CLASS DESCRIPTIONS—WEEK 3, MATHCAMP 2024

Contents

9:10 am classes

Commutative algebra/algebraic geometry (week 1 of 2). (Mark) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

In its classical form, algebraic geometry is the study of sets in n-dimensional space that can be described by polynomial equations (in n variables). This is both a very old and a quite active branch of mathematics, and for over a century now it has relied heavily on commutative algebra — that is, on the properties of commutative rings and related objects. We'll start by looking at some of those, including prime and maximal ideals and a review of quotient rings, and we'll see how the algebra can be used to give us information about the geometric sets. For instance, we'll use the algebra to show that if a set can be given by polynomial equations, then a finite number of such equations will do. We may also see how to translate the idea of dimension into the language of algebra. There may well be cameo appearances by the axiom of choice (in the guise of Zorn's lemma) and a bit of point-set topology (on a space whose points are ideals!), but you don't need to know any of those things going in. In the second week of the class, I hope, among other things, to prove Hilbert's famous Nullstellensatz ("Theorem of the Zeros"), arguably the starting point for modern algebraic geometry, at least for the case of two variables. (The theorem will presumably be stated and used in the first week.)

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Basic familiarity with rings (including polynomial rings) and fields. It will help if you have seen quotient rings, but they will be covered briefly.

Roots of unity and cyclotomic fields. (Chloe) $\boxed{\text{TW}}\Theta\text{FS}$ 50 minutes

The humble number 1, you know 1, you love 1, but it's hiding a terrible secret. We know $1^2 = 1 = (-1)^2$, so one and negative one are square roots of 1. But that's not the whole story, what if we consider i? Then $i^4 = 1$. Gasp! And i isn't the only one, these sorts of special numbers are called roots of unity! We will spend Day 1 getting acquainted with roots of unity and some of their properties at more of a 1 chili pace. On Day 2 we will consider what happens to $\mathbb{Q}$ when we attach these roots of unity. Day 2 will be firmly a 2 chili class considering the properties of cyclotomic fields.

Homework: None.

Class format: Lecture with a groupwork component.

Prerequisites: Complex numbers, definition of field for second day.

Hilbert's third problem. 🌶️🌶️🌶️ (Narmada) TW $\boxed{\Theta\text{FS}}$ 50 minutes

Welcome to the Pythagoras's Theorem → Hilbert's Third Problem → Banach–Tarski pipeline. The main goal of this class is to take an IBL approach to proving the two-dimensional case of Hilbert's Third Problem: any two polygons of the same area are dissection-congruent. I will tell you about the history of the problem and give you a little information to get started, but for most of the class you will be working to discover the proof from scratch! So if you want to take this class, don't look up the proof!

If time permits, after we prove the two-dimensional case, we'll switch back to a lecture-style class. I'll talk about the solution in higher dimensions (without a proof) and related problems.

Homework: [**HW**] Required.

Class format: IBL + lecture mix.

Prerequisites: None.

Kolmogorov Complexity. 🌶️🌶️ (Krishan) $\boxed{\text{TW}}$ ΘFS 50 minutes

Take a look at the following 500 digit numbers:

5126167610322753045752998802297090688992372125640787538977967726042087605407537499192711001514703200
2826572423385866868426679868161820686186701867368083354905795743554889761430701515904058503179442896
2774248831416820701517082761385423633219566050283349346878972291018086416457487067305963052668357728
6018022988207669603112840140090636501450567950993222106117796523549514470108196529446231552071850870
3015636811962678195564593358260264389216169842899719652285901079696837762360909211844216423737982976

2100716344740626275840340344563432536114523065334724323881603829353097699172215101654553200106681455
6757955164890534284045139920384527652469246248738390876715604633914664747485840261569508425358111862
8685414209483888317948751419499061834893959680078978789887468012609214536695335130918210273581438484
0496987778759788852462469882846053070988205498725835380265554554125241611789894906056789374043591283
8399221426446976457809405557961992525181240290015476084821047986294589055450611476984969572795282747

Even though both of these contain the same number of digits, the first is much simpler to describe. We can succinctly describe the first number by saying that it is equal to 2^{1660}. However, the second number is just a string of 500 random digits, so there shouldn't be a shorter way to represent it.

