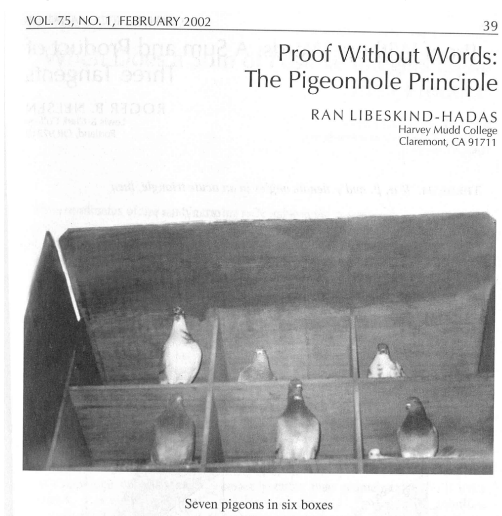
*Homework:* Recommended.

*Class format:* Lecture

*Prerequisites:* Basic facts about cardinality, or have a half-hour conversation with Susan at TAU.

**Intro to combinatorics (graph flavored).** (Kailee) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Perhaps you've heard of the pigeonhole principle. Here's my favorite proof of it: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Proof Without Words: The Pigeonhole Principle

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Seven pigeons in six boxes

In this class, we'll cover several classic combinatorial proof techniques including the basics of counting and proof by double counting, the pigeonhole and averaging principles, the principle of inclusion exclusion, and Hall's Theorem. We'll use these techniques to prove all kinds of things about graphs, from subgraphs to graph colorings, perfect matchings, and more. Whether you're learning these tools for the first time, or have seen them before but want more practice, there will be a range of problems to work on in class each day, so you can *count* on finding the right fit for you!

*Homework:* Optional.

*Class format:* Groupwork

*Prerequisites:* None.

*This class may help prepare you for:*

- **Evolution of random graphs** (Misha, in Week 2) — Prerequisites: Introduction to graph theory, or comfort with basic graph theory concepts such as trees, subgraphs, vertex degrees, distance.
- **Topological Graph Theory** (Marisa, in Week 3) — Prerequisites: Introduction to graph theory, or comfort with graph theory concepts like complete graph, complete bipartite graph, subgraph, and tree.

## 10:10 AM CLASSES

**Linear algebra (intro) (week 2 of 2).** 🌶🌶🌶 (Mark) T $\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

This is a continuation of last week's class. If you didn't take that class but you'd like to join now, please check with Mark at Friday's TAU to find out roughly what was covered and what you might need to catch up on.

For reference, here's some of last week's blurb:

. . .

In this class we'll deal with questions such as: . . . What happens to geometric concepts (such as lengths and angles) if you're not in the plane or 3-space, but in higher dimensions? . . . What happens to areas (in the plane), volumes (in 3-space), etc., when we carry out a linear change of coordinates? If after a sunny day somewhere the next day has an 80% probability of being sunny and a 20% probability of being rainy, while after a rainy day the next day has a 60% probability of being sunny and a 40% probability of being rainy, and if today is sunny, how can you (without taking 365 increasingly painful steps of computation) find the probability that it will be sunny exactly one year from now? If you are given the equation $8x^2 + 6xy + y^2 = 19$, how can you quickly tell whether this represents an ellipse, a hyperbola, or a parabola, and how can you then (without technology) get an accurate sketch of the curve? How do astronomers know the chemical composition of distant stars? (We may not get to all these things . . . , but we should cover most of them at least to some extent.)

*Homework:* Recommended.

*Class format:* Interactive lecture

*Prerequisites:* Last week's class (see also the start of the blurb, above)

*This class may help prepare you for:*

- **The real MCSP (Markov Chains and Stochastic Processes)** (Alyona & Arya, in Week 2) — Prerequisites: Basics of matrices and linear transformations (from intro to linear algebra), and basic definitions of probability (but the probability aspect we can cover during the class).
- $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ (Kevin, in Week 2) — Prerequisites: Comfort with linear algebra (eigenvalues, inner products)
- **Alice and Bob go quantum** (Narmada, in Week 2) — Prerequisites: Linear algebra: a good understanding of linear independence, span, and basis (for vectors in $\mathbb{R}^n$; familiarity with multiplying non-square matrices; know what the inner product is for $\mathbb{R}^n$ and $\mathbb{C}^n$; be comfortable with the words 'orthogonal' and 'projection'.