The Kolmogorov complexity of a natural number is (more-or-less) the length of the shortest computer program that can output that number. So the Kolmogorov complexity of the first number is much smaller than that of the second. In fact, we'll see that Kolmogorov complexity gives us a good way tell whether a number is "random".

We can extend this notion of complexity to the real numbers, which leads to a surprising result known as the point-to-set principle. We can determine the dimension of a subset of $\mathbb{R}^n$ using the Kolmogorov complexity of individual points in the set. Somehow individual, zero dimensional, points in a set "know" about the dimension of the set as a whole!

This is going to be a class more about ideas than proofs. My goal is to introduce you to these topics and give you a conceptual understanding, but I'd be happy to get into more of the details in TAU!

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: None.

The random graph and 0-1 laws. 🌶️🌶️🌶️ (Krishan) TW $\boxed{\Theta\text{FS}}$ 50 minutes

Given a property P that may or may not hold in a given graph, and a natural number n, we can compute the probability that P holds in a randomly generated graph on n vertices. We can see what happens to this probability as we send n to infinity.

In this class, we'll prove that this probability will always tend to 0 or 1 so long as P is a first-order property[1]! This result is known as the 0-1 law for graphs. This is a theorem about finite graphs and probability, but we'll see the proof mainly consists of ideas from a branch of mathematical logic called model theory.

On our way to proving the 0-1 law, we'll cover a number of important model theory concepts such as the compactness theorem and categoricity.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Basic probability (you should know how to compute the probability $P(A \text{ and } B)$, if A and B are independent events). +1 chili if you haven't seen graphs before.

The house always wins. (Misha) TWΘFS 50 minutes

In this class, you will learn about how to apply probability to the kind of problem that probability was first invented to solve: gambling.

Mostly, we will be interested in gambling in a sequence of repeated games, where you can pursue different long-term strategies. We'll talk about how to pick a strategy to maximize your probability of success depending on the kind of game you're playing, and what "success" means to you.

No actual gambling involved.

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: None.

Topology to prove calculus. (*Ruthi Hortsch*) TWΘFS 50 minutes

The theorems about continuous functions from calculus rely on a notion of what it means to "get closer and closer" to a point. If you've studied more advanced calculus, you've been introduced to how this is formalized with the "epsilon–delta" approach. This can sometimes get, how shall we put it, messy?

In this class we will discuss how notions of closeness can be generalized to defining a topology on a set, focusing on relating it back to calculus. In particular, we will aim to give alternative (very slick!) approaches to the proofs of the intermediate and extreme value theorems.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Calculus.

What do we do when we do math? (Maya) TWΘFS 50 minutes

In this course we'll look at some of the big questions about math, such as the relationship between truth and provability. (What could be more fundamental than that?) This also means we'll look at the very definitions of mathematical truth and the provability of a statement. Oh, and that means we also have to first say what a statement is, exactly. Once we have a handle on these things, we'll prove things about proofs. For example, if you've ever seen a proof by contradiction, and wondered how or why it's legitimate — well, the fact that it's legitimate is a theorem. That maybe we'll prove. (But not by contradiction.)

[1] Don't worry if you don't know what this means yet.

Homework: Optional.

Class format: Lecture plus IBL.

Prerequisites: This class will make better sense to those who have seen a few contexts of abstract math, such as group theory. Though if you haven't, you're still welcome!

10:10 am classes

Field extensions and Galois theory (week 1 of 2). (Mark) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

We'll begin by defining what field extensions are, and seeing some of what they can be used for. As an example, if you were never comfortable with defining the complex numbers by postulating the existence of a square root of -1, we'll see early on how you can "make" a square root of -1 using polynomials. We'll also see why some ancient and (in)famous construction problems, such as squaring the circle, are provably impossible.

You may know the story of the brilliant mathematician and societal misfit Galois, who died tragically young in a duel after developing an exciting new area of mathematics that was beyond what most of his contemporaries could even follow. In this class we will build up to the fundamental theorem of Galois theory, which gives an unexpected and beautiful correspondence allowing us to find and describe field extensions in terms of subgroups of a certain group. If time permits, we'll move on (perhaps with some side trips to scenic overlooks of other material) to sketch a proof that there is no general way of solving polynomial equations of degree 5 or more by radicals; that is, there is no analog of the quadratic formula for degree 5 and higher. (There are such analogs for degree 3 and 4.)

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Basic familiarity with groups, rings (especially polynomial rings), and fields.

Eigenvalues and eigenvectors through an engineer's eyes. (Elizabeth) T$\boxed{\text{W}\Theta\text{FS}}$ 50 minutes

Suppose you want to know what happens if you take a stick and squish both ends together. Or maybe you balance a stick across a gap and try to stand in the middle of it. If you try to answer these questions, you will quickly find yourself in the domain of solid mechanics, which is more generally the study of how things squish, bend, break, and deform when loads are applied to them. To engineers, understanding solid mechanics is absolutely critical for being able to build devices, buildings, and machines that don't break or fall down.

It's not *too* hard to answer these kinds of questions for simple shapes like nice rectangular or cylindrical beams, but once you consider more complicated shapes like spheres or cubes or chairs or entire airplanes, life gets very difficult very fast. We will find ourselves dealing with stresses as matrices, and then realize that we've fallen down a rabbithole full of linear algebra quite naturally. If you'd like to explore this rabbithole together, understand a lot more solid mechanics, and gain some intuition about eigenvectors and eigenvalues, this will be a great class for you.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: You should know how to multiply two matrices. If you are comfortable with linear algebra, you won't learn any new linear algebra, but you might find this alternative perspective interesting.

This class can continue as a project: "Make Squishy Solid Things"—I think there are some fun physical objects we can make to help ourselves understand solid mechanics better. Taking this class is highly recommended for doing this project, but not strictly necessary.

Bernoulli numbers. (*Dave Savitt*) T WΘFS 50 minutes

Here are three facts:

- The sum of the fourth powers $1^4 + 2^4 + \cdots + (m-1)^4$ is equal to $\frac{m^4}{4} - \frac{m^3}{2} + \frac{m^4}{4}$;
- We have infinite sums $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$ and $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$;
- The special case of Fermat's last theorem $x^7 + y^7 = z^7$ has no integer solutions with x, y, z nonzero.

In this class we'll explain how these three facts (and generalizations of them involving higher powers) are related, and we'll see what this has to do with the mysterious sequence

$$1, -1/2, 1/6, 0, -1/30, 0, 1/42, 0, -1/30, 0, 5/66, \ldots$$

of *Bernoulli numbers.*

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Elementary number theory, generating functions, calculus.

Topological graph theory. (Marisa) T WΘFS 50 minutes

Topological graph theory studies embeddings of graphs on surfaces, and that's what we'll be doing this week. Suppose you wanted to draw a graph on the plane with no edge crossings, but your graph is not planar. Well, you could give up. Or you could change the rules of the game: start drawing, and before two edges cross, add an overpass to your plane, and then send one of those edges up the overpass. Problem solved, under your new rules (a.k.a. on the torus)!

This class will explore the question: given a graph, on what surfaces can we draw it without crossings? Our toolkit will be highly combinatorial, even including our definitions of surfaces. Our goal will be to fully answer this question for several families of graphs by the end of the week. We'll also prove an analogue to the Four-Color Theorem for every closed surface *except* the plane.

Homework: Optional.

Class format: Mix of interactive lecture and group work.

Prerequisites: Introduction to graph theory, or comfort with graph theory concepts like complete graph, complete bipartite graph, subgraph, and tree.

Wallpaper patterns (week 1 of 2). (Susan) T WΘFS 50 minutes

Your wallpaper is a fascinating mathematical object! Well, maybe not your wallpaper in particular—you may not even have wallpaper.[2] However, any repeating pattern that we use to decorate a wall is an example of a mathematical object called a "wallpaper pattern."

In this class we will be discussing the classification of wallpaper patterns. Our journey will take us through arguments in combinatorics, topology, and geometry, to show that there are exactly seventeen distinct types of wallpaper pattern.