Graph theory: be comfortable with the terms ‘vertex’, ‘edge’, ‘independent set’, and ‘connected component’.

Other: please don’t ask me what a quantum computer is.

- **How to multiply numbers realllllly fast** (Eric, in Week 3) — Prerequisites: Intro linear algebra, at the level of knowing that linear transformations are invertible iff the determinants of their corresponding matrices are non-zero. Basic computer science (big O notation) is helpful context but is not necessary, consider the class as 3 chilis if you don’t have CS background.

  Alternatively if we have an intro CS course this could have that as a formal prereq and I’d spent less time talking about big O notation and complexity analysis.
- **Numerical Analysis: How Computers Do Calculus & Differential Equations** (Sonya, in Week 3) — Prerequisites: Linear algebra (actually, just matrix multiplication). Also calculus.
- **LED3D: The math of The Ball** (Tim!, in Week 3) — Prerequisites: Linear Algebra
- **Representation theory** (*Aaron Landesman*, in Week 4) — Prerequisites: Linear algebra and group theory. From group theory, familiarity with group actions is essential. One should also be familiar with symmetric groups, conjugacy classes, commutators, and quotient groups, among other things. From linear algebra, one should be familiar with linear transformations, eigenvalues, trace, and direct sum, among other things.
- **Error-Correcting Codes and Sphere Packing** (Kailee, in Week 4) — Prerequisites: Linear algebra (matrix multiplication, linear independence), Combinatorics (counting rules, binomial coefficients)
- **The first black hole: Schwarzschild spacetime** (Laithy, in Week 4) — Prerequisites: Linear algebra and calculus. Special relativity is recommended but not required.
- **Totally positive, dude** (*Mia Smith*, in Week 4) — Prerequisites: Linear algebra (familiarity with bases, matrices, and determinants)
- **Ghostly graphs** (Travis, in Week 4) — Prerequisites: You should know: the dot product for real vectors, what an orthonormal basis is, and the spectral theorem for real matrices. (The Introduction to Linear Algebra class at Mathcamp will be sufficient, if you’ve never seen linear algebra before.) If you’ve seen some of these things but not others, talk to me—I might be able to catch you up.
- **Topological Tverberg’s Theorem** (*Viv Kuperberg*, in Week 4) — Prerequisites: Linear algebra. Enough group theory to know what a group action on a set is.
- **Arithmetic complexity** (*Yuval Wigderson*, in Week 4) — Prerequisites: It would be helpful, but not strictly necessary, to know what determinants are and to have a rough sense of what the P vs. NP question asks. It would also be helpful to have seen big-O notation.

**Regular languages & word problems.** 🌶🌶🌶 (Sonya) T $\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

Is $2+3$ the same as $4+1$? How about $rsr$ and $r^2sr^2$ in the dihedral group $D_3$? If we want to answer questions like these for more complicated groups (and groups can get much more complicated), we need a system. We might even want to teach a computer to do the computations for us. In this course, we use the theory of regular languages to study group representations. This approach allows us to find simple proofs for some important group theory facts, but it also creates new questions, for instance about the optimal number of group generators/letters to describe a regular language.

*Homework:* Recommended.

*Class format:* Interactive lecture

*Prerequisites:* Group theory

**The systems of equations you weren't taught.** (Glenn) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

In school, you learned how to solve a basic system of linear equations. If you took linear algebra at Mathcamp, you learned how to solve really big systems of linear equations. But I bet no one ever taught you how to solve a system of *polynomial* equations!

That's because it's actually a significantly longer and more difficult process. And in fact, the vast majority of colleges and universities don't even have courses that teach this, in my experience.

In this class, we'll learn the method of Gröbner bases for solving systems of polynomials, and apply this to see how to (or how to have a computer) bash out proofs for practically all problems in Olympiad geometry. This is a real technique that can even solve some IMO-level problems in reasonable time limits! If we have time, I can also talk about some more recent work in which computers solve IMO geometry problems at the level of a gold medalist.

*Homework:* Recommended.

*Class format:* Worksheets (to be finished as homework)

*Prerequisites:* None! The derivative will make a guest appearance for one day, but it'll be accessible even if you haven't seen it before.