Expect lots of drawing, cutting, pasting, folding, and smershing—this is a hands-on class!

Homework: Optional.

[2] Is wallpaper retro now? Is retro still cool?

Class format: Lecture.

Prerequisites: None.

11:10 AM CLASSES

Continued fractions and Pell's equation. 🌶🌶 (Athina) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

What can the continued fraction expansion

$$\sqrt{198} = 14 + \cfrac{1}{14 + \cfrac{1}{28 + \cfrac{1}{14 + \cfrac{1}{28 + \cfrac{1}{14 + \cfrac{1}{28 + \cdots}}}}}}$$

tell us about the integer solutions to the equation $x^2 - 198y^2 = 1$? Why does the above expression seem to be repeating itself periodically? What other hidden patterns can we unveil? In this class we will try to answer these questions and explore the fascinating world of continued fractions.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Euclidean algorithm.

Numerical analysis: how computers do calculus & differential equations. 🌶🌶🌶 (Sonya) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

If you type $\displaystyle\int_2^4 \frac{\log x}{x}\,\mathrm{d}x$ into Wolfram Alpha, it will tell you the integral equals $\frac{3}{2}\log^2 2$. But if you ask it to compute $\displaystyle\int_1^{1.1} x\tan x\,\mathrm{d}x$, you will only get a numerical answer (which is about 0.184006). This is because the antiderivative of $x\tan x$ is not an elementary function, so you cannot use FTC to express the integral as a formula. So, how does Wolfram Alpha find the numerical result? If you remember your calculus course, the obvious answer is to compute Riemann sums with smaller and smaller intervals. However, using intervals of the same length is not always optimal — the approximation will converge to the correct answer more slowly in areas where the function oscillates a lot, and once your intervals are small enough to create a good approximation, the volume of computation will be high. Join this class to learn about smarter methods for approximating integrals and derivatives and solving basic differential equations. Some of the topics include stability and computational efficiency, proving that a numerical method converges in the first place and the difference between explicit and implicit methods. We will stick to the one-variable case, but if you already know multivariable calculus, this class is a good starting point for multivariable numerical analysis as well.

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: Linear algebra (specifically, matrix multiplication). Also calculus.

🔧 *This class can continue as a project:* Implement one of the numerical analysis methods using a programming language of your choice (something like MATLAB might work best, but you can use Python as well). Some good options are the finite differences method or finite elements if you want a challenge. Then, you can test its limitations and see if your program does as well as Wolfram Alpha.

Impossible integration, also the vegan kind. (Glenn) TWΘFS 50 minutes

Every January, all of MIT's brightest students come together to compete in the annual MIT Integration Bee. They integrate monstrosities like $\displaystyle\int \frac{e^{x/2}\cos x}{\sqrt[3]{3\cos x + 4\sin x}}\,\mathrm{d}x$ in mere minutes. But I've got a problem that'll stump even the fastest MIT students — let them try $\displaystyle\int e^{x^2}$.

Innocuous at first glance, my devilish function will actually have them struggling forever! In fact, there is just no expression using our familiar vocabulary of polynomials, nth roots, exponentials, logarithms, and trigonometric functions whose derivative is e^{x^2}. But how do you even prove something like that?

Not only is this function impossible to integrate, much like how an Impossible Burger tastes like beef but is made of soy, this question that tastes like calculus and analysis is actually made up of nothing but algebra!

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: You should be able to take derivatives of any functions involving things like $\sqrt{\ }$, exp, log, etc. You should know how to think about polynomials as forming a ring, and what a homomorphism is.

The axiom of choice. (Laithy & Narmada) TWΘFS 50 minutes

Are you confused about the axiom of choice? Then don't worry, we will confuse you even further! In this class, we will learn about the history of the axiom of choice, study the many formulations of it, some of which are very strange, and learn why it's important.

The statement seems simple, but it's a subtle idea. To a non-mathematician, we might summarize this statement as, "If you have a bunch of choices, each of which is possible to make, then you can make all of them." Two main questions should arise here.