**Points on a line, really?** (Maya) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

This course is about the real numbers! You've definitely seen them before, and you've done computations and manipulations with them for years, and have definitely drawn the real line many, many times. A drawing of a line turns out to be a very good way to *represent* the real numbers, but what *are* the real numbers? *Are* they points on a line? And if they are, what does it mean to add, or multiply, two points on a line? Should we go about it through the route of decimal representation? And how about exponentiation: we can explain what we mean by raising something to 2, but what are we doing when we are raising something to $\sqrt{2}$? We'll explore these questions, and in doing so we'll get to re-create for ourselves the real numbers in all their glorious abstraction, and touch upon some beautiful ideas that get us on the road to real analysis.

*Homework:* Optional.

*Class format:* Lecture plus IBL

*Prerequisites:* You don't need any background: if you've drawn a real line and know what the length of the hypotenuse of a right angled triangle is, you're good.

**VC-Dimension.** (*Aaron Anderson*) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

In 1972, mathematicians on three continents, working in logic, combinatorics, and statistical learning theory, came across the same idea, which we now call VC-dimension.

In this class, we'll learn what VC-dimension is, why it was interesting to people working in all of these different fields, and why it is useful for convex-ish geometry, machine learning, and catering.

*Homework:* Recommended.

*Class format:* Some IBL, some lecture: We'll explore the basic ideas of VC-dimension together through IBL worksheets, and I'll lecture about some applications, particularly in the second half of the week.

*Prerequisites:* Probability: There will be one big probability-theory proof. I'll make it as self-contained as possible, but the more you know about expectation and variance, the better.

## 11:10 AM CLASSES

**Problem solving.** 🌶🌶 → 🌶🌶🌶🌶 (Mark) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

This is *not* a class that systematically covers problem-solving techniques. Instead, each day you'll see a variety of attractive (I hope) problems, and just like in "real life", part of the challenge of each problem will be deciding how to approach it and what technique(s) might be relevant. The chili level varies by problem, usually from 2 to 4 chilis. The "optional" listing for homework simply means that you are, of course, welcome to keep working on any problems you don't finish during the class period.

*Homework:* Optional.

*Class format:* Mostly work by campers, in groups or individually as they wish; I circulate, offer hints when desired, and may occasionally lecture on something for a few minutes.

*Prerequisites:* Vary by problem, but most often either "none" or "some calculus". If a problem has a prerequisite that isn't obvious (for example, the problem doesn't contain an integral sign, but integration is needed to solve it), I intend to label the problem accordingly.

**$\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$.** 🌶🌶🌶 (Kevin) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

You may know that $x^2y^2 = (xy)^2$ and maybe also that $(x_1^2+x_2^2)(y_1^2+y_2^2) = (x_1y_1 - x_2y_2)^2 + (x_1y_2 + x_2y_1)^2$. There's even an identity for 4 squares, but did you know that

$$\begin{aligned}&(x_1^2 + \cdots + x_8^2)(y_1^2 + \cdots + y_8^2)\\ &\quad = (x_1y_1 - x_2y_2 - x_3y_3 - x_4y_4 - x_5y_5 - x_6y_6 - x_7y_7 - x_8y_8)^2\\ &\quad + \cdots\\ &\quad + (x_1y_8 - x_2y_7 + x_3y_6 + x_4y_5 - x_5y_4 - x_6y_3 + x_7y_2 + x_8y_1)^2?\end{aligned}$$

(There are 8 squares on the RHS.)

In this class, we'll learn good reasons for these identities to be true, and we'll learn why the pattern doesn't continue past 8. (Spoiler: It has something to do with quaternions and octonions.)

*Homework:* Recommended.

*Class format:* Lecture

*Prerequisites:* Comfort with linear algebra (eigenvalues, inner products)

**An overly convoluted process.** 🌶🌶🌶 (Ben) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Some integrals are practical[1] to solve in the sense that you can use some combination of $u$-substitution, memorized integrals, and sensible clever tricks to work out an exact answer. However, some integrals, while easy to write down, are not quite as practical to solve, such as

$$\int_0^\infty \frac{\sin(x)}{x}dx,\ \int_0^\infty \frac{\sin(x)}{x}\frac{\sin(x/3)}{x/3}dx,\ \ldots,\ \int_0^\infty \frac{\sin(x)}{x}\frac{\sin(x/3)}{x/3}\cdots\frac{\sin(x/15)}{x/15}dx.$$

(OK, the last one there isn't quite so easy to write down either, since we're taking a product of eight terms of the form $\frac{\sin(blah)}{blah}$, for $blah = x, x/3, x/5, \ldots, x/13, x/15$.)