First: Isn't this obvious? Why does this have to be an axiom? Second: Why do so many people care about not using it? Hopefully by the end of this class, you will have different questions.

Homework: Recommended.

Class format: Lecture.

Prerequisites: None.

Does the order matter when we sum up infinitely many numbers? (Laithy) TWΘFS 50 minutes

Finite sums enjoy many convenient properties. For instance, the order doesn't matter when we add finitely many numbers. But does this property hold when we extend our consideration to infinite sums?

It turns out that's very far from the truth. Rearranging the terms of an infinite series can lead to drastically different results. We will prove the surprising and counterintuitive fact that under some conditions on a sequence $\{a_n\}_{n\in\mathbb{N}}$ in $\mathbb{R}$, one can rearrange the order of the sequence so that the sum can be any real number you want!

Through proofs and examples, we'll uncover this surprising behavior of infinite sums and develop a deeper understanding of their properties.

Homework: Optional.

Class format: Interactive lecture.

Prerequisites: Sequences and series.

Two topological theorems; or, How to tell if something is $\mathbb{Q}$ in disguise. 🌶🌶🌶🌶 (Ben) $\boxed{\text{TW}\Theta\text{F}}$S 50 minutes

One of the trickiest homework problems I've ever been asked is the following:

Are $(0,1) \cap \mathbb{Q}$ and $[0,1] \cap \mathbb{Q}$ homeomorphic?

So, "homeomorphic" means "not only is there a bijection between these sets, but also there's a *continuous* bijection, where the inverse function is also continuous." For example, the sets $\mathbb{N}$ and $\mathbb{Q}$ have the same size (there is a bijection), but there's no homeomorphism, because any such bijection is very discontinuous.

"Homeomorphism" means roughly[3] "the same shape." So, this question is basically asking something about how these (very weird) spaces are "shaped."

There are three major things I want to go over here—

(1) What's the actual answer to this question?
(2) Why is the question a lot easier if we don't intersect with the rational numbers? (Hint: Let $f\colon [0,1] \to (0,1)$ be a continuous function. Consider $f(0)$, do some work, and find Ben if you want to chat about your thinking.)
(3) Why is Ben going to spend the entire class talking about the Cantor set?

This class should be quite approachable whether or not you've seen topology before—we'll be building up all of the necessary machinery in the first few days. This will include some very normal topological notions... and a few that are quite a bit more unusual.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: None.

1:10 PM CLASSES

Dynamics, mostly complex. 🌶🌶🌶 (*Scott Kaschner*) $\boxed{\text{TW}\Theta\text{FS}}$ 50 minutes

Lots of things are called *dynamical systems*. Any mathematical system that changes could arguably wield the monicker. In every case I know of, they're mostly harmless unless there are a bunch of them together and you try to organize them. Classification of dynamical behavior (mathematically speaking) has been annoyingly difficult for centuries. There's documentation.

In this class, we'll explore a variety of types of dynamical systems, from systems of differential equations to iterated function systems, and how they are all related. Speaking of things being related, we'll also look at *families* of dynamical systems. If all goes according to plan, we'll figure out what people mean by things like *bifurcation* and *chaos*.

Homework: Recommended.

Class format: Lecture.

Prerequisites: None.

Mathematical billiards. 🌶 (Arya) $\boxed{\text{TW}\Theta\text{FS}}$ ◔ 80 minutes

Suppose you have a point-sized ball gliding on a billiard table with a frictionless surface. The trajectory ends if it goes into a hole, and if it hits the boundary of the table, the ball follows the standard laws of reflection (the angle of incidence is the same as the angle of reflection). Depending

[3]I.e. this is a literal translation of the Greek pieces that make up the word.

on the shape of the table, we can ask several questions—how many times can the ball hit a wall before it goes into a hole? Can it come back to where it started, and keep looping its path in a periodic motion? Is the trajectory of the ball dense inside your shape? In this class, we shall try to answer some of these questions and discuss some related open questions.

Homework: Recommended.

Class format: IBL (guided problem-solving).

Prerequisites: None.