In this class, we'll learn one way to solve these exactly (…except for the last one, which is a lot harder). If you'd like to skip the hard work of taking this class, they're all $\frac{\pi}{2}$, except the last one, which is *very slightly less* than $\frac{\pi}{2}$.

[1] Note that I do not say "easy," because some of these integrals are pretty hard.

Why does this pattern of $\frac{\pi}{2}$s break down? What does French mathematician Joseph Fourier have to do with this? And how does it all relate to the convolution product—whatever that is? We'll discuss all this, and more!

*Homework:* Optional.

*Class format:* Interactive lecture, with significant classroom discussion

*Prerequisites:* Calculus (e.g. you should know how to take derivatives, integrals, and improper integrals). Having seen $\epsilon$–$\delta$ proofs in the past will help but should not be strictly necessary.

**The real MCSP (Markov Chains and Stochastic Processes).** 🌶 (Alyona & Arya) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Picture this: a camper is gambling with the cookie fairy to win carrots. They play a game, which they win with probability $p$. If the camper wins, they get a carrot, and if the camper loses, the camper must obtain a carrot (somehow) and return it to the cookie fairy. The benevolent cookie fairy has already bestowed the camper with 6 juicy carrots. If the camper can win 10 carrots in total, they may rest happy; but if the camper ever reaches zero carrots, they must go hungry for TAU.

This seemingly silly situation of carrot-craving campers is related to several modern problems such as Google's Page Rank algorithm, predicting weather and other forms of statistical modeling. For another application — if a camper starts wandering away from camp into Tacoma, they will return to UPS with a probability 1; but if a particularly spacey camper starts pacing into outer space, they will return with probability zero — so "probably" not advisable :) In this class, we shall introduce Markov chains and random walks, and present a whole slew of applications (both in math and in life). Sit back, relax and enjoy the ride!

*Homework:* Optional.

*Class format:* Interactive lecture.

*Prerequisites:* Basics of matrices and linear transformations (from intro to linear algebra), and basic definitions of probability (but the probability aspect we can cover during the class).

**Graph on, graph off, or: Why bother with combinatorics when you could do analysis instead?** 🌶🌶🌶 (Travis) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

A sequence of real numbers converges if, more or less, the terms of the sequence get closer and closer to a single real number called the *limit* of the sequence. But what could it mean for a sequence of *graphs* (of the combinatorial variety) to converge? And what would the limit of a convergent sequence of graphs be?

The mind-bending answers to these strange questions bring graph theory into the land of analysis. We'll see what integrals have to do with counting subgraphs; several ways of measuring the "distance" between two graphs; and what it mean for two graphs with different number of vertices to "look similar" to each other. And by building up this theory of graph limits, we'll see that Szemerédi's Regularity Lemma, a fundamental and powerful theorem in the area of structural graph theory, is *really* a statement about the set of *limits* of graph sequences.

*Homework:* Recommended.

*Class format:* mixed interactive lecture + groupwork

*Prerequisites:* You should be comfortable working with integrals and limits, and know some basic probability (how to calculate expected value, how to apply the union bound, and what independence of random variables is. You can pick this up by a quick talk with a mentor). You do not need to know any real analysis.

*This class can continue as a project:* Learn more about graph limits! We'll talk about what aspects of graph limits you want to learn more about, and based on that, I'll direct you to the right parts of a textbook to learn about it, and we'll meet regularly to discuss. (Since the project builds on the class, taking this class is required to do the project.)

## 1:10 PM CLASSES

**Algorithms for large primes.** (Zach) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Much of modern internet security relies on a counterintuitive principle: **testing** whether large numbers are prime is fast, but **factoring** those same numbers is believed to be infeasible, even with state-of-the-art supercomputers and factoring algorithms.

For example, consider this 617-digit number $n$:

3049393803 9064098204 6257224329 8853574672 1496643781 0821538918 8696453420 2146997229 6758419947 0131652491
3849210517 4158750767 8519631211 9495759970 8592524343 0912930217 3156352106 8467091704 3042905675 3647687903
1227528692 0589276904 8370921428 5585719241 1019900737 7816113198 1122159963 1064596622 5416780223 2291640108
9348914343 2024811908 9653390042 0837116144 9456532221 2395483082 5359910625 7243375192 3565957069 9858976093
3034168762 8457872080 4811538402 6599867498 1094692572 8808367980 5389339036 5915012815 2428549483 2182868787
4342301743 0194193066 8801385061 2219622243 0101198484 7699115272 5406666046 4440567481 0600472360 7644097968
6192546646 5327459.

This number $n$ is **not** prime and $n+8$ **is** prime, and a typical laptop can **verify** both of these facts in fractions of a second. By contrast, the technology to **factor** $n$ (and numbers like it) into primes likely does not yet exist, and most encrypted communications (in particular, most internet traffic) depends on this fact! The example $n$ above is copied directly from the public certificate that protects `https://www.amazon.com`, but this security could be breached by anyone who can factor $n$ into primes, so Amazon and all of its users rely on this not being feasible.

To factor a large number and/or test whether it is prime, the naïve "trial division" algorithm considers all potential factors individually: "is it divisible by 2? 3? 4? 5? etc.". But for numbers with hundreds of digits, this is way too slow, since the universe will literally suffer heat death before this algorithm makes noticeable progress.

So how is it possible to conclude that a large number (like $n$) is composite *without* factoring it? How can we be sure that a large number (like $n+8$) is prime *without* testing all of its possible prime factors? We'll explore clever algorithms that enable efficient tests like these, and the elegant underlying number theory.

Topics may include: primality certificates; probable vs provable primes; the Great Internet Mersenne Prime Search; generating large primes; the AKS primality test.

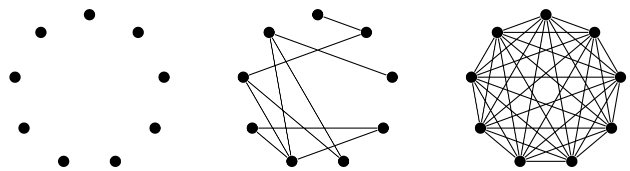
*Homework:* Recommended.

*Class format:* Lecture

*Prerequisites:* Modular arithmetic: should understand modular inverses and Fermat's Little Theorem. I plan *not* to assume or use any knowledge of abstract algebra.

**Evolution of random graphs.** (Misha) $\boxed{\text{TW}\Theta\text{FS}}$ 80 minutes

The first diagram you see below is a bit boring: it's just a bunch of points in a circle. To make it more exciting, let's repeat the following: pick two points at random and draw a line segment to connect them. After a few steps, you get the diagram in the middle—though we don't want to keep going too far, or else we end up at the last diagram, which is boring again.



How many steps will it take for the diagram to be connected? When will triangles first appear? At what stage will we be able to find a path that visits every point once? And how do these answers depend on $n$, the number of points—especially when $n$ is large?

We study the evolution of random graphs to learn the answers to these questions. Along the way, we'll learn some tricks for describing how random variables behave, and use random graphs to construct some exciting counterexamples in graph theory.

*Homework:* Recommended.

*Class format:* A mix of interactive lecture and learning through working on problems.

*Prerequisites:* Introduction to graph theory, or comfort with basic graph theory concepts such as trees, subgraphs, vertex degrees, distance.

**Introduction to ring theory.** (Eric) TWΘFS 80 minutes

A ring is a set of objects that you can "add" and "multiply." Many examples of rings come from two sources: arithmetic (rings whose elements are like numbers) and geometry (rings whose elements are like functions). We'll explore many familiar concepts in the abstract (like modular arithmetic, prime factorization, division with remainder), what they mean in these two worlds in particular, and the varied and interesting ways in which they can break down.

We'll start by recontextualizing how we think of the 4 arithmetic operations ($+, -, \times, \div$), and figuring out how to make "modular arithmetic" work in any setting where we can "add" and "multiply." Then we'll work through a hierarchy of rings with extra structure, seeing how various nice properties die out as we relax how much extra structure we have, but also keeping some things alive by expanding our idea of what prime factorizations should be.

This class will run 80 minutes long with the last half or so dedicated to working on some required problems. There will also be many optional homework problems geared towards helping you explore other directions in ring theory that we won't focus on in class.

*Homework:* Recommended.

*Class format:* Mixture of interactive lecture and worksheet type problem-solving.

*Prerequisites:* None. Group theory can be helpful context but is not necessary.