How to multiply numbers reallllly fast. 🌶️🌶️ (Eric) TWΘFS 50 minutes

How quickly can we multiply large integers? You can probably do it by hand faster than I can, but in this class we'll tackle this question for numbers way too big for a human to work with by hand. Fortunately we have computers to handle tasks like this for us; this class will be about the algorithms that computers use when multiplying very large integers.

We'll use a bunch of algebraic tools (Vandermonde matrices, roots of unity, modular arithmetic among them) to see some of the ideas behind ways for multiplying integers "faster" than the algorithm we typically use by hand. We'll take a detour into the world of polynomials where we'll learn about the Fast Fourier Transform, which is the algorithm underlying all the state-of-the-art techniques. Our priority will be picking apart how (abstract) algebra is the basis of these techniques, rather than getting to the most state-of-the-art algorithms.

Homework: Recommended.

Class format: Interactive lecture.

Prerequisites: Intro linear algebra, at the level of knowing that linear transformations are invertible iff the determinants of their corresponding matrices are non-zero. Basic computer science (big O notation) is helpful context but is not necessary; consider the class as 3 chilis if you don't have CS background.

Measure and Martin's axiom (week 2 of 2). 🌶️🌶️🌶️🌶️ (Susan) TWΘFS 50 minutes

Continuation of Martin's axiom week 1.

Homework: Recommended.

Class format: Lecture.

Prerequisites: None.

Inspecting gadgets. 🌶️🌶️🌶️ (Della) TWΘFS 80 minutes

If you know how to build a one-way path, and you know how to build a path that flips direction every time someone takes it, can you use them to build a one-way path that can be used only once? What about a path that can be used once in either direction?

Instead, suppose you know how to build a path which, when traversed, toggles a different path between passable and blocked. Which kinds of paths in the previous paragraph can you build with this? Can you make anything else interesting?

We will investigate questions like these through 'motion-planning gadgets', a model of this kind of behavior that arose in computational complexity theory—building the right gadgets in a video game can prove that the game is hard. You will spend this class solving engineering puzzles, and eventually proving that some of these engineering puzzles are impossible.

Homework: Optional.

Class format: IBL.

Prerequisites: None.

Colloquia

Fractals and other lies told by mathematicians. (*Scott Kaschner*) $\boxed{\text{T}}$WΘFS 50 minutes

What is a fractal? What does democracy (or really any other form of governmental organization) have to do with mathematics? This talk aims to pretend to think about considering non-answers to both of these questions. It will mostly be fractals, though. Many examples and rigorous definitions will be given. Some nonsense definitions will also be given. The glory and shortcomings of these definitions will be explored. If everyone is willing to abide Robert's Rules of Order, votes will be taken.

Group theory: try not to laugh challenge (GONE WRONG). (Narmada) T$\boxed{\text{W}}$ΘFS 50 minutes

AHA HAH. This is the key step in solving a special case of a big open problem in group theory. I will talk a little bit about the history of the problem and some related work, and we will actually see the proof of this special case! You don't need to know any group theory, but some familiarity will help you.

All curves intersect nicely. (Glenn) TW$\boxed{\Theta}$FS 50 minutes

I'll show you my favorite theorem in mathematics! And like many theorems, the hard work comes from stating the theorem with the right hypotheses, not the proof. I will draw a lot of pictures. You will find this theorem in textbooks called "Algebraic Geometry" (scary) but I promise it's actually pretty easy to understand!

The theorem formerly known as the Mordell conjecture. (*Ruthi Hortsch*) TWΘ$\boxed{\text{F}}$S 50 minutes

The premise of arithmetic geometry is this: we can turn questions from algebraic number theory into problems about geometry, and use our geometric knowledge to solve the problem there. The beautiful and surprising way these interplay changed the way people think about number theory and has formed modern research in the field. We'll start by giving an overview of what this means formally and why you should care.

This will lead us to a key theorem in arithmetic geometry: Faltings' Theorem (né the Mordell Conjecture), which states that no curve of genus 2 or higher has infinitely many rational points. We will discuss what this means, some results we can get from it, and why proving it landed Faltings the Fields Medal in 1986.