*This class may help prepare you for:*

- **Roots of Unity and Cyclotomic Fields** (Chloe, in Week 3) — Prerequisites: Complex Numbers, definition of field for second day.
- **Impossible Integration, also the vegan kind** (Glenn, in Week 3) — Prerequisites: You should be able to take derivatives of any functions involving things like $\sqrt{\ }$, exp, log, etc. You should know how to think about polynomials as forming a ring, and what a homomorphism is.
- **Commutative algebra/algebraic geometry (week 1 of 2)** (Mark, in Week 3) — Prerequisites: Basic familiarity with groups, rings (including polynomial rings), and fields
- **Field extensions and Galois theory (week 1 of 2)** (Mark, in Week 3) — Prerequisites: Basic familiarity with groups, rings (especially polynomial rings), and fields
- **What do we do when we do math?** (Maya, in Week 3) — Prerequisites: You must have seen a few contexts of abstract math, such as group theory

- **Commutative algebra/algebraic geometry (week 2 of 2)** (Mark, in Week 4) — Prerequisites: Groups, rings (esp. polynomial rings; if it's 2 weeks I can work that in, if it's 1 week I really need comfort with them in advance), a bit about fields
- **Field extensions and Galois theory (week 2 of 2)** (Mark, in Week 4) — Prerequisites: Groups, rings (esp. polynomial rings), fields

**Stupid games on infinite sets.** 🌶🌶🌶🌶 (Susan) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Let's play a game. I'll pick a countable ordinal number, then you pick a bigger one, and then I'll pick one that's even bigger. We'll continue this for infinitely many turns, and when we're done we'll check to see who's won. Sound like fun?

As it turns out, these "games" can be a powerful tool for studying important ideas in set theory. In this class, we will learn about the ordinal numbers, clubs, and stationary sets. We will prove the existence of a stationary set which is also co-stationary, and see how this results in a game which has a clear winner and loser, but no winning strategy.

*Homework:* Recommended.

*Class format:* Lecture. This class starts out gentle and escalates in difficulty, reaching a full four chilis by day four or five.

*Prerequisites:* None.

## COLLOQUIA

**What's in a name? Definable combinatorics.** (*Aaron Anderson*) $\boxed{\text{T}}\text{W}\Theta\text{FS}$ 50 minutes

Ramsey theory, Szemerédi Regularity Lemmas, and a bunch of other combinatorial theorems tell us about the behavior of large graphs, but the numerical bounds involved are horrible. This is because they tell us about *all* graphs, and some graphs look random and confusing.

But what if we don't care about those graphs, and actually care about graphs that come from specific, interesting problems? In this case, we can do much better, with logic! In this talk, we'll see how we can use definability and model theory to separate crazy, intractable, random-looking graphs from reasonable ones, and understand the latter much, much better.

**Obtaining freedom via ping pong.** (Arya) $\text{T}\boxed{\text{W}}\Theta\text{FS}$ 50 minutes

Groups, like people, are defined by their actions. In this talk, we shall delve into the world of geometric group theory, by studying free groups and thinking about which groups are free. The purpose of this talk is to convince the audience that abstract algebra can also be studied by simply drawing pictures. Arya's meta-goal (as always) is to talk about hyperbolic geometry. All of this somehow relates to ping pong. Some familiarity with groups or linear algebra would be useful, but not necessary. Come to the talk to find out more!

**Project selection fair.** (in Thompson) $\text{TW}\boxed{\Theta}\text{FS}$ 50 minutes

This is not a colloquium.

Many Mathcampers enjoy working on *some* kind of long-term project throughout camp: on their own, or in groups, and possibly with guidance from a staff member.

These projects range from reading math papers to folding origami to doing original research to writing programs to baking. They can take lots of time every day or just some planning once or twice a week.

If this sounds appealing to you, and you have a project you'd like to work on, just talk to any of the Mathcamp staff about it! We'd be happy to help out.

If this sounds appealing to you, but you don't have a project in mind yet, then come to this event: the project selection fair! Staff will have their own project ideas for you to sign up for.

We'll provide you with a packet of project proposals on Wednesday evening at sign-in, and you can talk to the project advisors and submit a preference form at the fair on Saturday.

The project selection fair is held in the classrooms on the first floor of Thompson, *not* the usual colloquium room.

**Algebraic topology and social choice theory.** (Athina) TWΘ$\boxed{\text{F}}$S 50 minutes

Mathcamp is over and the Mentors decided to go on a camping trip near a lake! They are trying to determine a good spot to set up their tent, but they can't seem to agree — Arya wants the tent to be near the big tree, Athina prefers a spot on the lake's shore, Chloe would like the tent to be on top of the hill overlooking the lake, and so on. Could math help them with this problem?? Come to find out, and also to learn a bit of algebraic topology!